LECTURE NOTE

Introduction to
SUPERCONDUCTIVITY

by Michael Tinkham

LECTURED BY

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Introduction to Superconductivity

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Note prepared by Sung-Ik Lee

total 11 chapter copy major portion from Hu Jong Lee's lecture note.

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제1장

Historical Overview

1911. 초전도 발견 Kamerlingh Onnes in Leiden

1950-60

Satisfactory theoretical picture of the classical superconductor

1986 Bednorz and Müller

이책 : 발견된지 80년이 넘었지만
아직도 fascinating한 field 이다

1.1. Basic Phenomena

① Perfect Condudivity Mercury, lead, tin

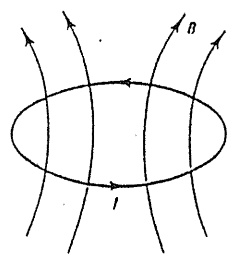


FIGURE 1.1
Schematic diagram of persistent current experiment.

실험 : Nuclear Resonance

과연 $R=0$ 인가?

lower bound 10^5year.

이론 $10^{10^{10}}$ years

② Perfect diamagnetism

1933 년 Meissner & Ochsenfeld

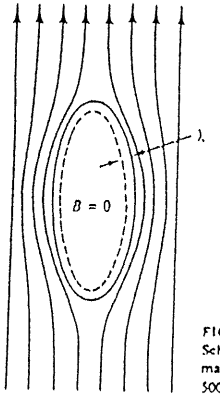


FIGURE 1.2
Schematic diagram of exclusion of magnetic flux from interior of massive superconductor. λ is the penetration depth, typically only 500 Å.

Meissner effect

Magnetic field is excluded from entering a superconductor.

- perfect conductivity 에서 설명이 되지 않는다

초전도가 깨지는 field 는 ?

$$\frac{H_c^2(T)}{8\pi} = f_n(T) - f_s(T)$$

Helmholtz free energies per unit volume in the respective phases in zero field

현상적으로 또는 경험적으로

$$H_c(T) \approx H_c(0) \left[1 - \left(\frac{T}{T_c}\right)^2 \right]$$

Transition in zero field at T_c

: 2nd Order

Transition in $H \neq 0$: 1st order

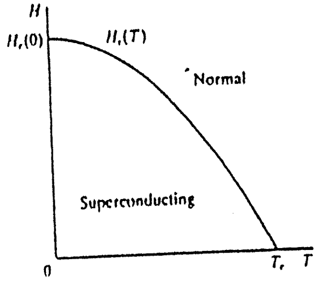


FIGURE 1.3
Temperature dependence of the critical field.

1.2. The London equation

F. London, H. London 의 proposal. 1935.

Two equations to govern the microscopic electric & Magnetic field.

$$\vec{E} = \frac{\partial}{\partial t}(\Lambda \vec{J}_s) \qquad (1\text{-}3)$$

$$\vec{h} = -c\, \nabla \times (\Lambda \vec{J}_s) \qquad (1\text{-}4)$$

$$\text{where } \Lambda = \frac{4\pi\lambda^2}{c^2} = \frac{m}{n_s e^2} \qquad (1\text{-}5)$$

(1-3) Perfect Conductivity

$$\vec{E} \neq 0 \,, \quad J_s \rightarrow \infty$$

2nd eq. 과 Maxwell eq. $\nabla \times \vec{h} = \frac{4\pi}{c}\vec{J}$ 합치면

$$\nabla^2 \vec{h} = \frac{1}{\lambda^2}\vec{h}$$

λ : penetration depth

$$\lambda(T) \approx \lambda(0) \left[1 - \left(\frac{T}{T_c}\right)^4 \right]^{-\frac{1}{2}}$$

$E = \frac{\partial}{\partial t}(\Lambda \vec{J}_s)$ 의 유도

만약 $\frac{d}{dt}(m\vec{v}) = e\vec{E}$

$\vec{J} = ne\vec{v}$ 이면 成立

$\therefore \ l \Rightarrow \infty$

그러나 $\vec{E}$ field 가 시료 전체에 골고루 분포 되어 있지 않다. E field 의 average (길이 l) 에 의해 J 가 결정.
interface에만 있다.

$l = \infty$ 이어도 effective conductivity 는 remains finite.

High-frequency current 인 경우

— Surface resistance 가 remains finite

Profound motivation for the London equation

$$\vec{p} = m\vec{v} + e\frac{\vec{A}}{c}$$

absence of $\vec{E}$ field.

ground state 에서는 ZERO momentum

local average velocity in the presence of the field $\langle \vec{V}_s \rangle = \frac{-e\vec{A}}{mc}$

Superconducting electron is rigid.

$$\vec{J}_s = n_s e \langle V_s \rangle$$

$$= \frac{-n_s e^2 \vec{A}}{mc} = -\frac{\vec{A}}{\Lambda c}$$

T=0 일때는 n_s 가 n 으로 된다.

$$\therefore \ \lambda_L(0) = \left(\frac{mc^2}{4\pi n e^2} \right)^{1/2}$$

실험하면 실험의 $\lambda_L(0)$ 가 위값보다 항상 크다 Pippard 의 Nonlocal electrodynamics.

1.3. The Pippard 의 Nonlocal Electrodynamics.

$$\vec{J}(\vec{r}) = \sigma \vec{E}(\vec{r})$$

$$\vec{J}(\vec{r}) = \frac{3\sigma}{4\pi l} \int \frac{\vec{R}\,[\vec{R} \cdot \vec{E}(\vec{r}')]\, e^{-R/l}}{R^4} d\vec{r}'$$

where $\vec{R} = \vec{r} - \vec{r}'$

Volume l 정도의 average.

Pippard 의 Argument

Superconducting wave function should have a similar characteristic dimension ξ_0 which could be estimated by an uncertainty principle Argument.

$$\Delta x \geq \frac{\hbar}{\Delta p} \approx \frac{\hbar v_F}{k T_c}$$

$$\cdot\ \xi_0 = a \cdot \frac{\hbar v_F}{k T_c}$$

a : numerical constant of order unity to be determined

Aluminum $\xi_0 \gg \lambda_L(0)$

이 ξ_0 는 nonlocal electrodynamics of normal metal 에서의 ℓ 에 비교된다.

따라서 Pippard 의 proposal 은

$$J_s(\vec{r}) = -\frac{3}{4\pi \xi_0 \Lambda c} \int \frac{\vec{R}\,[\vec{R}\cdot\vec{A}(\vec{r}\,')]}{R^4} e^{-R/\xi} d\vec{r}\,'$$

$$\vec{R} = \vec{r} - \vec{r}\,'$$

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}$$

$a = 0.15$ 일때 실험과 잘 맞는다.

BCS 에서는 $a = 0.18$ 이었다.

왜 λ 가 $\lambda_L(0)$ 보다 훨씬 더 컸나?

왜냐하면 $\vec{A}(\vec{r})$ 이 $\lambda \ll \xi_0$ 에서 매우 빨리 떨어진다. Supercurrent 의 response 가 작다.

field penetration이 커진다.

1.4. The energy gap and the BCS theory

Gap 이 kT_c 정도이다

- Daunt and Mendelssohn 주장

Thermoelectric effect ~~Effect~~ ZERO에서 hint

처음 정확한 실험

Corak 의 electronic Specific heat 에 의해

$$C_{es} \approx \gamma T_c\, a\, e^{-bT_c/T}$$

$$C_{en} = \gamma T$$

거의 같은 시대

Measurement of electromagnetic absorption in the region of $\hbar\omega \sim kT_c$

millimeter-microwave technique

Biondi - aluminum $T_c \simeq 1.2K$

Small gap 발견

그러나 온도를 T_c 아래로 많이 내리진 못했다.

Glover and Tinkham

thin lead film $T_c \sim 7.2K$

온도 매우 낮게 까지 내렸다

energy gap $\sim (3\sim4)\,kT_c$

Spectroscopic measurement

E_g to create the pair of excitations

Thermal measurement $\frac{E_g}{2}$ per statistically independent particle.

BCS.

even a weak attractive interaction between electrons,

$\Rightarrow$ Causes an instability of the ordinary Fermi-Sea ground state of the electron gas w.r.t the formation of bound pairs of electrons occupying states with equal and opposite momentum and spin.

Cooper pairs ξ_0 의 길이

$$E_g(0) = 2\Delta(0) = 3.528\ kT_c$$

for $T \ll T_c$

Gap 과 absorption edge above $\hbar\omega_g = E_g$

Glover 와 Tinkham 의 실험 data 와 in qualitatively agree.

1.5. The Ginzburg - Landau theory.

Ginzburg - Landau theory of superconductivity

pseudowave function ψ as an order parameter within Landau's general theory of 2nd order phase transition.

local density of superconducting electrons

$$n_s = |\psi(x)|^2$$

Variational principle

$$\frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)^2\psi + \beta|\psi|^2\psi = -\alpha(T)\psi$$

$$\vec{J}_s = \frac{e^*\hbar}{i2m^*}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{e^{*2}}{m^*c}|\psi|^2\vec{A}$$

London 의 예측보다 더 많은 정보

1) n_s 의 spatial variation

2) non linear effect 있다.

3) intermediate state 가 있음을 예측

H_c 근처에서 superconductor 와 normal domain 이 같이 있다.

1959. Gor'kov

GL 이론은 limiting form of the microscopic theory of BCS, valid near T_c

$\Psi \sim \Delta$

Ψ : wave ft. of center of mass motion of the Cooper pairs.

이 이론에서 GL coherence length 처음으로 소개

$$\xi(T) = \frac{\hbar}{|2m^* \alpha(T)|^{1/2}}$$

$\Psi(\vec{r})$ can vary without undue energy increase

$T < T_c$, Puro superconductor

$\xi(T) \approx \xi_0$ pippard coherence length

$T \sim T_c$

$\xi(T) \sim (T_c - T)^{-\frac{1}{2}}$

α vanishes as $T-T_c$

$\kappa = \frac{\lambda}{\xi}$ dimensionless near T_c

typical classic pure superconductors,

$\lambda \approx 500\,Å$ $\xi \approx 3{,}000\,Å$ so $\kappa << 1$

1.G. Type Ⅱ Superconductors

1957. Abrikosov

만약 κ 가 커지면 어떤일이 일어나나?

Negative surface energy

- 계속 표면을 만든다

microscopic length ξ 까지 가게 된다

이것은 classical intermediate state와 다르다.

Abrikosov 는 이것을 type Ⅱ 라 한다

$\kappa > \frac{1}{\sqrt{2}}$ instead of discontinuous breakdown of superconductivity in a first order transition at H_c,

Continuous increase in the flux penetration. at $H = H_{c1}$ 에서 시작

- 조금씩 들어가기에 Energy Cost 가 매우 적어진다.
  High field magnet 가 가능한 원인

- Schubnikov phase
  quantum of flux

$$\Phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7}\ G \cdot cm^2$$

- Magnetic decoration technique
  Scanning tunneling microscopic measurement
  - existence of the vortex array

Bardeen - Stephen의 법칙

Reduction by a factor B/H_{c2}.

초전도성 을 크게하기 위해 pinning을 만든다

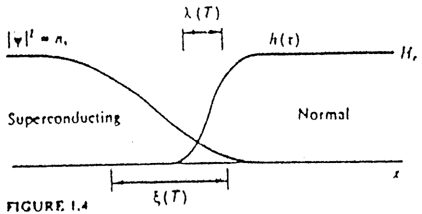


FIGURE 1.4
Interface between superconducting and normal domains in the intermediate state.

1.7. Phase, Josephson tunneling and fluxoid quantization.

Essential universal characteristic of the superconducting state.

- Existence of the many particle condensate wave function $\psi(\vec{r})$.

amplitude 와 phase 기술

phase coherence를 macroscopic distance 유지시킨다.

analogous, but not identical to the familior Bose-Einstein condensate

phase, particle number의 duality.

uncertainty relation

$\Delta N \cdot \Delta\varphi \gtrsim 1$

$N \sim 10^{22}$

both N, φ is known to within small fraction of uncertainty → phase를 semi classical variable로 간주

Josephson Relation

$$J = J_c \sin(\varphi_1 - \varphi_2)$$

DC voltage에 대해서는

$$\text{phase difference} \sim \frac{2eV_{12}t}{\hbar}$$

응용 ① ultrasensitive voltmeter, magnetometer

② $\frac{h}{e}$의 정확한 측정

Superconducting ring에서의 일

$$\Phi' = \Phi + \frac{m^*c}{e^{*2}} \oint \frac{\vec{J}_s \cdot d\vec{s}}{|\psi|^2}$$

where $\Phi = \oint \vec{A} \cdot d\vec{s}$

ring 안쪽에서는

$$\Phi' = \Phi \quad \because \vec{J}_s = 0$$

$J_s \neq 0$ 이면

양쪽항이 모두 중요

Φ'만 quantized

1. 8.

Fluctuations and Nonequilibrium Effects.

KT 만큼의 thermal fluctuation이 있을것이다

T_c 근처에서 ψ fluctuation이 매우 커진다

$T > T_c$

fluctuations cause some vestiges of superconductivity to remain

실험 ① Glover

Conductivity of amorphous films of superconductors

diverges $\propto$ $(T-T_c)^{-1}$ as $T \to T_c$.

같은 시기에 이론적으로 예측

② diamagnetic susceptibility of pure bulk

sample $\sim$ $(T-T_c)^{-\frac{1}{2}}$

이 효과는 $T \sim 2T_c$ 정도까지 나타난다.

재래식 초전도체에서는 이 효과 매우 작으나

고온 " " 크다.

$\because$ coherence length $\gg$ atomic dimension 이다.

Nonequilibrium regime

Schmid and Schön

T_c 가 factor 2 만큼 커질수 있다.

energy gap Δ 는 $T=0$ equilibrium 보다 변화없다.

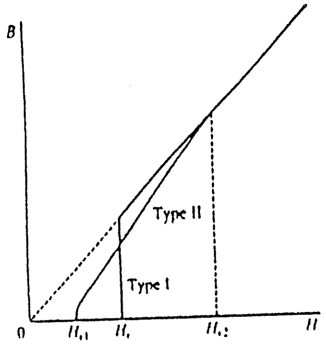


FIGURE 1.5
Comparison of flux penetration behavior of type I and type II superconductors with the same thermodynamic critical field H_c. $H_{c2} = \sqrt{2}\kappa H_c$. The ratio of B/H_{c2} from this plot also gives the approximate variation of R/R_n, where R is the electrical resistance for the case of negligible pinning, and R_n is the normal-state resistance.

1.9. High temperature Superconductivity

Resistance - causing fluctuation이 매우 크다.

(1) T 가 매우 큰데서 작동

(2) low electron density $\Rightarrow$ low v_F $\Rightarrow$ low ξ.

$$\xi_0 = a \cdot \frac{\hbar v_F}{KT}$$

(3) high anisotropy

— pinning is less effective

Basic mechanism - Not clear

Nature of pairing - remains controversal

favor d wave pairing

Magnetic properties, melting of flux-line lattice

- measurable resistance over a substantial range of fields below $H_{c2}(T)$

Josephson Coupling

Lawrence - Doniach model

Chap 9 에서

Larkin - Ovchinnikov 의 Collective pinning theory

flux creep 소개

Experimental review of unconventional pairing

Chapter 2.

Introduction to Electrodynamics of Superconductors.

Electrodynamic behavior of the type I superconducto

: London equation 으로 설명

London eq. 은 GL의 good approximation 이다

n_s 가 equilibrium 상태이면

2.1 장 The London equations

Normal Metal

Standard Drude Model

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \frac{m\vec{v}}{\tau}$$

Steady state drift velocity

$$\vec{v} = \frac{e\vec{E}\tau}{m}$$

$$\therefore \vec{J} = ne\vec{v}$$

$$= \frac{ne^2\tau}{m}\vec{E}$$

$$= \sigma\vec{E}$$

초전도

초전도 전자만 기여

$$\frac{d\vec{J}}{dt} = \frac{n_s e^2}{m}\vec{E} \qquad \left(\because \frac{d\vec{v}_s}{dt} = \frac{e}{m}\vec{E}\right)$$

$$= \frac{1}{\Lambda}\vec{E}$$

$$= \frac{c^2}{4\pi\lambda^2}\vec{E}$$

Time dependent Maxwell eq.

$$\nabla \times \vec{h} = \frac{4\pi}{c}\vec{J} \quad , \quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{h}}{\partial t}$$

pf.

$$\frac{\partial}{\partial t}(\nabla \times \vec{h}) = \frac{4\pi}{c}\frac{\partial \vec{J}}{\partial t}$$

$$= \frac{c}{\lambda^2}\vec{E}$$

$$\therefore \nabla \times \left(\frac{1}{c}\frac{\partial}{\partial t}\vec{h}\right) = \frac{1}{\lambda^2}\vec{E}$$

$$\therefore \nabla \times (\nabla \times \vec{E}) = -\frac{1}{\lambda^2}\vec{E}$$

Note 1. 시간이 포함안된 경우 즉 static 한 경우도 성립.

2. Time varying magnetic field 에도 成立

" indep. "

$$\nabla^2 \vec{h} = \frac{1}{\lambda^2}\vec{h} \qquad \text{where} \quad \lambda^2 = \frac{mc^2}{4\pi n_s e^2}$$

2.2. Screening of a static Magnetic field.

$$h(x) = h(0)\, e^{-x/\lambda}$$

1. $\lambda \sim 200\text{Å}$ in typical classic metallic conductors

   500Å pure sample $T \ll T_c$

2. short electronic mean free path

   short coherence length

   ⇒ electrodynamics become local, as in the London theory.

   이 경우 λ는 매우 커진다. typically 1500Å

   Corresponding to a much smaller value of the phenomenological parameter n_s.

3. BCS 에서는 정확히 λ, ξ, ℓ 관계로 계산하겠지만

   경험학상

$$\lambda(T) \simeq \lambda(0)\frac{1}{(1-t^4)^{1/2}}$$

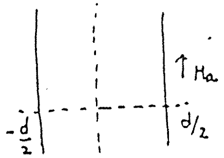
이것을 two-fluid temperature dependence라 부른다.

— earlier model of Gorter, Casimir.

Condensed (superconducting) electron

normal

2.2.1. Flat Slab in Parallel Magnetic field



$$h = H_a \cdot \frac{\cosh(x/\lambda)}{\cosh(d/2\lambda)}$$

$$B = \bar{h}$$
$$\equiv H_a + 4\pi M$$
$$= H_a \cdot \frac{2\lambda}{d} \tanh \frac{d}{2\lambda}$$

$d \gg \lambda$, 이면 $B \to 0$ and $M = -\frac{H_a}{4\pi}$

Meissner effect limit of perfect diamagnetism of bulk superconductors.

$d \ll \lambda$

$$\tanh x \sim x - \frac{1}{3}x^2 \cdots$$

$$B \longrightarrow H_a \left(1 - \frac{d^2}{12\lambda^2}\right)$$

$$M \to -\frac{H_a}{4\pi} \frac{d^2}{12\lambda^2}$$

Note.

$$(F_n - F_s)\Big|_{H=0} = -\int_0^{H_m} M(H)\,dH$$

$$\frac{H_c^2}{8\pi} \equiv (F_n - F_s)\Big|_{H=0}$$

$$\parallel \uparrow \quad H_{c\parallel} = \sqrt{12}\, H_c \frac{\lambda}{d}$$

100 Å thick film of tin

$$\lambda \approx 500\text{Å}$$
$$H_c \approx 300\,\text{Oe} \quad \Rightarrow H_{c\parallel} \approx 7500\ \text{Oe} \gg H_c$$

이 경우 Magnetization 이 Meissner value 보다 매우 더 낮을수 있다.

2.2.2. Critical Current of Wire

$a \gg \lambda$

$$H = \frac{2I}{ca}$$

if $H = H_c$, 초전도가 깨진다.

Silsbee 의 Criterion

$$I_c = \frac{caH_c}{2}$$

전류는 단지 표면에만 흐른다.

면적 $2\pi a\lambda$

$$\therefore J_c = \frac{I_c}{2\pi a\lambda}$$
$$= \frac{c}{4\pi} \cdot \frac{H_c}{\lambda}$$

$H_c = 500\,\text{Oe}$, and $\lambda = 500\text{Å}$

$$J_c \sim 10^8\ \text{A/cm}^2$$

2.3.

Type I superconductors in strong magnetic fields :

The intermediate state

field를 가해 초전도 깨자.

가장 쉬운 경우 : $h = 0$ demagnetization factor zero

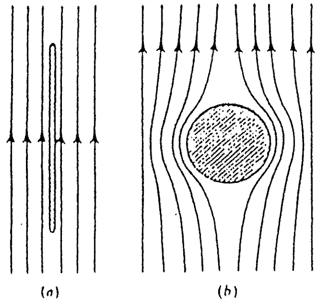


FIGURE 2.1
Contrast of exterior-field pattern (a) when demagnetizing coefficient is nearly zero and (b) when it is ⅓ for a sphere. In (b) the equatorial field is three halves the applied field for the case of full Meissner effect, which is shown.

$H < H_c$ 에서도
초전도 깨질수있다.

Helmholtz free energy

$$F_n = \underline{V f_{no}} + V \cdot \frac{H_a^2}{8\pi} + V_{ext} \cdot \frac{H_a^2}{8\pi}$$

field 없을때
free energy

$$F_s = V f_{no} + V_{ext} \cdot \frac{H_a^2}{8\pi}$$

$$\therefore \quad F_n - F_s = V (f_{no} - f_{so}) + \frac{V H_a^2}{8\pi}$$

$$= V \frac{H_c^2}{8\pi} + V \cdot \frac{H_a^2}{8\pi}$$

$$f_{no} - f_{so} = \frac{H_c^2}{8\pi}$$

만약 $H_a = H_c$ 이면

$$F_n - F_s \Big|_{H_c} = V \left(\frac{H_c^2}{4\pi} \right)$$

해석 : H_c 에서의 free energy 차이는 $\frac{H_c^2}{4\pi}$ 이다.

~~Ex~~

awkward : Helmholtz energy 써서
이 경우는 B 가 constant 일때 유용하다

Gibbs free energy
: H 가 일정할때

$$g = f - \frac{hH}{4\pi}$$

This leads to

$$G_n = V f_{no} - \frac{V H_a^2}{8\pi} - \frac{V_{ext} H_a^2}{8\pi}$$

· Since $h = B = H$ in the normal & outside the sample,

$$G_s = V f_{so} - \frac{V_{ext} \cdot H_a^2}{8\pi}$$

$h = B = 0$ in the superconducting state

$$G_n - G_s = V (f_{no} - f_{so}) - \frac{V H_a^2}{8\pi}$$

2.3.1.

Nonzero Demagnetization Factor.

복잡하다.

$B = 0$ at inside. (macroscopic scale)

Sphere인 경우

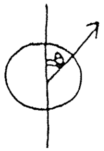
$$\nabla \cdot \vec{B} = \nabla \times \vec{B} = \nabla^2 \vec{B} = 0$$

with boundary condition.

$B \to H_a$ as $r \to \infty$

$B_n \doteq 0$ at $r \to R$

$$\therefore \vec{B} = H_a + \frac{H_a R^3}{2} \nabla \left(\frac{\cos\theta}{r^2} \right)$$



$$(B_\theta)_R = \frac{3}{2} H_a \sin\theta$$

$42° \le \theta \le 138°$ $H > H_a$ 이다.

Equator $B_\theta = \frac{3}{2} H_a$

따라서 $H > H_a$ 이면 Sphere가 normal 되는 부분이 생겨난다. 그러나 모든 부분이 ZERO가 되지는 않는다.

$\frac{2}{3} H_c < H_a < H_c$ 에서 일어나는 현상이다.

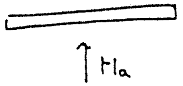
초전도, Normal이 같이 존재한다.

inte...

ellipsoidial shapes

$$1 - \eta \le \frac{H_a}{H_c} < 1$$

demagnetization factor η



이 경우는 항상 intermediate state 보인다.

2.3.2. Intermediate State in a Flat Slab

Landau가 처음 풀다.

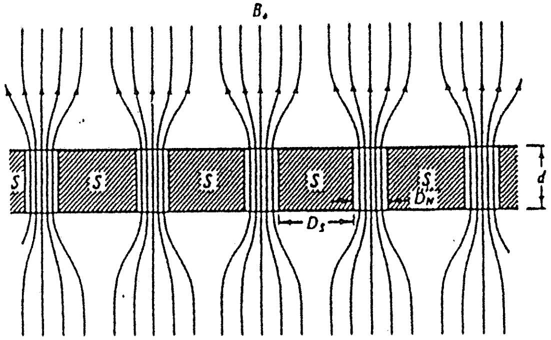


FIGURE 2.2
Schematic diagram showing magnetic flux channeling through the normal laminae in the intermediate state of a type I superconductor. Flux density is B_a at large distances and zero or $h_n (\approx H_c)$ in the cross section of the slab. The normal regions are macroscopic, in contrast to the vortices in a type II superconductor, which contain only a single quantum of flux.

B_a : fixed (average 값)

$h = 0$ in the superconducting region

ρ_n : normal 부분의 fraction이다. $\left(\rho_n = \frac{B_a}{h_n} \right)$

현상론적 해설

표면 에너지 계산해야 한다.

Additional surface energy per unit area of interface

$$\gamma = \frac{H_c^2}{8\pi}\delta \qquad \delta \approx \xi - \lambda$$

$\sim 10^{-5} \sim 10^{-4}$ cm for type I

만약 γ 가 Negative 면 무수히 쪼개진다.

⇒ type II 초전도체

$\gamma > 0$ 인 경우

F_1 : interface energy within the sample

F_2 : Field energy just outside the sample

Configuration : free energy 차이가 매우 작아서 geometry 가 radically different 가능.

Laminar model of the intermediate state

그림 2.2.

$$D = D_n + D_s$$

$$F_1 = \frac{2d\gamma}{D} = \frac{2d\delta}{D}\cdot\frac{H_c^2}{8\pi}$$

interface energy F_1 per unit area of the slab

표면이 생겼기에 차이나는 에너지

$$F_2 = \frac{\rho_n h_n^2}{8\pi} - \frac{B_a^2}{8\pi}$$

표면때문에 Field 가 Normal로 갔다

표면없이 모두 Normal

$$= \frac{\rho_n h_n^2}{8\pi} - \frac{\rho_n^2 h_n^2}{8\pi}$$

$$= \frac{\rho_n \rho_s h_n^2}{8\pi} \qquad \text{where} \quad \rho_s = 1 - \rho_n$$

Let $L \equiv (D_n^{-1} + D_s^{-1})^{-1}$

$$= \frac{D}{\rho_n^{-1} + \rho_s^{-1}}$$

$$= D\rho_n\rho_s$$

$$F_2 = \frac{2\rho_n^2\rho_s^2 D h_n^2}{8\pi}, \quad F_1 = \frac{2d\delta}{D}\frac{H_c^2}{8\pi}$$

$F = F_1 + F_2$ 의 minimum 점을 찾자.

$D \approx 10^{-2}$ cm

특정

1. moving a tiny magnetoresistive or Hall effect probe over the surface
2. making powder patterns with ferromagnetic powder

δ 재면 GL 이론과 잘 맞는다.

시작점이 H_c 보다 당연히 작게 나온다.

2.3.3. Intermediate State of a sphere.

$\frac{2}{3} < \frac{H_a}{H_c} < 1$ 인 경우

S, N laminae 구조 보인다.

어떻게 푸나?

구의 안에서 Macroscopic Maxwell eq. 생각

Flux density in the N laminae is always exactly H_c.

Normal fraction $\rho_n = \frac{B}{H_c}$

H의 tangent 는 연속이다.

Macroscopic field inside the sphere

: uniform

outside — dipole field

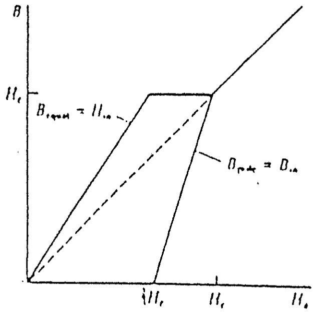


FIGURE 2.3
Internal values of B and H in a superconducting sphere in an applied field H_a. As indicated, these can be measured externally by measuring the surface field B at the pole and the equator, respectively. The sphere is in the intermediate state for $2H_c/3 < H_a < H_c$.

$$\vec{B} = H = H_a + \frac{H_1 R^3}{2} \nabla \left(\frac{\cos\theta}{r^2} \right)$$

$r \geq R$, $H \leq \frac{2}{3} H_c$

$r = R$ 에서 $B_n = 0$.

$H_1 = H_a$

만약 $H \geq \frac{2}{3} H_c$ 이면

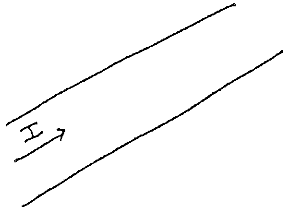
$$B_n = B\cos\theta$$
$$= H_a\cos\theta - H_1\cos\theta$$

$$\therefore \quad H_{tan} = H_c \sin\theta$$
$$= H_a \sin\theta + \frac{1}{2} H_1 \sin\theta$$

$\therefore \quad H_1 = \frac{2(H_c - B)}{3}$ so that

$B = 3H_a - 2H_c$, $\frac{2}{3} \leq \frac{H_a}{H_c} \leq 1$

2.4. Intermediate State above Critical current of a Superconducting wire



$$B = H = \frac{2I}{ca}$$

$H = H_c$ 가 되면 wire 는 no longer superconductor

$I_c = \frac{H_c ca}{2}$ Silsbee's rule

만약 모두 normal 이 된다면

$H(r) = \frac{2Ir}{ca^2} < H_c$ normal 안된다
$r \to 0$

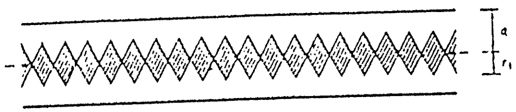


FIGURE 2.4
London's model of the intermediate-state structure in a wire carrying a current in excess of I_c. The shaded region is superconducting. The core radius r_1 is a at I_c and ideally approaches zero only asymptotically as $I \to \infty$.

λrω.

r_1 을 중심으로 달라진다

$$H(r) = H_c \quad \text{for} \quad r \leq r_1$$

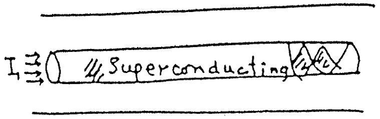
$$H(r) = \frac{2 I(r)}{c r} \qquad r \geq r_1$$

$I(r)$: r 이내에 흐르는 total Current

$$\therefore \; J(r) = \frac{1}{2\pi r} \frac{dI}{dr}$$

$$= \frac{C H_c}{4\pi r}$$

또한 $\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$ 위의 그림과 reconcile by London.



$$J(r) = \frac{E r_1}{\rho r} \qquad r < r_1$$

따라서 $r_1 = \frac{\rho c H_c}{4\pi E}$

$$I_1 = \frac{C r_1 H_c}{2} = \frac{C^2 H_c^2 \rho}{8\pi E}$$

$$I_2 = \frac{E}{\rho}\pi(a^2 - r_1^2) = \frac{\pi a^2 E}{\rho} - \frac{C^2 H_c^2 \rho}{16\pi E}$$

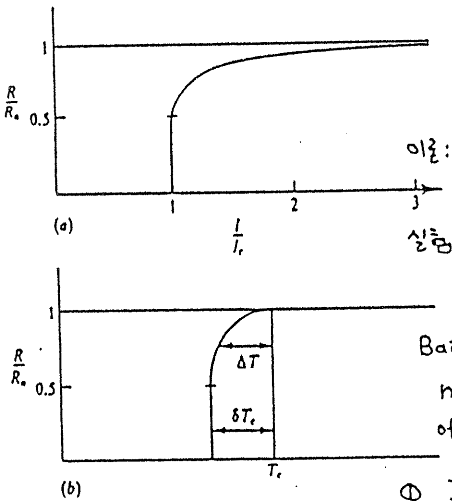
$I_c = \frac{H_c C a}{2}$ 삽입하여 H_c 소거

$$E = \frac{\rho I}{2\pi a^2}\left\{1 \pm \left[1 - \left(\frac{I_c}{I}\right)^2\right]^{1/2}\right\}$$

$$\frac{R}{R_n} = \begin{cases} 0 & I < I_c \\ \frac{1}{2}\left\{1 + \left[1 - \left(\frac{I_c}{I}\right)^2\right]^{1/2}\right\} & I > I_c \end{cases}$$

이 이론에 의하면 갑자기 저항이 증가한다.

intermediate state pattern suddenly fills the entire wire.



갑자기 증가.

이론: $\frac{1}{2}R_n$ 까지 증가

실험 : 0.7 - 0.8 R_n 까지
증가 ⇒ London 이론은 너무 간단.

Baird, Mukherjee
more detailed numerical studies
of static model.

① jump $\frac{R}{R_n}$ = 0.69

FIGURE 2.5
Resistance of a wire in the intermediate state. (a) Current dependence at constant temperature. (b) Temperature dependence at constant current, showing the broadening and depression of the apparent transition temperature. The parameter $\delta T_c = I(dI_c/dT)^{-1}$.

Andreyev, Sharvin

$\frac{R}{R_n}$ 좀더 정확

$I_c \propto H_c \propto (T_c - T)$ T_c 근처에서

$$I_c = \left.\frac{dI_c}{dT}\right|_{T_c} \Delta T \approx c\,a\,H_c(0)\cdot\frac{\Delta T}{T_c}$$

이것으로 부터 $\frac{R}{R_n}$ 을 온도의 함수로 바꾸면

$$\frac{R}{R_n} = \frac{1}{2}\left\{ 1 + \left[1 - \left(\frac{\Delta T}{\delta T_c}\right)^2 \right]^{1/2} \right\}$$

$\Delta T = T_c - T$

$\delta T_c = I \cdot \left(\frac{dI_c}{dT}\right)^{-1}$ 그림 2.5 (b)

모양은 어떠한가

Current induced intermediate state in thin film superconductors.

여러갈다. 왜냐하면 edge effect 때문이다.

intermediate state structure 를 magneto-optic technique으로 볼수있다.

Huebener

: Resistance increases in discrete increment

∵ additional channel이 나타난다.

moving domain structure 는 motion picture 로 찍다.

2.5. High frequency Electrodynamics

frequency Response 는 매우 중요하다.

low frequency in power line

high frequency in microwave and computer application

초전도체는 A.C. 에 대하여 finite dissipation이 있다.

E field 가 시간에 대해 변화하면 저항 생긴다.

Two fluid model 로 기술하자.

2.5.1. Complex conductivity in Two-Fluid Approximation.

Two fluid model

$$n = n_s + n_n$$

τ : τ_s , τ_n different relaxation time

$$\sigma(\omega) \equiv \sigma_{1i}(\omega) - i\,\sigma_{2i}(\omega)$$

$$= \frac{n_i e^2 \tau_i}{m} \frac{1}{1 + i\omega\tau_i} \qquad i = n, s$$

$$\sigma_{1i}(\omega) = \sigma_{0i} / (1 + \omega^2 \tau_i^2)$$

$$\sigma_{2i}(\omega) = \sigma_{0i}\,\omega\,\tau_i / (1 + \omega^2 \tau_i^2)$$

where $\sigma_{0i} \equiv n_i e^2 \tau_i / m$

$\tau_s \rightarrow \infty$ 인 경우 분수

$$\sigma_{1s} = \sigma_{os} / \omega^2 \tau_s^2$$

결국 delta function이 된다.

$$= \frac{\pi}{2} \frac{n_s e^2}{m} \delta(\omega)$$

같은 limit 에서

$$\sigma_{2s}(\omega) = \frac{n_s e^2}{m\omega}$$

. Oscillator Strength sum rule

$$\int \sigma_{1i}(\omega)\, d\omega = \frac{\pi}{2} \frac{n_i e^2}{m} \quad \text{independent of } n_i, \tau_i$$

.. Historical Gorter - Casimir two fluid model

$$n_n \sim t^4, \quad n_s \sim (1-t^4)$$

두 개를 합하면

$$\sigma_1(\omega) = \frac{\pi n_c e^2}{2m} \delta(\omega) + n_n e^2 \tau_n / m$$

$$\sigma_2(\omega) = \frac{n_s e^2}{m\omega}$$

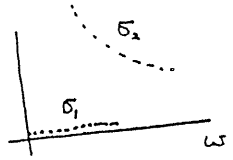
2.5.2.

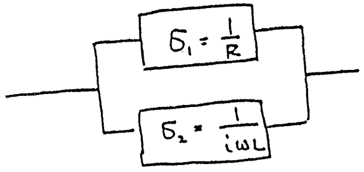
High frequency Dissipation in Superconductors

High freq.

$$\sigma_1 = n_n e^2 \tau_n / m$$

$$\sigma_2 = n_s e^2 / m\omega$$





Resonance frequency

$$\omega_0 = \frac{R}{L}$$

$\omega < \omega_0$ σ_2 dominant

$\omega > \omega_0$ σ_1 "

Ratio of current in the two channel

$$\frac{J_s}{J_n} = \frac{n_s e^2 / m\omega}{n_n e^2 \tau_n / m} = \frac{n_s}{n_n \omega \tau_n}$$

Crossover freq

$$\omega \approx \left(\frac{n_s}{n_n}\right) \frac{1}{\tau_n}$$

. τ_n is typically 10^{-12} sec

crossover freq 10^{11} Hz if $\left(\frac{n_s}{n_n} \sim 1\right)$

옛날 $\frac{n_s}{n_n} \approx \frac{1-t^4}{t^4}$

BCS $\sim \frac{1-t^4}{}$

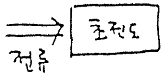
결론: below high microwave range,

most of the current ~ supercurrent

그러나 dissipation 있다.

Realistic experimental arrangement

Current bias



power dissipated per unit volume

$$\rho J^2 = R_e\left(\frac{1}{\sigma}\right) J^2$$

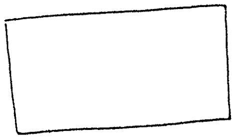
$$= \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2} J^2$$

$$\approx \frac{\sigma_1}{\sigma_2^2} J^2 \quad \left(\begin{array}{l} \sim \omega^2 \\ \sim \sigma_1 \quad \text{normal electronal density} \end{array} \right.$$

실험의 예:

Surface resistance and absorptivity of a Superconducting Surface

Cavity



Metal – σ 가 커서 대부분 반사

여러번 반사된다

정상파가 생긴다.

$h = 2H_{inc}$ 와 $h = 0$ 사이에서

전류가 흐른다.

$$\vec{H} = \frac{1}{c} 4\pi \vec{J}$$

$$\mathcal{J} = \frac{CH_{inc}}{2\pi} \text{ flowing in the skin depth } \delta.$$

dissipating power는

$\mathcal{J}^2 R_s$ where R_s is a surface resistance.

즉 resistance per square of the surface layer of thickness δ

Skin depth problem 으로 general complex conductivity

$$\delta = c\,[\,2\pi\omega\,(|\sigma| + \sigma_2)]^{-\frac{1}{2}}$$

$$R_s = \delta^{-1} R_e\left(\frac{1}{\sigma}\right)$$

$$= \delta^{-1} \cdot \frac{\sigma_1}{|\sigma|^2}$$

$$\cong \delta^{-1} \cdot \frac{\sigma_1}{\sigma_2^2}$$

Absorption Coefficient

$$A = \frac{P_{abs}}{P_{inc}} = \frac{\mathcal{J}^2 R_s}{CH_{inc}^2/4\pi} = \frac{c}{\pi} R_s$$

$$\sigma_1 = \sigma_n, \ \sigma_2 = 0$$

$$A_n = \left(\frac{2\omega}{\pi\sigma_n}\right)^{1/2} \qquad \sim \omega^{1/2}$$

$$A_s = \frac{2\sigma_1\omega^{1/2}}{\sigma_2^{3/2}\pi^{1/2}} \quad \propto \quad \omega^2\sigma_1$$

ω 가 커지면 A_s 가 take over.

$A_s \sim 10^{-3}$ 또는 더 작다. 너무 작다.

실험은 Cavity 내에서 한다.

Quality factor 로 dissipation이 기술된다

$$Q = \frac{\text{stored energy}}{\text{loss per radian}}$$

$$= \frac{(H^2/8\pi)\, V}{\left(\frac{c}{4\pi\omega}\right) H^2 A S}$$

$$= \frac{\omega}{2c} \frac{V}{S} \frac{1}{A} \approx \frac{L}{\lambda} \frac{1}{A} \approx \frac{1}{A}$$

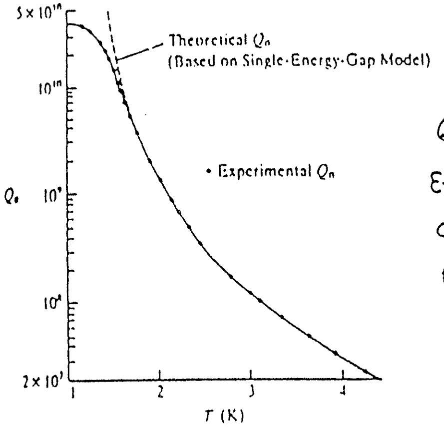


FIGURE 2.6
Temperature dependence of Q_n for a 11.2-GHz niobium cavity. [After Turneaure and Weissman, J. Appl. Phys. 39, 4417 (1968).]

$Q \sim 10^{10}$

Extremly low absorption

Compared to normal metal

typically $Q \leq 10^4$

Narrow band microwave filter

사용하면 excellent

$Q = 10^{10}$ 이면

a frequency resolution of 1 Hz in a cavity resonant at 10^{10} Hz

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Chapter 3

The BCS theory

2nd quantization 개념이용 하여 푼다.

어린이에게는 무리

3.1. Cooper pairs

1956. 아무리 작아도 attractive interaction이 있으면 bound pair를 이룬다.

만약 bound pair를 이룬다면 ground state wave ft.은

$$\psi_0(\vec{r}_1, \vec{r}_2) = \sum_{\vec{k}} g_k \, e^{i\vec{k}\cdot\vec{r}_1} \cdot e^{-i\vec{k}\cdot\vec{r}_2}$$

Spin 부분 Singlet

그래야 Space part 가 가까와 질수 있다

$$\psi_0(\vec{r}_1 - \vec{r}_2) = \left[\sum_{k > k_F} g_k \cos \vec{k} \cdot (\vec{r}_1 - \vec{r}_2)\right] (\alpha_1\beta_2 - \beta_1\alpha_2)$$

$$\therefore H\psi_0(\vec{r}_1 - \vec{r}_2) = E\,\psi_0(\vec{r}_1 - \vec{r}_2) \text{ 에 대입}$$

$$(E - 2\epsilon_k)\, g_k = \sum_{k' > k_f} V_{kk'}\, g_k$$

where $V_{kk'} = \Omega^{-1} \int V(\vec{r})\, e^{i(k'-k)\cdot\vec{r}}\, d\vec{r}$

$V_{kk'}$ 을 구체적으로 모른다

$V_{kk'} = -V$ 가정 $E_F + \hbar\omega_c$ 까지. otherwise ZERO

$$g_k = V \frac{\sum g_{k'}}{2\epsilon_k - E}$$

Summation $\sum g_k$

$$\therefore \frac{1}{V} = \sum_{k > k_F} (2\epsilon_k - E)^{-1}$$

$$= N(0) \int_{E_F}^{E_F + \hbar\omega_c} \frac{d\epsilon}{2\epsilon - E}$$

$$= \frac{1}{2} N(0) \ln \frac{2E_F - E + 2\hbar\omega_c}{2E_F - E}$$

$N(0)V \ll 1$

weak coupling approximation

$$\underline{E \approx 2E_F - 2\hbar\omega_c \, e^{-2/N(0)V}}$$

Negative energy w.r.t the Fermi Surface

made up entirely of electrons with $k > k_F$

i.e with $k > E_F$

Origin of the attractive interaction

$$V(r) = \frac{e^2}{r}$$

$$V(q) = V(\vec{k} - \vec{k}') = V_{\vec{k}\vec{k}'}$$

$$= \Omega^{-1} \int v(\vec{r}) \, e^{i\vec{q}\cdot\vec{r}} \, d\vec{r}$$

We find $V(\vec{q}) = \dfrac{4\pi e^2}{\Omega q^2} = \dfrac{4\pi e^2}{q^2}$

Ω : unit normalization volume.

dielectric constant

Fermi - Thomas approximation ϵ

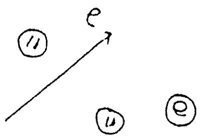
$$\epsilon = 1 + \frac{k_s^2}{q^2}$$

$$\therefore V(q) = \frac{4\pi e^2}{q^2 + k_s^2}$$

Electronic screening has eliminated the divergence at $q = 0$

but still $V(q) > 0$

repulsive



ⓔ로 attractive force 느낄수 있다

1950 년 Fröhlich

electron lattice interaction in explaining superconductivity

Isotope effect로 Confirm이 된다.

$$T_c \sim M^{-\frac{1}{2}}$$

Jellium model (Pines)

Solid is approximated by a fluid of electrons and point ions, with complete neglect of crystal structure and Brillouin zone effect as well as of the finite ion-core size

$$V(\vec{q},\omega) = \frac{4\pi e^2}{q^2 + k_s^2} + \underbrace{\frac{4\pi e^2}{q^2 + k_s^2}\,\frac{\omega_q^2}{\omega^2 - \omega_q^2}}_{\text{phonon mediated interaction}}$$

$\omega < \omega_q$ 이면 attractive 이다.

detailed 계산 : Carbotte에 의해

band structure & electron phonon coupling

Schrödinger eq.

$$-\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2)\Psi_0 + V(\vec{r}_1,\vec{r}_2)\Psi_0 = E\Psi_0$$

$$\vec{r}_1 - \vec{r}_2 = \vec{r}$$

$$\sum_{k>k_F} 2\cdot\frac{\hbar^2 k^2}{2m} g_k \cos\vec{k}\cdot\vec{r} + \sum_{k>k_F} g_k V(\vec{r}_1,\vec{r}_2)\cos\vec{k}\cdot\vec{r}$$

$$= E\sum_{k>k_F} g_k \cos\vec{k}\cdot\vec{r}$$

양변 ⊗ $\int \cos\vec{k}'\cdot\vec{r}\, d^3\vec{r}$ 이면

$$2\cdot\frac{\hbar^2 k'^2}{2m} g_{k'} + \sum_{k'>k_F} g_k V_{kk'} = E g_{k'}$$

$$V_{kk'} = \frac{1}{\Omega}\int V(\vec{r}_1,\vec{r}_2)\cos\vec{k}\cdot\vec{r}\,\cos\vec{k}'\cdot\vec{r}\, d^3\vec{r}$$

$$(E - 2\varepsilon_{k'})g_{k'} = \sum_{k'>k_F} g_k V_{kk'}\ , \quad \varepsilon_{k'} = \frac{\hbar^2 k'^2}{2m}$$

$$\therefore\ g_k = \frac{\sum_{k>k_F} V_{kk'} g_{k'}}{E - 2\varepsilon_k}$$

Cooper assumed

$$V_{kk'} = \begin{cases} -V & \varepsilon_F < \varepsilon_k < \varepsilon_F + \hbar\omega_c \\ \text{const} & \omega_D \\ 0 & \varepsilon_K > \varepsilon_F + \hbar\omega_c \end{cases}$$

$$\frac{1}{V} = \sum_{k>k_F} \frac{1}{2\varepsilon_k - E}$$

$$= \sum_{k>k_F} \frac{1}{2\underbrace{(\varepsilon_k - E_F)}_{\xi_k} + \underbrace{(2E_F - E)}_{\text{binding energy}}}$$

$$= N(0) \cdot \int_{\varepsilon_F}^{\varepsilon_F + \hbar\omega_c} \frac{d\varepsilon}{2\varepsilon - E}$$

$$= \frac{N_0}{2} \ln \frac{2(\varepsilon_F + \hbar\omega_c) - E}{2\varepsilon_F - \varepsilon}$$

$N(0) \cdot V \ll 1$ weak coupling limit

→ Strong coupling limit : Eliashberg theory

ex. Pb

$$\therefore \frac{1}{N(0)V} = \frac{1}{2} \ln \frac{2\varepsilon_F - E + \hbar\omega_c}{2\varepsilon_F - E} \simeq \frac{1}{2} \ln \frac{\hbar\omega_c}{2\varepsilon_F - E}$$

$\therefore$ E = Cooper pair Energy

$$= 2E_F - \underbrace{\hbar\omega_c\, e^{-2/N_0V}}_{\text{binding energy part}}$$

① 항상 $E < 2E_F$ whenever $V > 0$

② $E(V)$ 가 $V=0$ 에서 Singular 하므로 perturbation method 를 쓸 수 없다.

wave function.

$$g_k = \frac{V \sum_{k>k_F} g_k}{2\varepsilon_k - E} \sim \frac{1}{2E_k - E}$$

$$= \frac{1}{2\xi_k + 2E_F - E}, \quad \xi_k \sim k^2$$

Attractive interaction

(i) $V = \frac{e^2}{r}$

$$V_{kk'} = \frac{1}{\Omega} \int V(r)\, e^{i\vec{k}\cdot\vec{r}}\, d^3r$$

$$= \frac{1}{\Omega} \frac{4\pi e^2}{q^2} \qquad q = k - k'$$

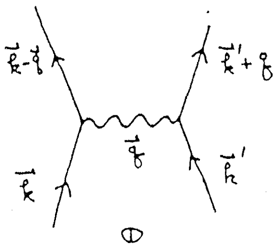
Not negative

(ii) Screening included

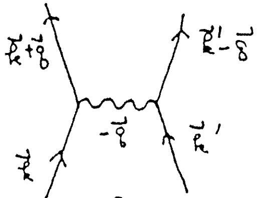
$$V = \frac{1}{\epsilon} \frac{e^2}{r} \qquad \epsilon = 1 + \frac{k_s^2}{q^2} \qquad k_s^{-1} \simeq 1\text{Å}$$

$$V_{kk'} = \frac{1}{\Omega} \frac{4\pi e^2}{q^2 + k_s^2} \qquad \text{Not Negative}$$

(iii) Electron - lattice interaction



①



②

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

$$V^{①}_{kk'} = \frac{M_{k'+q,k'}\ M_{k-q,k}}{\varepsilon(k) - \varepsilon(k-q) - \hbar\omega_q} = \frac{|M_q|^2}{\varepsilon(k) - \varepsilon(k-q) - \hbar\omega_q}$$

$$V^{②}_{kk'} = \frac{|M_q|^2}{\varepsilon(k') - \varepsilon(k'+q) - \hbar\omega_q} = -\frac{|M_q|^2}{\varepsilon(k) - \varepsilon(k-q) + \hbar\omega_q}$$

$$V_{kk'} = V^{①}_{kk'} + V^{②}_{kk'}$$

$$= \frac{2\hbar\omega_q\,|M_q|^2}{\{\varepsilon(k) - \varepsilon(k-q)\}^2 - \hbar^2\omega_q^2}$$

따라서 만약

$$|\varepsilon(k) - \varepsilon(k-q)| < \hbar\omega_q \quad \text{OR}$$

$$|\varepsilon(k') - \varepsilon(k'+q)| < \hbar\omega_q$$

In other words

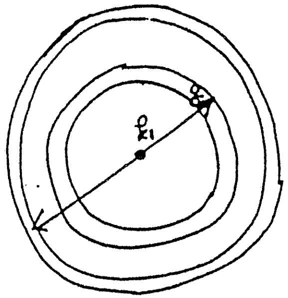
$$\left|\frac{\hbar^2 k_F\, q}{m}\right| \lesssim \hbar\omega_q \quad \cdots \quad ⊛$$

$$q = |\Delta k| < \frac{m\omega_q}{\hbar k_F}$$

$$|\Delta k'| < \frac{m\omega_q}{\hbar k_F} \quad \text{이면} \quad V_{kk'} < 0 \quad \text{Negative}$$

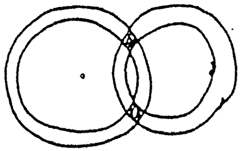
언제 윗식을 가장 잘 만족을 하나?

$$\vec{k}_{total} = \vec{k} + \vec{k}' = \text{ZERO} \ \text{일때}$$

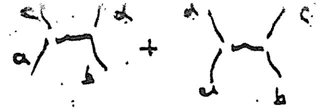


Maximum probability

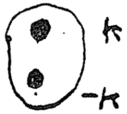
$$\vec{K} - k = \Delta q$$
$$\vec{K}' - k' = \Delta q$$



BCS 에서



$T_c \sim \frac{1}{\sqrt{M}}$ approximate : $-V$ over a range of energies near E_F.



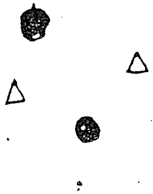
$\boxed{10^{23}}$

$$\frac{1}{\sqrt{2}} \left| \chi_i(1)\chi_j(2) - \chi(2)\chi(1) \right|$$

$$= \frac{1}{\sqrt{N!}} \left|{}^{1\ 2\ 3 \cdots N}\right|$$

이후종 교수 Note 참조.

3. 3. BCS ground state



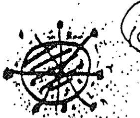
이런 경우 어떻게 binding 에너지 얻나?

Slater 의 determinant 쓸까?

너무 복잡

BCS wave function

$$| \psi_0 \rangle = \sum_{k > k_F} g_k\, C^+_{k\uparrow}\, C^+_{-k\downarrow}\, | F \rangle$$



$\boxed{10^{23}}$

10^{23} / $\frac{M!}{\frac{N}{2}! \cdot (\ )!}$

$|F\rangle$ represents the Fermi sea with all states filled up to k_F

pairs of time-reversed states are guaranteed to be occupied

Electron operators

$$[C_{k\sigma}, C^+_{k'\sigma'}]_+ \equiv C_{k\sigma} C^+_{k'\sigma'} + C^+_{k'\sigma'} C_{k\sigma} = \delta_{kk'} \delta_{\sigma\sigma'}$$

$$[C_{k\sigma}, C_{k'\sigma'}]_+ = [C^+_{k\sigma}, C^+_{k'\sigma'}] = 0$$

the electron number operator :

$$n_{k\sigma} = C^+_{k\sigma} C_{k\sigma}$$

The most general N-electron wave function expressed in terms of momentum eigen fts and with the Cooper pairing built in is

$$\boxed{| \Psi_N \rangle = \sum g(k_i, \cdots, k_\ell)\, C^+_{k_i\uparrow} C^+_{-k_i\downarrow} \cdots C^+_{k_\ell\uparrow} C^+_{-k_\ell\downarrow}\, | \phi_0 \rangle}$$

$| \psi_0 \rangle$: vacuum state with no particle present

몇개의 term이 있나

$$\frac{M!}{(M-\frac{N}{2})!\,(\frac{N}{2})!} \simeq \boxed{10^{10^{20}}}$$

항이 너무 많다.

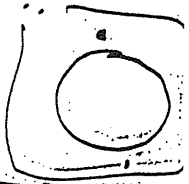
Hopeless

BCS 에서 particle이 많으니 Mean field로 풀자.

⇒ No serious error.

Grand Canonical ~~Ensemble~~로 시작

[illegible]



BCS wave function.

$$\boxed{| \Psi_G \rangle = \prod_{\vec{k} = \vec{k}_1, \cdots, \vec{k}_M} (u_k + v_k\, C^+_{k\uparrow} C^+_{-k\downarrow})\, | \phi_0 \rangle}$$

Where $|u_k|^2 + |v_k|^2 = 1$

Some Algebra with pair creation operators

$$\left.\begin{aligned} B_k &\equiv C_{-k\downarrow} C_{k\uparrow} \\ B_k^+ &= C_{k\uparrow}^+ C_{-k\downarrow}^+ \end{aligned}\right\}$$ quasi- Bose operators

$$[B_k, B_{k'}^+]_- = B_k B_{k'}^+ - B_{k'}^+ B_k$$

$$= C_{-k\downarrow} C_{k\uparrow} C_{k'\uparrow}^+ C_{-k'\downarrow}^+ - C_{k'\uparrow}^+ C_{-k'\downarrow}^+ C_{-k\downarrow} C_{k\uparrow}$$

$$= C_{k\uparrow} C_{k'\uparrow}^+ C_{-k\downarrow} C_{-k'\downarrow}^+ - C_{k'\uparrow}^+ C_{k\uparrow} C_{-k'\downarrow}^+ C_{-k\downarrow}$$

$$= (\delta_{kk'} - C_{k'\uparrow}^+ C_{k\uparrow})(\delta_{kk'} - C_{-k'\downarrow} C_{-k\downarrow})$$

$$- C_{k'\uparrow}^+ C_{k\uparrow} C_{-k'\downarrow}^+ C_{-k\downarrow}$$

$$= \delta_{kk'} (1 - C_{k\uparrow}^+ C_{k\uparrow} - C_{-k\downarrow}^+ C_{-k\downarrow})$$

$$= \delta_{kk'} (1 - n_{k\uparrow} - n_{-k\downarrow})$$

같은 방법으로

$$[B_k, B_{k'}]_- = [B_k^+, B_{k'}^+]_- = 0$$

$$\bar{N} = \langle \sum_{k\sigma} n_{k\sigma} \rangle$$

$$= \langle \Psi_G | \sum_k (C_{k\uparrow}^+ C_{k\uparrow} + C_{k\downarrow}^+ C_{k\downarrow}) | \Psi_G \rangle$$

$$= 2 \sum_k \langle \Psi_G | C_{k\uparrow}^+ C_{k\uparrow} | \Psi_G \rangle$$

$$= 2 \sum_k \langle \Psi_G | C_{k\uparrow}^+ C_{k\uparrow} C_{-k\downarrow}^+ C_{-k\downarrow} | \Psi_G \rangle$$

$$= 2 \sum_k \langle \Psi_G | C_{k\uparrow}^+ C_{-k\downarrow}^+ C_{k\uparrow} C_{-k\downarrow} | \Psi_G \rangle$$

OR

$$= 2 \sum_k \langle \Psi_G | B_k^+ B_k | \Psi_G \rangle$$

$$= 2 \sum_k \langle \phi_0 | (u_k^* + v_k^* B_k) B_k^+ B_k (u_k + v_k B_k^+)$$

$$\prod_{l \neq k} \underbrace{(u_l^* + v_l^* B_l)(u_l + v_l B_l^+)}_{|u_l|^2 + u_l^* v_l B_l^{*} + v_l^* u_l B_l + |v_l|^2 B_l B_l^+} | \phi_0 \rangle$$

$$\boxed{\text{이쪽은 } 1}$$

$$= 2 \sum_k \langle \phi_0 | \, |u_k|^2 B_k^+ \overset{0}{\cancel{B_k}} + u_k^* v_k \overset{0}{\cancel{B_k}}{}^+ B_k B_k^+$$

$$+ v_k^* u_k B_k B_k^+ \overset{0}{\cancel{B_k}} + |v_k|^2 B_k B_k^+ B_k B_k^+ | \phi_0 \rangle$$

오른쪽 $B_k | \phi_0 \rangle \longrightarrow$ ZERO

왼쪽 B_k^+

$$= 2 \sum_k \langle \phi_0 | \, |v_k|^2 B_k B_k^+ B_k B_k^+ | \phi_0 \rangle$$

$$= 2 \sum_k |v_k|^2 \langle \phi_0 | (1 - n_{k\uparrow} - n_{-k\downarrow})(1 - n_{k\uparrow} - n_{-k\downarrow}) | \phi_0 \rangle$$

$$= 2 \sum_k |v_k|^2$$

$$\therefore \quad \bar{N} = 2 \sum_k |v_k|^2$$

Note.

For ground state

$$\sum_{k\sigma} n_{k\sigma} | \Psi_G \rangle = 2 \sum_k B_k^\dagger B_k | \Psi_G \rangle$$

$$\overline{N^2} = 4 \sum_{k,k'} \langle \Psi_G | B_{k'}^\dagger B_{k'} B_k^\dagger B_k | \Psi_G \rangle$$

$$= 4 \sum_{k,k'} \langle \Phi_0 | \prod_{\ell'} (u_{\ell'}^* + v_{\ell'}^* B_\ell) B_{k'}^\dagger B_{k'} B_k^\dagger B_k$$

$$\cdot \prod (u_\ell + v_\ell B_\ell^\dagger) | \Phi_0 \rangle$$

$$= 4 \sum_k \langle \Phi_0 | (u_k^* + v_k^* B_k') B_k^\dagger B_k B_k^\dagger B_k (u_k + v_k B_k^\dagger)$$

$$\prod_{\ell \neq k} (u_\ell^* + v_\ell^* B_\ell) (u_\ell + v_\ell B_\ell^\dagger) | \Phi_0 \rangle$$

$$+ 4 \sum_{k \neq k'} \langle \Phi_0 | (u_{k'}^* + v_{k'}^* B_{k'}) (u_k^* + v_k^* B_k) B_{k'}^\dagger B_{k'} B_k^\dagger B_k$$

$$\cdot (u_k + v_k B_k^\dagger) (u_{k'} + v_{k'} B_{k'}^\dagger)$$

$$\prod_{\ell \neq k, k'} (u_\ell^* + v_\ell^* B_\ell) (u_\ell + v_\ell B_\ell^\dagger) | \Phi_0 \rangle$$

$$= 4 \sum_k | v_k |^2 + 4 \sum_{k \neq k'} \langle \Phi_0 | u_{k'}^* u_k^* + u_{k'}^* v_k^* B_k + v_{k'}^* u_k B_{k'}$$

$$+ (v_{k'}^* v_k^* B_{k'} B_k)$$

$$B_{k'}^\dagger B_{k'} B_k^\dagger B_k (u_k u_{k'} + u_k v_{k'} B_{k'}^\dagger + v_k u_{k'} B_k^\dagger + v_k v_{k'} B_k^\dagger B_{k'}^\dagger) | \Phi_0 \rangle$$

$$= 4 \sum_k | v_k |^2 + 4 \sum_{k \neq k'} \langle \Phi_0 | | v_{k'} |^2 | v_k |^2 B_{k'} B_k B_{k'}^\dagger B_{k'} B_k^\dagger B_k B_k^\dagger B_{k'}^\dagger | \Phi_0 \rangle$$

$$\overline{N^2} = 4 \sum_k | v_k |^2 + 4 \sum_{\cdots} | v_k |^2 | v_{k'} |^2$$

$$\therefore \overline{N^2} - \overline{N}^2 = 4 \Big(\sum_k | v_k |^2 + \sum_{k \neq k'} | v_k |^2 | v_{k'} |^2 - \sum_{k,k'} | v_k |^2 | v_{k'} |^2 \Big)$$

$$= 4 \sum_k (| v_k |^2 - | v_k |^4)$$

$$= 4 \sum_k | v_k |^2 (1 - | v_k |^2)$$

$$= 4 \sum_k | v_k |^2 | u_k |^2$$

Both $\overline{N}$ and $\overline{N^2} - (\overline{N})^2$ scale with the volume

$$\therefore \delta N_{rms} \equiv \left[\overline{N^2} - (\overline{N})^2 \right]^{1/2} \sim (\overline{N})^{1/2}$$

$$\sim 10^{10}$$

fractional # fluctuations :

$$\frac{\delta N_{rms}}{\overline{N}} \simeq \frac{1}{\overline{N}^{1/2}} \sim 10^{-10}$$

extremly small

- We can ignore exact particle number conservation.

- $\Delta N \cdot \Delta \varphi \gtrsim 1$

3. 4. Variational Method.

- Original BCS paper.
- later somewhat more modern form

3.4.1. Determination of the Coefficients

Pairing Hamiltonian or Reduced Hamiltonian

$$H = \sum_{k\sigma} \epsilon_k n_{k\sigma} + \sum_{k\ell} V_{k\ell} C^*_{k\uparrow} C^*_{-k\downarrow} C_{-\ell\downarrow} C_{\ell\uparrow}$$

이 pair 말고 다른 항 없나?

BCS ground state 에서는 ZERO expectation value but maybe important in other applications.

Mean number를 regulate 하기 위하여

$-\mu N_{op}$ 도입. μ: chemical potential

BCS Ground State:

phase coherent superposition of many body state with $(k\uparrow, -k\downarrow)$ occupied or unoccupied as units.

So that $b_k = \langle C_{-k\downarrow} C_{k\uparrow} \rangle \neq 0$ in S.C.

$$C_{-k\downarrow} C_{k\uparrow} = b_k + \left(C_{-k\downarrow} C_{k\uparrow} - b_k \right)$$ fluctuati

---

pairing Hamiltonian: Only terms involving paired electron

$$H = \sum_{k\sigma} \xi_k n_{k\sigma} + \sum_{k\ell} V_{k\ell} C^+_{k\uparrow} C^+_{-k\downarrow} C_{-\ell\downarrow} C_{\ell\uparrow}$$

$$= \sum_{k\sigma} \xi_k n_{k\sigma} + \sum_{k\ell} V_{k\ell} \{ b_k^* + (C^+_{k\uparrow} C^+_{-k\downarrow} - b_k^*)\} \{ b_\ell + (C_{-\ell\downarrow} C$$

$$\simeq \sum_{k\sigma} \xi_k n_{k\sigma} + \sum_{k,\ell} V_{k\ell} (C^+_{k\uparrow} C^+_{-k\downarrow} b_\ell + b_k^* C_{-\ell} C_{\ell\uparrow} - b_k^* b_\ell$$

Define

$$\Delta_k = - \sum_\ell V_{k\ell} b_\ell$$

$$= \sum_{k\sigma} \xi_k C^+_{k\sigma} C_{k\sigma} - \sum_k \left(\Delta_k C^+_{k\uparrow} C^+_{-k\downarrow} + \Delta_k^* C_{-k\downarrow} C_{k\uparrow} - \Delta_k b_k^* \right)$$

Take a linear transformation to diagonalize the Hamiltonian:

$$C_{k\uparrow} = U_k^* \gamma_{k\uparrow} + V_k \gamma^+_{-k\downarrow}$$

$$C^+_{-k\downarrow} = - V_k^* \gamma_{k\uparrow} + U_k \gamma^+_{-k\downarrow}$$

where $|U_k|^2 + |V_k|^2 = 1$

γ_k's : (quasi particle) Fermi Operator

then $$H = \sum_{k\sigma} \xi_k C^+_{k\sigma} C_{k\sigma} - \sum_k (\Delta_k C^+_{k\uparrow} C^+_{-k\downarrow} + \Delta_k^* C_{-k\downarrow} C_{k\uparrow} - \Delta$$

$$= \sum_k (\xi_k C^+_{k\uparrow} C_{k\uparrow} + \xi_k C^+_{-k\downarrow} C_{-k\downarrow}) - \sum_k \text{두째항}$$

$$= \sum_k \{ \xi_k (u_k \gamma^+_{k\uparrow} + v_k^* \gamma_{-k\downarrow})(u_k^* \gamma_{k\uparrow} + v_k \gamma^+_{-k\downarrow})$$

$$+ \xi_k (-v_k^* \gamma_{k\uparrow} + u_k \gamma^+_{-k\downarrow})(-v_k \gamma^+_{k\uparrow} + u_k^* \gamma_{-k\downarrow}) \}$$

$$- \sum_k \Delta_k (u_k \gamma^+_{k\uparrow} + v_k^* \gamma_{-k\downarrow})(-v_k^* \gamma_{k\uparrow} + u_k \gamma^+_{-k\downarrow})$$

$$- \sum_k \Delta_k^* (-v_k \gamma^+_{k\uparrow} + u_k^* \gamma_{-k\downarrow})(u_k^* \gamma_{k\uparrow} + v_k \gamma^+_{-k\downarrow}) + \sum \Delta_k b_k^*$$

$$= \sum_k \xi_k (|u_k|^2 \gamma^+_{k\uparrow} \gamma_{k\uparrow} + u_k v_k \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow} + v_k^* u_k^* \gamma_{-k\downarrow} \gamma_{k\uparrow} + |v_k|^2 \gamma_{-k\downarrow} \gamma^+_{-k\downarrow})$$

$$+ \sum_k \xi_k (|v_k|^2 \gamma_{k\uparrow} \gamma^+_{k\uparrow} - v_k^* u_k^* \gamma_{k\uparrow} \gamma_{-k\downarrow} - u_k v_k \gamma_{-k\downarrow} \gamma^+_{k\uparrow} + |u_k|^2 \gamma^+_{-k\downarrow} \gamma_{-k\downarrow})$$

$$- \sum_k \Delta_k (-u_k v_k^* \gamma^+_{k\uparrow} \gamma_{k\uparrow} + u_k^2 \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow} - v_k^{2*} \gamma_{-k\downarrow} \gamma_{k\uparrow} + v_k^* u_k \gamma_{-k\downarrow} \gamma^+_{-k\downarrow})$$

$$- \sum_k \Delta_k^* (-u_k^* v_k \gamma^+_{k\uparrow} \gamma_{k\uparrow} - v_k^2 \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow} + u_k^{2*} \gamma_{-k\downarrow} \gamma_{k\uparrow} + u_k^* v_k \gamma_{-k\downarrow} \gamma^+_{-k\downarrow})$$

$$+ \sum_k \Delta_k b_k^*$$

$$= \sum_k \xi_k \{ |v_k|^2 + (|u_k^2| - |v_k|^2) \gamma^+_{k\uparrow} \gamma_{k\uparrow} + |v_k|^2 + (|u_k|^2 - |v_k|^2) \gamma^+_{-k\downarrow} \gamma_{-k\downarrow}$$

$$+ 2 u_k v_k \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow} + 2 u_k^* v_k^* \gamma_{-k\downarrow} \gamma_{k\uparrow} \}$$

$$+ \sum_k \{ (\Delta_k u_k v_k^* + \Delta_k^* u_k^* v_k) \gamma^+_{k\uparrow} \gamma_{k\uparrow} + (-\Delta_k u_k^2 + \Delta_k^* v_k^2)$$

$$\cdot \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow} + (\Delta_k v_k^{2*} - \Delta_k^* u_k^{2*}) \gamma_{-k\downarrow} \gamma_{k\uparrow}$$

$$+ (-\Delta_k v_k^* u_k - \Delta_k^* v_k u_k^*) \underbrace{\gamma_{-k\downarrow} \gamma_{-k\uparrow}} \qquad + \sum_k \Delta_k b_k^*$$

$$= \sum_k \{ \xi_k (|u_k|^2 - |v_k|^2)(\gamma^+_{k\uparrow} \gamma_{k\uparrow} + \gamma^+_{-k\downarrow} \gamma_{-k\downarrow}) + 2|v_k|^2$$

$$+ 2 u_k v_k \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow} + 2 u_k^* v_k^* \gamma_{-k\downarrow} \gamma_{k\uparrow} \}$$

$$+ \sum_k (\Delta_k u_k v_k^* + \Delta_k^* u_k v_k)(\gamma^+_{k\uparrow} \gamma_{k\uparrow} + \gamma^+_{-k\downarrow} \gamma_{-k\downarrow} - 1)$$

$$+ (\Delta_k v_k^{2*} - \Delta_k^* u_k^{2*}) \gamma_{-k\downarrow} \gamma_{k\uparrow} + (\Delta_k^* v_k^2 - \Delta_k u_k^2) \gamma^+_{k\uparrow} \gamma^+_{-k\downarrow}$$

$$+ \Delta_k b_k^*$$

Choose u_k and v_k so that the coefficients of $\gamma_{-k\downarrow} \gamma_{k\uparrow}$ and $\gamma^+_{k\uparrow} \gamma^+_{-k\downarrow}$ vanish.

$$2 \xi_k u_k v_k + \Delta_k^* v_k^2 - \Delta_k u_k^2 = 0 \qquad \cdots \quad \circledast$$

then

$$H = \sum_k \xi_k \{ (|u_k^2| - |v_k|^2)(\gamma^+_{k\uparrow} \gamma_{k\uparrow} + \gamma^+_{-k\downarrow} \gamma_{-k\downarrow}) + 2|v_k|^2 \}$$

$$+ \sum_k \{ (\Delta_k u_k v_k^* + \Delta_k^* u_k^* v_k)(\gamma^+_{k\uparrow} \gamma_{k\uparrow} + \gamma_{-k\downarrow} \gamma_{-k\downarrow} - 1) + \Delta_k b_k^* \}$$

diagonalized.

$\circledast$ 식 $\times \; \Delta_k^*$

$$v_k^2 \Delta_k^{*2} + 2 \xi_k u_k v_k \Delta_k^* - |\Delta_k|^2 u_k^2 = 0$$

$$\therefore\ \Delta_k^* = -\xi_k u_k v_k + \sqrt{\xi_k^2 u_k^2 v_k^2 + |\Delta_k|^2 u_k^2 v_k^2} \,/\, v_k$$

$$= \left(\sqrt{\xi_k^2 + |\Delta_k|^2} - \xi_k\right)\frac{u_k}{v_k}$$

OR $$\Delta_k^* = (E_k - \xi_k)\frac{u_k}{v_k}$$

$$E_k \equiv \sqrt{\xi_k^2 + |\Delta_k|^2}$$

따라서 $$\left|\frac{v_k}{u_k}\right| = \frac{E_k - \xi_k}{|\Delta_k|}$$

$$|u_k|^2 + |v_k|^2 = 1$$

$$\therefore\ |v_k|^2 = 1 - |u_k|^2$$
$$= 1 - |v_k|^2 \cdot \frac{|\Delta_k|^2}{(E_k - \xi_k)^2}$$

OR $$|v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$$

OR $$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$$

Noting that

$$\Delta_k v_k^* = (E_k - \xi_k) u_k^*$$

$$\Delta_k |v_k|^2 = (E_k - \xi_k) u_k^* v_k$$

$$= \Delta_k \cdot \frac{1}{2} \cdot (1 - \xi_k/E_k)$$

$$= \frac{\Delta_k}{2E_k}(E_k - \xi_k)$$

$$\rightarrow u_k^* v_k = \frac{\Delta_k}{2E_k}$$

then

$$H = \sum_k \xi_k \left\{ (2|u_k|^2 - 1)(\gamma_{k\uparrow}^+ \gamma_{k\uparrow} + \gamma_{-k\downarrow}^+ \gamma_{-k\downarrow}) + 2|v_k|^2 \right\}$$

$$+ \sum_k \left\{ \underbrace{2E_k |u_k|^2 |v_k|^2}_{4E_k \cdot \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)|u_k|^2} \cdot 2(\gamma_{k\uparrow}^+ \gamma_{k\uparrow} + \gamma_{-k\downarrow}\gamma_{-k\downarrow} - 1) + \Delta_k b_k^* \right\}$$

$$= \sum_k \left\{ \xi_k (2|u_k|^2 - 1) + 2(E_k - \xi_k)|u_k|^2 \right\} (\gamma_{k\uparrow}^+ \gamma_{k\uparrow} + \gamma_{-k\downarrow}^+ \gamma_{-k\downarrow})$$

$$+ \sum_k \left\{ 2\xi_k \underbrace{|v_k|^2}_{1 - |u_k|^2} - 2(E_k - \xi_k)|u_k|^2 + \Delta_k b_k^* \right\}$$

$$= \sum_k \left\{ -\xi_k + 2E_k \cdot \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) \right\} (\gamma_{k\uparrow}^+ \gamma_{k\uparrow} + \gamma_{-k\downarrow}^+ \gamma_{-k\downarrow})$$

$$+ \sum_k \left\{ 2\xi_k - 2E_k \cdot \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) + \Delta_k b_k^* \right\}$$

$$H = \underbrace{\sum_k (\xi_k - E_k + \Delta_k b_k^*)}_{\text{Condensation Energy}} + \underbrace{\sum_k E_k (\gamma_{k\uparrow}^+ \gamma_{k\uparrow} + \gamma_{-k\downarrow}^+ \gamma_{-k\downarrow})}_{\text{quasi particle excitation energy}}$$

$$\Delta_k = -\sum_\ell V_{k\ell} \langle c_{-\ell\downarrow} c_{\ell\uparrow} \rangle$$

$$= -\sum_\ell V_{k\ell} \langle (-v_\ell \gamma^+_{\ell\uparrow} + u_\ell^* \gamma_{-\ell\downarrow})(u_\ell^* \gamma_{\ell\uparrow} + v_\ell \gamma^+_{-\ell\downarrow}) \rangle$$

$$= -\sum_\ell V_{k\ell} u_\ell^* v_\ell \langle -\gamma^+_{\ell\uparrow}\gamma_{\ell\uparrow} + 1 - \gamma^+_{-\ell\downarrow}\gamma_{-\ell\downarrow} \rangle$$

T=0 에서는

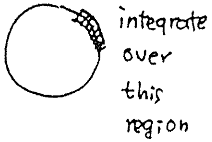
$$= -\frac{1}{2} \sum_\ell V_{k\ell} \left(1 - \frac{\xi_\ell^2}{E_\ell^2}\right)^{1/2}$$ if u_ℓ, v_ℓ are real.

$$= -\frac{1}{2} \sum_\ell V_{k\ell} \frac{\Delta_\ell}{E_\ell}$$

BCS approximation

$$V_{k\ell} = \begin{cases} -V & |\xi_\ell| < \hbar\omega_c \\ 0 & \text{Otherwise} \end{cases}$$

$$\Delta_k = \begin{cases} \Delta & |\xi_\ell| < \hbar\omega_c \\ 0 & \text{Otherwise} \end{cases}$$



then

$$1 = \frac{V}{2} \sum_k \frac{1}{E_k}$$

OR $$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c} \frac{d\xi}{(\Delta^2+\xi^2)^{1/2}} = \sinh^{-1}\left(\hbar\omega_c/\Delta\right)$$

따라서

$$\Delta(0) = \frac{\hbar\omega_c}{\sinh\left(\frac{1}{N(0)V}\right)} \simeq 2\hbar\omega_c \cdot e^{-\frac{1}{N(0)V}}$$

Finite temperature 에서

E_k : an excitation energy of a fermion quasi - particles

$$\langle 1 - \gamma^+_{k\uparrow}\gamma_{k\uparrow} - \gamma^+_{-k\downarrow}\gamma_{-k\downarrow} \rangle = 1 - 2\underline{f(E_k)}$$

$$\frac{1}{e^{\beta E_k}+1}$$

$$\Delta_k = -\sum_\ell V_{k\ell} u_\ell^* v_\ell \langle 1 - \gamma^+_{k\uparrow}\gamma_{k\uparrow} - \gamma^+_{-k\downarrow}\gamma_{-k\downarrow} \rangle$$

$$= -\sum_\ell V_{k\ell} u_\ell^* v_\ell \{ 1 - 2f(E_k) \}$$

$$= -\sum_\ell V_{k\ell} u_\ell^* v_\ell \tanh\left(\frac{\beta E_\ell}{2}\right)$$

OR $$= -\sum_\ell V_{k\ell} \frac{\Delta_\ell}{2E_\ell} \tanh\left(\frac{\beta E_\ell}{2}\right)$$

Taking BCS approximation, where $V_{k\ell} = -V$, $\Delta_k = \Delta$

$$\frac{1}{V} = \frac{1}{2} \sum_k \frac{\tanh\left(\frac{\beta E_k}{2}\right)}{E_k}$$

To determine T_c

$$\Delta(T_c) \rightarrow 0$$

$$E_K \longrightarrow |\xi_K|$$ The excitation spectrum becomes the same as in the normal metal

$$\frac{1}{V} = \frac{1}{2}\sum_K \frac{\tanh(\beta_c|\xi_K|/2)}{|\xi_K|}$$

$$\rightarrow \frac{1}{2}\int_{-\hbar\omega_c}^{\hbar\omega_c} \frac{\tanh(\frac{\beta_c \xi}{2})}{\xi}\, d\xi$$

$\because$ $|\xi_K|$ symmetric w.r.t. E_F

OR

$$\frac{1}{N(0)V} = \int_0^{\frac{\beta_c\hbar\omega_c}{2}} \frac{\tanh x}{x}\, dx$$

$$= \ln(1.13\,\beta_c\,\hbar\omega_c)$$

$$k_B T_c = 1.13\,\hbar\omega_c \cdot e^{-\frac{1}{N(0)V}}$$

$$\Delta(0) = 2\hbar\omega_c\, e^{-\frac{1}{N(0)V}}$$

$$\frac{\Delta(0)}{k_B T_c} = 1.764$$

Temperature dependence of the gap : $\Delta(T)$

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c} \frac{\tanh\{\frac{1}{2}\beta(\xi^2+\Delta^2(T))^{1/2}\}}{(\xi^2+\Delta^2(T))^{1/2}}\, d\xi$$

in the weak coupling limit.

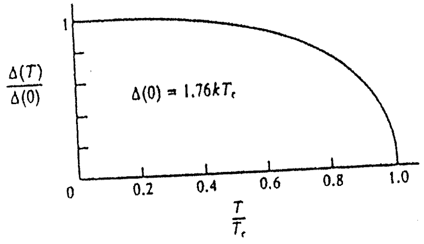


FIGURE 3.2
Temperature dependence of the energy gap in the BCS theory. Strictly speaking, this universal curve holds only in a weak-coupling limit, but it is a good approximation in most cases.

- $\frac{\Delta(T)}{\Delta(0)}$: a universal function of T
- $T \simeq 0$: $\Delta(T)$ insensitive to T
- $T \simeq T_c$: $\frac{\Delta(T)}{\Delta(0)} \simeq 1.74\,(1-\frac{T}{T_c})^{1/2}$

the typical mean field behavior

Remark :

$$\Delta = V \sum_k \frac{\Delta}{2E_k} (1 - 2f_k)$$

$$1 = V \sum_k \frac{1}{2E_k} - V \sum_k \frac{f_k}{E_k}$$

$$= V N(0) \cdot 2 \int_0^{\hbar\omega_c} d\xi \cdot \frac{1}{2E} - V \sum_k \frac{f_k}{E_k}$$

$$= N(0) V \left\{ \ln(\Lambda + \xi) \Big|_0^{\hbar\omega_c} \right\} - V \sum_k \frac{f_k}{E_k}$$

$$= N(0) V \left(\ln 2\hbar\omega_c - \ln \Delta \right) - V \sum_k \frac{f_k}{E_k}$$

At $T = 0$, no quasiparticle excitation → $f_k = 0$

$$\frac{1}{N(0)V} = \ln 2\hbar\omega_c - \ln \Delta(0)$$

Then

$$\ln \Delta(T) = \ln 2\hbar\omega_c - \frac{1}{N(0)V} - \frac{1}{N(0)} \sum_k \frac{f_k}{E_k}$$

$$\therefore \frac{\Delta(T)}{\Delta(0)} = e^{-\frac{1}{N(0)} \sum_k (f_k / E_k)}$$

1) at $T = 0$, $\Delta(T) \longrightarrow \Delta(0)$

2) $\Delta(T)$ decreases as quasi particles are added

3) This eg tells us how $\Delta(T)$ Changes with f_k for nonequilibrium

신석기 시대 접근 방법 - 새책에서 시도

$$\langle \Psi_G | H - \mu N_{op} | \Psi_G \rangle = 2 \sum \xi_k v_k^2 + \sum_{kl} V_{kl} u_k v_k u_l v_l$$

Constraint $u_k^2 + v_k^2 = 1$

$$\therefore u_k = \sin\theta_k , \quad v_k = \cos\theta_k$$

$$\therefore \text{윗식} = \sum \xi_k (1 + \cos 2\theta_k) + \frac{1}{4} \sum_{kl} V_{kl} \sin 2\theta_k \sin 2\theta_l$$

whence

$$0 = \frac{\partial}{\partial\theta_k} \langle \Psi_G | H - \mu N_{op} | \Psi_G \rangle$$

$$= -2 \xi_k \sin 2\theta_k + \sum_l V_{kl} \cos 2\theta_k \sin 2\theta_l$$

양쪽 미분 때문에 $\frac{1}{4} \longrightarrow \frac{1}{4} \cdot 2 \cdot 2$

따라서 $\tan 2\theta_k = \dfrac{\sum_l V_{kl} \sin 2\theta_l}{2 \xi_k}$

Now we define the quantities

$$\Delta_k = - \sum_l V_{kl} u_l v_l$$

$$= -\frac{1}{2} \sum_l V_{kl} \sin 2\theta_l \qquad ⊛$$

and $E_k = \left(\Delta_k^2 + \xi_k^2 \right)^{1/2}$

위식을 다시 표현

$$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k}$$

$$2u_k v_k = \sin 2\theta_k = \frac{\Delta_k}{E_k}$$

$$v_k^2 - u_k^2 = \cos 2\theta_k = -\frac{\xi_k}{E_k}$$

이것을 ⊛ 에 代入

$$\Delta_k = -\frac{1}{2}\sum_l V_{kl} \sin 2\theta_l$$

$$= -\sum_l V_{kl} u_l v_l$$

$$= -\frac{1}{2}\sum_l \frac{\Delta_k}{E_k} \cdot V_{kl}$$

$$= -\frac{1}{2}\sum_l \frac{\Delta_l}{(\Delta_l^2 + \xi_l^2)^{1/2}} V_{kl}$$

Trivial solution

$\Delta_k = 0$ 이므로 $u_k = 1$ for $\xi_k < 0$

$v_k = 0$ for $\xi_k > 0$

T= 0 에서 $|\psi\rangle$ 는 Singlet Slater determinant

with all states up to k_F

$V_{kl} < 0$ 이면 Nontrivial solution을 얻게되어

BCS 에서는

$$V_{kl} = \begin{cases} -V & \text{if } |\xi_k| \text{ and } |\xi_l| \le \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$$

이 식을 넣으면

$$\Delta_k = \begin{cases} \Delta & \text{for } |\xi_k| < \hbar\omega_c \\ 0 & \text{for } |\xi_k| > \hbar\omega_c \end{cases}$$

따라서

$$1 = \frac{V}{2}\sum_k \frac{1}{E_k}$$

$$\therefore \frac{1}{N(0)V} = \int_0^{\hbar\omega_c} \frac{d\xi}{(\Delta^2 + \xi^2)^{1/2}} = \sinh^{-1}\frac{\hbar\omega_c}{\Delta}$$

$$\text{Thus} \quad \Delta = \frac{\hbar\omega_c}{\sinh\left[\frac{1}{N(0)V}\right]}$$

$$\approx 2\hbar\omega_c \, e^{-\frac{1}{N(0)V}}$$

$$V_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left[1 - \frac{\xi_k}{(\Delta^2 + \xi_k^2)^{1/2}}\right]$$

$$U_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) = 1 - v_k^2$$

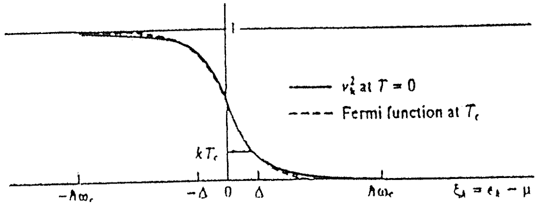


FIGURE 3.1
Plot of BCS occupation fraction v_k^2 ($= 1 - u_k^2$) vs. electron energy measured from the chemical potential (Fermi energy). To make the cutoffs at $\pm\hbar\omega_c$ visible, the plot has been made for a strong-coupling superconductor with $N(0)V = 0.43$. For comparison, the Fermi function for the normal state at T_c is also shown on the same scale using the BCS relation $\Delta(0) = 1.76kT_c$.

3.6.3. Thermodynamic Quantities

$$E_k = \sqrt{\xi_k^2 + \Delta^2(T)}$$

$$f_k = \frac{1}{1 + e^{\beta E_k}}$$

Entropy를 계산하자.

∵ Specific heat를 알수 있으니까.

$$S_{es} = -2k \sum_k \left[(1 - f_k)\ln(1 - f_k) + f_k \ln f_k\right]$$

$$C_{es} = T\frac{dS_{es}}{dT} = -\beta\frac{dS_{es}}{d\beta}$$

$$C_{es} = 2\beta k \sum_k \frac{\partial f_k}{\partial \beta} \ln\frac{f_k}{1 - f_k}$$

$$= -2\beta^2 k \sum_k E_k \frac{\partial f_k}{\partial \beta}$$

$$= -2\beta^2 k \sum_k E_k \frac{df_k}{d(\beta E_k)}\left(E_k + \beta\frac{dE_k}{d\beta}\right)$$

$$= 2\beta k \sum_k -\frac{\partial f_k}{\partial E_k}\left(E_k^2 + \frac{1}{2}\beta\frac{d\Delta^2}{d\beta}\right)$$

첫 term : Redistribution of quasi-particles among the various energy states as the temperature changes.

둘째 term : Unusual

Effect of the temperature - dependent gap in changing the energy level themselves.

위식 증명

$$① \quad \frac{\partial S_{es}}{\partial \beta} = \frac{\partial}{\partial \beta}\left[-2k_B \sum_k \left\{(1 - f_k)\ln(1 - f_k) + f_k \ln f_k\right\}\right]$$

$$= -2k_B \sum_k \left\{-\frac{\partial f_k}{\partial \beta}\ln(1 - f_k) + \frac{\partial f_k}{\partial \beta}\ln f_k\right\}$$

$$② \quad E_k\frac{\partial E_k}{\partial \beta} = \frac{1}{2}\frac{\partial}{\partial \beta}E_k^2 = \frac{1}{2}\frac{\partial}{\partial \beta}\left(\underset{\substack{\uparrow \\ \text{indep of } \beta}}{\xi^2} + \Delta^2(T)\right) = \frac{1}{2}\frac{\partial \Delta^2}{\partial \beta}$$

$$③ \quad \frac{\partial f_k}{\partial(\beta E_k)} = -\frac{1}{(e^{\beta E_k} + 1)^2}e^{\beta E_k} = \frac{1}{\beta}\frac{\partial f_k}{\partial E_k}$$

Note: Normal state.

$$E = 2 \sum_k f(\mathcal{E}_k)\, \mathcal{E}_k$$

$$C_{en} = \frac{\partial E}{\partial T} = - k_B \beta^2 \frac{\partial E}{\partial \beta}$$

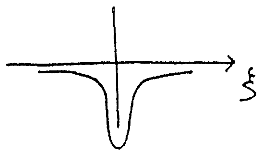
$$= - k_B \beta^2 \frac{\partial}{\partial \beta} \left(2 \sum_k f(\mathcal{E}_k) \mathcal{E}_k \right)$$

$$= - 2 k_B \beta^2 \sum_k \frac{\partial f(\mathcal{E}_k)}{\partial \beta} \mathcal{E}_k$$

$$= - 2 k_B \beta^2 \sum_k \frac{\partial f(\mathcal{E}_k)}{\partial (\beta \mathcal{E}_k)} \frac{\partial \beta \mathcal{E}_k}{\partial \beta} \mathcal{E}_k$$

$$= - 2 k_B \beta \sum_k \frac{\partial f_k}{\partial (\beta \mathcal{E}_k)} \mathcal{E}_k^2$$

$$\longrightarrow \frac{2\pi^2}{3} N(0)\, k_B^2\, T \quad \text{at low temp.}$$



T_c 에서 의 jump

$$\Delta C = (C_{es} - C_{en}) \big|_{T_c}$$

$$= k_B \beta_c^2 \sum_k - \frac{\partial f_k}{\partial |\xi_k|} \cdot \frac{\partial \Delta^2}{\partial \beta} \bigg|_{T_c}$$

∵ T_c 에서는 Δ가 ZERO 이니

OR

$$= k_B \beta_c^2 \left(\frac{\partial \Delta^2}{\partial \beta} \right)_{T_c} N(0) \cdot \int_{-\hbar\omega_c}^{\hbar\omega_c} \left(- \frac{\partial f(\xi)}{\partial |\xi|} \right) d\xi$$

$$\int_{-\infty}^{\infty} - \frac{\partial}{\partial |\xi|} \left(\frac{1}{e^{\beta\xi} + 1} \right) d\xi$$

$$= - \int_0^{\infty} \frac{\partial}{\partial \xi} \left(\frac{1}{e^{\beta\xi} + 1} \right) d\xi - \int_{-\infty}^{0} \frac{\partial}{\partial (-\xi)} \frac{1}{(e^{\beta\xi} + 1)} d\xi$$

따라서

$$\Delta C = K_B \beta_c^2 N(0) \left(\frac{\partial \Delta^2}{\partial \beta} \right)_{T_c}$$

$$= - N(0) \cdot \left(\frac{\partial \Delta^2}{\partial T} \right)_{T_c}$$

$$= 9.4\, N(0)\, k_B^2\, T_c$$

$$\therefore \frac{\Delta C}{C_{en}} = \frac{9.4}{2\pi^2/3} = 1.43$$

$$\Delta(T) = 1.74\, \Delta(0) \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}$$

$$\frac{2\Delta(0)}{K_B T_c} = 3.53$$

$$\Delta^2 = 2.89\, \Delta^2(0) \cdot \left(1 - \frac{T}{T_c}\right)$$

$$\therefore \left(\frac{\partial \Delta^2}{\partial T} \right)_{T_c} = 2.89\, \Delta^2(0) \cdot \left(- \frac{1}{T_c} \right)$$

$$\Delta(0) = 1.77\, K_B T_c$$

$$= - 2.89 \cdot (1.77)^2\, K_B^2\, T_c \qquad = - 9.4\, K_B^2\, T_c$$

$$\therefore \quad U_{es}(T)\Big|_{T_c}^{T} = \int_{T_c}^{T} C_{es}\, dT$$

$$U_{es}(T) = U_{es}(T_c) - \int_{T}^{T_c} C_{es}\, dT$$

$$= U_{en}(T_c) - \int_{T}^{T_c} C_{es}\, dT$$

$$\therefore \text{ Since } \quad U_{en}(T) = U_{en}(0) + \int_0^T \gamma T' dT'$$

$$= U_{en}(0) + \frac{1}{2}\gamma T^2$$

$$F_{en}(T) = U_{en}(T) - T\, S_{en}(T)$$

$$= U_{en}(0) + \frac{1}{2}\gamma T^2 - T(\gamma T)$$

$$= U_{en}(0) - \frac{1}{2}\gamma T^2$$

$$\because \quad S_{en}(T) - S_{en}(0) = \int_0^T \frac{C_{en}(T')}{T'}\, dT'$$

$$= \gamma T$$

한편

$$F_{en}(T) - F_{es}(T) = \frac{H_c^2(T)}{8\pi}$$

$$\therefore \quad F_{es}(T) = F_{en}(T) - \frac{H_c^2(T)}{8\pi}$$

$$= U_{en}(0) - \frac{1}{2}\gamma T^2 - \frac{1}{8\pi} H_c^2(T)$$

$$H_c^2(0)\left\{1 - \left(\frac{T}{T_c}\right)^2\right\}^2$$

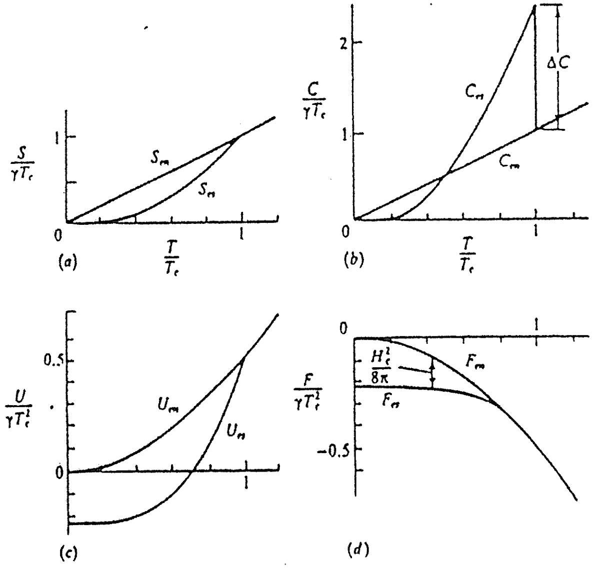


FIGURE 3.3
Comparison of thermodynamic quantities in superconducting and normal states. $U_m(0)$ is chosen as the zero of ordinates in (c) and (d). Because the transition is of second order, the quantities S, U, and F are continuous at T_c. Moreover, the slope of F_n joins continuously to that of F_m at T_c, since $\partial F/\partial T = -S$.

차이는 알았지만 U 를 알려면 H_c 의 온도 의존성을 알아야 한다.

H_c 의 온도 의존성은 매우 정확히 측정할 수 있다

3.7. State functions and the density of states.

$$\gamma_{k0}^{\dagger} = u_k^{*} c_{k\uparrow}^{\dagger} - v_k^{*} c_{-k\downarrow}$$

$$\gamma_{k1}^{\dagger} = u_k^{*} c_{-k\downarrow}^{+} + v_k^{*} c_{k\uparrow}$$

γ_k^{+} : operator quasi particle excitations of the two spin direction from the superconducting ground state in terms of electron creation operator c_k^{+}.

γ의 Vacuum 상태란?

$$\gamma_{k0}|\Psi_G\rangle = \gamma_{k1}|\Psi_G\rangle = 0$$

이것이 superconductor의 ground state 이다.

$$\gamma_{k0}|\Psi_G\rangle = (u_k c_{k\uparrow} - u_k c_{-k\downarrow}^{+}) \prod_{l} (u_l + v_l c_{l\uparrow}^{+} c_{-l\downarrow}^{+})|\phi_0\rangle$$

계산시 k th pair 는?

$$\underbrace{u_k^{2} c_{k\uparrow}}_{\text{annihilation operator 이기에 ZERO}} + \overset{\text{두개합 치 ZERO}}{(u_k v_k c_{k\uparrow} c_{k\uparrow}^{+} c_{-k\downarrow}^{+} - v_k u_k c_{-k\downarrow}^{+})} - v_k^{2} \underbrace{c_{-k\downarrow}^{\dagger} c_{k\uparrow}^{\dagger}}_{\text{ZERO}} c_{-k\downarrow}^{+}$$

그렇다면 excited state 는?

$$\gamma_{k0}^{+}|\Psi_G\rangle = (|u_k|^2 c_{k\uparrow}^{\dagger} + u_k^{*} v_k c_{k\uparrow}^{+} c_{k\uparrow}^{+} c_{-k\downarrow}^{+} - v_k^{*} u_k c_{-k\downarrow} - |v_k|^2 c_{-k\downarrow} c_{k\uparrow}^{+} c_{-k\downarrow}^{+}) \times \prod_{l\neq k} (u_l + v_l c_{l\uparrow}^{+} c_{-l\downarrow}^{+})|$$

가운데 두항 ZERO

마지막 항 $|v_k|^2 c_{k\uparrow}^{+} c_{-k\downarrow} c_{-k\downarrow}^{+}$ by using anticommutator

$$= (|u_k|^2 c_{k\uparrow}^{+} + |v_k|^2 c_{k\uparrow}^{+}) \prod_{l\neq k} (u_l + v_l c_{l\uparrow}^{+} c_{-l\downarrow}^{+}$$

$$= c_{k\uparrow}^{+} \prod_{l\neq k} (u_l + v_l c_{l\uparrow}^{+} c_{-l\downarrow}^{+})|\phi_0\rangle$$

Similarly

$$\gamma_{k1}^{\dagger}|\Psi_G\rangle = c_{-k\downarrow}^{+} \prod_{l\neq k} (u_l + v_l c_{l\uparrow}^{+} c_{-l\downarrow}^{+})|\phi_0\rangle$$

these are excited states called singles in the original BCS treatment

$\left(\begin{matrix} (k\uparrow, -k\downarrow) \text{ pair} \\ \text{확률 } 2v_k^2 \end{matrix} \right)$ 상태에서 1 인 상태로 보내다.

$$1 - 2v_k^{2} = u_k^{2} - v_k^{2}$$

Number가 Conserve 되나? Conserve 될 필요가 없다.

Density of States

γ_k^+, the quasi particle excitation operator, has one to one correspondence with C_k^+ of the normal metal.

$N_s(E)$: (Superconducting) quasi particle density of States

$$N_s(E)\,dE = N_n(\xi)\,d\xi$$
$$= N_n(0)\,d\xi \qquad \xi : \text{only} \sim \text{meV from } E_F$$

$$\therefore \frac{N_s(E)}{N_n(0)} = \frac{d\xi}{dE}$$
$$= \frac{d}{dE}\sqrt{E^2-\Delta^2} = \begin{cases} \dfrac{E}{\sqrt{E^2-\Delta^2}} & E \geq \Delta \\ 0 & E < \Delta \end{cases}$$

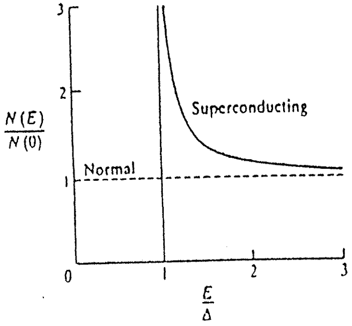
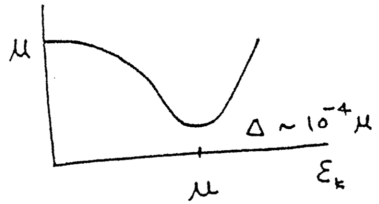
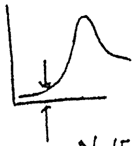


FIGURE 3.4
Density of states in superconducting compared to normal state. All k states whose energies fall in the gap in the normal metal are raised in energy above the gap in the superconducting state.

$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$
$$= \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}$$





$$N_s(E) \simeq \frac{\hbar}{\tau_{in}\Delta} N_n(0)$$

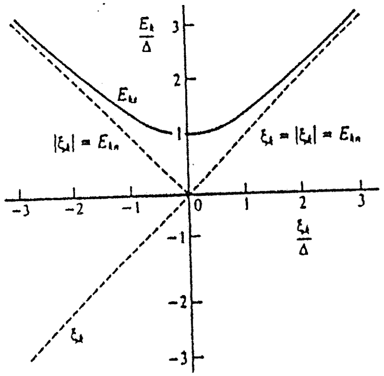


FIGURE 3.5
Energies of elementary excitations in the normal and superconducting states as functions of ξ_k, the independent-particle kinetic energy relative to the Fermi energy.

$$\simeq \frac{0.1\,mK}{3K} N_n(0)$$

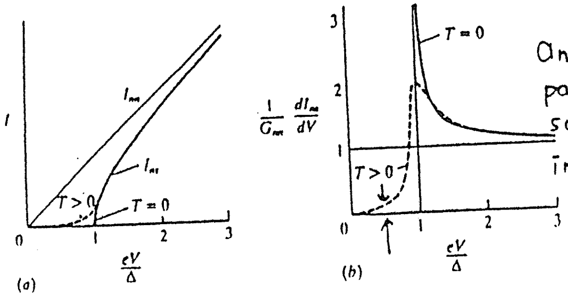


FIGURE 3.7
Characteristics of normal-superconductor tunnel junctions. (a) I-V characteristic. (b) Differential conductance. Solid curves refer to $T = 0$; dashed curves refer to a finite temperature.

Any inelastic Scattering or pair breaking process such as magnetic impurity or proximity effect quasiparticle lifetime effect :

The quasiparticles above the BCS ground state may condense back to Cooper pairs within the life time τ. The finite life time of the quasi particles can influence the gap structure, i.e, Smearing out the gap.

Qualitatively the finite lifetime of the quasi particles produces an uncertainty in their energy of an order

$$dE \sim \hbar/\tau$$

$$N_s(E,\gamma) = N_n(0)\ Re\left(\frac{|E-i\gamma|}{\sqrt{(E-i\gamma)^2-\Delta^2}}\right)$$

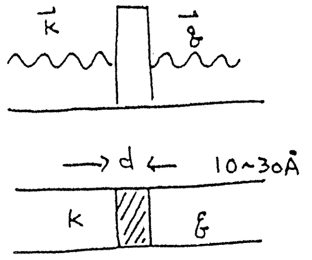
$$\gamma = \frac{\hbar}{2\tau}$$: "gap brodening factor"

The lifetime becomes very long at low temperatures due to low occupation of the quasi- particle states, reducing the finite lifetime effect.

3.8. Electron tunneling

Density of state : electron tunneling에 의해 측정

Giaever 에 의해 (pioneered)



SIS
SIS
NIN

$$H_T = \sum T_{kq}\, C^{+}_{k\sigma} C_{q\sigma} + \text{hermitian Conjugate}$$

1. Josephson tunneling 무시
   transition probability $\propto |T|^2$

2. No magnetic perturbations → No spin flip

3. Incident angle dependence is ignored, since the electrons scatter within short time ($\sim 10^{-20}$ sec) as they tunnel

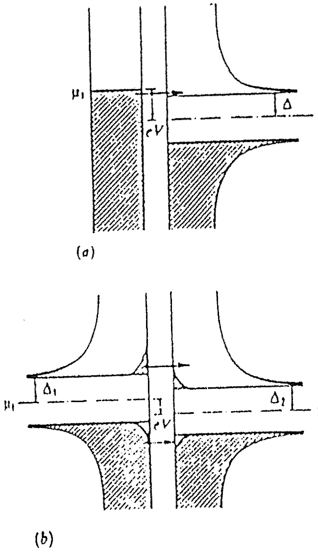


FIGURE 3.6
Example of semiconductor model description of electron tunneling. Density of states is plotted horizontally vs. energy vertically. Shading denotes states occupied by electrons. (a) N-S tunneling at $T = 0$, with bias voltage just above the conduction threshold, i.e., eV slightly exceeds the energy gap Δ. Horizontal arrow depicts electrons from the left tunneling into empty states on the right. (b) S-S tunneling at $T > 0$, with bias voltage below the threshold for conduction at $T = 0$, i.e., with $eV < \Delta_1 + \Delta_2$. Horizontal arrows depict tunneling involving thermally excited electrons or holes, respectively.

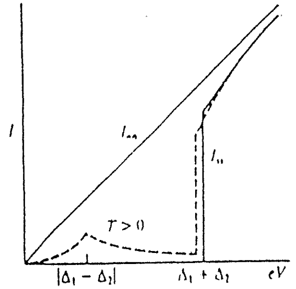


FIGURE 3.8
Superconductor-superconductor tunneling characteristic. Note that for $T > 0$ there are sharp features corresponding to both the sum and the difference of the two gap values. The peak at $|\Delta_1 - \Delta_2|$ would actually be a logarithmic singularity in the absence of gap anisotropy and level broadening due to lifetime effects.

$[C^+_{k\sigma}, C_{q\sigma}] = 0$

Valid even when $k = q$

$\because$ k : electron state in the left

q : " " right

Electron flow rate from k to q Fermi 의 Golden Rule.

$$= \frac{2\pi}{\hbar} \cdot 2 \iint_0^\infty d\varepsilon_k \, d\varepsilon_q \; N(\varepsilon_k) N(\varepsilon_q) \Big\{ |\langle q | H_T | k \rangle|^2 f_k (1 - f_q) \, \delta(\varepsilon_q - \varepsilon_k$$

$$- |\langle k | H_T | q \rangle|^2 f_q (1 - f_k) \, \delta(\varepsilon_k - \varepsilon_q + e|V|) \Big\}$$

NIN tunneling

$|\langle q | H_T | k \rangle|^2$ Not representing a transfer $k \to q$

$$= |\langle 0 | \cdot C_{q\uparrow} \sum_{\substack{k'q' \\ \sigma}} \Big(\cancelto{0}{T_{k'q'\sigma} \, C^+_{k'\sigma} C_{q'\sigma}} + T^*_{k',q'} C^+_{q'\sigma} C_{k'\sigma} \Big) C^+_{k\uparrow} | 0 \rangle |^2$$

$\uparrow$: choose one polarity

q, q' 이 같고

$k\,k'$ 이 같아야 non zero

$$= |\langle 0 | C_{q\uparrow} C^+_{q\uparrow} C_{k\uparrow} C^+_{k\uparrow} | 0 \rangle T^*_{kq} |^2 = |T_{kq}|^2$$

In a similar way

$$|\langle k | H_T | q \rangle|^2 = |T_{kq}|^2$$

$$\therefore I_{NN} \propto \frac{4\pi}{\hbar} |T|^2 N_k(0) N_q(0) \int_0^{\infty}\!\!\int (f_k - f_q)\, \delta(\varepsilon_q - \varepsilon_k - e|V|)\, d\varepsilon_k\, d\varepsilon_q$$

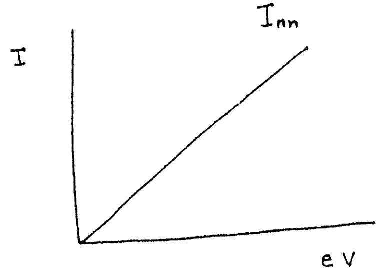
$$= \frac{4\pi}{\hbar} |T|^2 N_k(0) N_q(0) \int_0^{\infty} \{ f^0(\varepsilon_k) - f^0(\varepsilon_k + e|V|) \}\, d\varepsilon_k$$

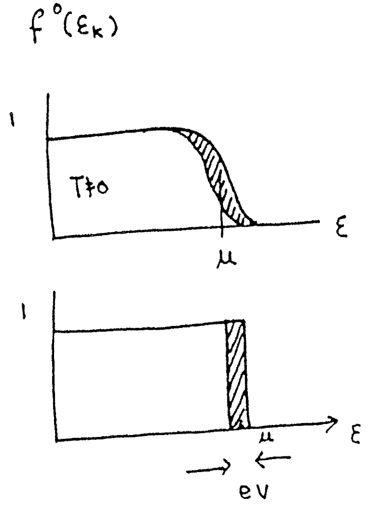
$$= \frac{4\pi}{\hbar} |T|^2 N_k(0) N_q(0)\, eV$$

$$= G_{nn} V \qquad \text{" Ohm's law"}$$

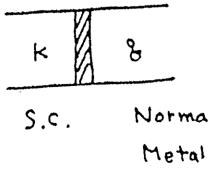
G_{nn} : Independent of V and temp.

(Coupling Strength) × (the DOS's)





SIN tunneling



$$H_T = \sum_{\substack{kq\\ \sigma}} \left(T_{kq}\, C^{+}_{k\sigma} C_{q\sigma} + T^{*}_{kq}\, C^{+}_{q\sigma} C_{k\sigma} \right)$$

$$= \sum_{kq} T_{kq} \left(C^{+}_{k\uparrow} C_{q\uparrow} + C^{+}_{k\downarrow} C_{q\downarrow} \right) + \sum_{kq} T^{*}_{qk} \left(C^{+}_{q\uparrow} C_{k\uparrow} + C^{+}_{q\downarrow} C_{k\downarrow} \right.$$

$$= \sum_{kq} T_{kq} \left\{ (U_k \gamma^{+}_{k\uparrow} + V^{*}_k \gamma_{-k\downarrow}) C_{q\uparrow} + (-V^{*}_{-k} \gamma_{-k\uparrow} + U_{-k} \gamma^{+}_{k\downarrow}) C_{q\downarrow} \right\}$$

$$+ \sum_{kq} T^{*}_{qk} \left\{ C^{+}_{q\uparrow} (U^{*}_k \gamma_{k\uparrow} + V_k \gamma^{+}_{-k\downarrow}) + C^{+}_{q\downarrow} (-V_{-k} \gamma^{+}_{-k\uparrow} + U^{*}_{-k} \gamma_{k\downarrow}) \right\}$$

These terms vanish, if we start with spin ↑ particle.

Electron flow rate from k to q

$$= \frac{4\pi}{\hbar} \int_{-\infty}^{\infty} dE_k\, N_s(E_k) \int_{-\infty}^{\infty} d\xi_q\, N_q(0) \left\{ |\langle q|H_T|k\rangle|^2 f_k (1 - f_q)\, \delta(\xi_q - E_k \right.$$

$$\left. - |\langle k|H_T|q\rangle|^2 f_q (1 - f_k)\, \delta(\xi_q - E_k - e|V|) \right\}$$

$$|\langle \underset{SC}{k\uparrow} | H_T | \underset{norm}{q\uparrow} \rangle|^2$$

$$= |\langle \underset{S.C}{k\uparrow} | \sum_{k',q'} T_{k'q'} (u_{k'} \gamma^+_{k'\uparrow} + v^*_{k'} \gamma_{-k'\downarrow}) C_{q'\uparrow} | \underset{norm}{q\uparrow} \rangle|^2$$

$$= |\sum_{k'q'} \langle 0 | (u^*_k \gamma_{k\uparrow} + v_k \gamma^+_{-k\downarrow})(u_{k'} \gamma^+_{k'\uparrow} + v_{k'} \gamma_{-k'\downarrow}) C_{q'\uparrow} C^+_{q\uparrow} | 0 \rangle T_{k'q'} |^2$$

$$= | \langle 0 | |u_k|^2 \gamma_{k\uparrow} \gamma^+_{k\uparrow} C_{q\uparrow} C^+_{q\uparrow} | 0 \rangle |^2 \, | T_{kq} |^2$$

$$= | T_{kq} |^2 \, (|u_k|^2 + |v_k|^2)^2$$

Note that there is always a state k' with exactly the same quasi particle energy $E_{k'} = E_k$, but with $\xi_{k'} = -\xi_k$

$$u(\xi_{k'}) = u(-\xi_k) = v(\xi_k) \qquad \because \; u_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right), \quad v_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$$

같은 방법으로

$$|\langle \underset{norm}{q\uparrow} | H_T | \underset{S.C.}{k\uparrow} \rangle|^2$$

$$= |\langle \underset{norm}{q\uparrow} | \sum_{k',q'} C^+_{q'\uparrow} (u^*_{k'} \gamma_{k'\uparrow} + v_{k'} \gamma^+_{-k'\downarrow}) | \underset{SC}{k\uparrow} \rangle|^2$$

$$= |\sum_{k'q'} T_{k'q'} \langle 0 | C_{q\uparrow} C^+_{q'\uparrow} (u^*_{k'} \gamma_{k'\uparrow} + v_{k'} \gamma^+_{-k'\downarrow})(u_k \gamma^+_{k\uparrow} + v_k \gamma^*_{-k\downarrow}) | 0 \rangle |^2$$

$$= |T|^2 (|u|^2 + |v|^2)^2$$

$$\therefore I_{SN} = \frac{4\pi e}{\hbar} \int_{-\infty}^{\infty} dE_k \, N_s(E_k) \int_{-\infty}^{\infty} d\xi_q \, N_q(0) \, |T_{kq}|^2 \cdot \{ f_k (1 - f_q) - f_q (1 - f_k) \} \cdot \delta (\xi_q - E_k - e|V|) \}$$

C.f.

$$I_{NN} = \frac{4\pi e}{\hbar} |T|^2 \, N_k(0) \, N_q(0) \int_{-\infty}^{\infty} d\xi_k \{ f^\circ(\xi_k) - f^\circ(\xi_k + e|V|) \}$$

" The Semiconductor Model "

Metal : as a continuous distribution of independent - particle energy states with density $N(0)$.

$$I_{1\to 2} = A \int |T|^2 \underbrace{N_1(E) f(E)}_{\text{the \# of occupied initial states}} \underbrace{N_2(E+eV) \{ 1 - f(E+eV) \}}_{\text{the \# of available final states}} dE$$

$$I_{2\to 1} = A \int |T|^2 N_1(E) \{ 1 - f(E) \} N_2(E+eV) f(E+eV) dE$$

$$I = I_{1\to 2} - I_{2\to 1}$$

$$= A \int^{\infty} |T|^2 N_1(E) \, N_2(E+eV) \, \{ f(E) - f(E+eV) \} dE$$

$$\therefore\ I_{SN} = \frac{4\pi e}{\hbar}\int_{-\infty}^{\infty} dE_k\, N_s(E_k)\int_{-\infty}^{\infty} d\xi_q\, N_q(0)\,|T_{kq}|^2$$

$$\cdot\{f_k(1-f_q) - f_q(1-f_k)\} \times \delta(\xi_q - E_k - e|V|)$$

$$= \frac{4\pi e}{\hbar}|T|^2 N_q(0)\int_{-\infty}^{\infty} dE_k\, N_s(E_k)\,\{f^\circ(E_k) - f^\circ(E_k + e|V|)\}$$

C.f.

$$I_{NN} = \frac{4\pi e}{\hbar}|T|^2 N_k(0)\, N_q(0)\int_{-\infty}^{\infty} d\xi_k\,\{f^\circ(\xi_k) - f^\circ(\xi_k + e|V|)\}$$

" The semiconductor model"

metal : as a continuous distribution of independent
- particle energy states with density $N(0)$

$$I_{1\to 2} = A\int |T|^2 N_1(E) f(E)\, N_2(E+eV)\{1 - f(E+eV)\}\, dE$$

$$I_{2\to 1} = A\int |T|^2 N_1(E)\{1 - f(E)\} N_2(E+eV)\, f(E+eV)\, dE$$

$$I = I_{12} - I_{2\to 1}$$

$$= A\int |T|^2 N_1(E)\, N_2(E+eV)\{f(E) - f(E+eV)\}\, dE$$

$$I_{NS} = A|T|^2 N_2(0)\int_{-\infty}^{\infty} N_{1S}(E)\{f(E) - f(E+eV)\}\, dE$$

$$= \frac{G_{NN}}{e}\int_{-\infty}^{\infty}\frac{N_{1S}(E)}{N_1(0)}\{f(E) - f(E+eV)\}\, dE$$

$$= \frac{G_{NN}}{e}\int_{\Delta}^{e|V|}\frac{E}{\sqrt{E^2-\Delta^2}}\{f(E) - f(E+eV)\}\, dE$$

$$\xrightarrow{T=0} -\frac{G_{NN}}{e}\sqrt{E^2-\Delta^2}\,\Big|_{\Delta}^{e|V|}$$

$$= -G_{NN}|V|\sqrt{1-\left(\frac{\Delta}{e|V|}\right)^2}$$

$$= I_{NN}\cdot\sqrt{1-(\Delta/e|V|)^2}$$

differential conductance

$$G_{NS} = \frac{dI_{NS}}{dV}$$

$$= G_{NN}\int_{-\infty}^{\infty}\frac{N_{1S}(E)}{N_1(0)}\underbrace{\left\{-\frac{\partial f(E+eV)}{\partial(eV)}\right\}}\, dE$$

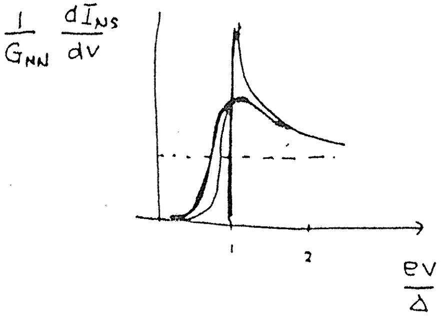
$$\overset{T\to 0}{=} G_{NN}\frac{N_1(e|V|)}{N_1(0)}$$

a ball shaped weighting
function peak at
$E = -eV$

Thus

$T\to 0$ G_{NS} : a direct measure of DOS

$T\neq 0$ G_{NS} measures a DOS smeared by $\sim k_BT$
in energy due to the width of the



SIS tunneling

$$I_{SS} = \frac{G_{NN}}{e}\int_{-\infty}^{\infty} \frac{N_{S_1}(E)}{N_1(0)} \cdot \frac{N_{S_2}(E+eV)}{N_2(0)} \{f(E) - f(E+eV)\}\, dE$$

$$= \frac{G_{NN}}{e}\int_{-\infty}^{\infty} \frac{|E|}{\sqrt{E^2-\Delta_1^2}} \frac{|E+eV|}{\sqrt{(E+eV)^2-\Delta_2^2}} \{f(E) - f(E+eV)\}\, dE$$

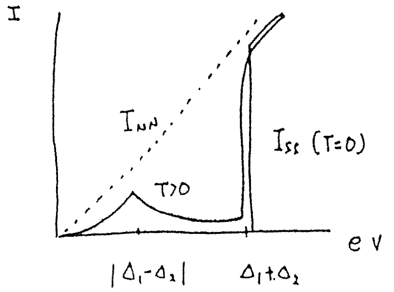
$|E| > \Delta_1$

$|E+eV| > \Delta_2$

The negative resistance region for

$|\Delta_1-\Delta_2| < eV < \Delta_1+\Delta_2$: observable only for

Voltage-bias arrangement



3.8.5.

Phonon Structure

: Strong electron-phonon structure인 경우의 tunneling data. - Lead, Mercury 에서 phonon의 structure 보인다.

이론 : Schrieffer, Scalapino & Wilkins

$$N_s(E) = N(0)\ Re \frac{E}{[E^2-\Delta^2(E)]^{1/2}}$$

Δ 가 complex # 이다.

damping of the quasi particle excitations by decay with creation of real phonons : hence, it is large when $E \approx \hbar\omega_{ph}$

Eliashberg Eq. 풀면 더 정확.

Electron phonon mechanism을 더욱 확인

Central quantity : $\alpha^2 F(\omega)$

α : electron-phonon coupling strength

$F(\omega)$: density of phonon states

계산 (microscopic calculation) : Carbotte

실험 : McMillan, Rowell.

실험 (tunneling) 에서는 phonon의 density of state 에서의 Van Hove singularity 등을 잘 보여준다.

tunneling current 의 2nd derivative 에서

3. 9. Transition Probabilities and Coherence effects

$$H_1 = \sum_{k\sigma, k'\sigma'} \underline{B_{k'\sigma', k\sigma}} \, C^+_{k',\sigma'} \, C_{k\sigma}$$

Matrix elements of the perturbation operator

Metal : 각각의 항들은 independent.

$|B_{k'\sigma', k\sigma}|^2$ 이 transition probability에 비례한다.

초전도 상태 : phase coherent superposition of occupied one electron state

따라서 interference term이 존재한다.

$$C^+_{k'\uparrow} C_{k\uparrow} = U_{k'} U^*_k \gamma^+_{k'0} \gamma_{k0} - U^*_{k'} U_k \gamma^+_{\vec{k}1} \gamma_{\vec{k}'1}$$

$$+ U_{\vec{k}'} U_{\vec{k}} \gamma^+_{\vec{k}'0} \gamma^+_{k1} + U^*_{\vec{k}'} U^*_{\vec{k}} \gamma_{\vec{k}'1} \gamma_{\vec{k}0}$$

$$C^+_{-k\downarrow} C_{-k'\downarrow} = - U^*_k U_{k'} \gamma^*_{\vec{k}'0} \gamma_{k0} + U_k U^*_{k'} \gamma^+_{k1} \gamma_{k'1}$$

$$+ U_k U_{k'} \gamma^+_{k'0} \gamma^+_{k1} + U^*_k U^*_{k'} \gamma_{\vec{k}'1} \gamma_{\vec{k}0}$$

이 두개의 항이 자승하기 전에 더해져야 한다

- $B_{k'\sigma', k\sigma}$ 와 $B_{-k,-\sigma,-k',-\sigma'}$ 은 sign 만이 다르다.

$$\Delta \vec{k} = \vec{k}' - \vec{k}$$

$$\Delta \sigma = \sigma' - \sigma$$

Thus these terms can be combined as

$$B_{k'\sigma', k\sigma} \left(C^+_{k'\sigma'} C_{k\sigma} \pm C^+_{-k-\sigma} C_{-k'-\sigma'} \right)$$

where the sign choice depends on the nature of H_1

Case I : For electron - Phonon interaction such as

⊕ ultrasonic attenuation ⊕ - electron - phonon interaction Simple scalar deformation potential

Depending only on $\Delta \vec{k}$, independent of the sense of $\vec{k}$ or σ

(Interaction with a scalar deformation)

Case II : For int. of the electrons with the e.m. field

⊖ Via a term $\vec{p} \cdot \vec{A}$, which changes sign on replacing $\vec{k}$ by $-\vec{k}$.

ultrasonic, em. field 모두 spin 과는 무관

$I_+ S_-$: spin change , signs are formally reversed.

$\Theta_{\sigma\sigma'} = \pm 1$ for $\sigma' = \pm \sigma$ → 책 볼것

$\gamma_{k\sigma} = \gamma_{k0}$ for $\sigma = \uparrow$, $\gamma_{k\sigma} =$

$$B_{k'\sigma',k\sigma}\,\left(C^+_{k'\sigma'}C_{k\sigma} \pm C^+_{-k-\sigma}\,C_{-k'-\sigma'}\right)$$

$$= B_{k'\sigma',k\sigma}\,\left(u_{k'}u_k\,\gamma^+_{k'\sigma'}\gamma_{k\sigma} - v_{k'}v_k\,\gamma^+_{-k-\sigma}\gamma_{-k'-\sigma'} + u_{k'}v_k\,\gamma^+_{k'\sigma'}\gamma_{-k-\sigma}\right.$$
$$\left. + v_{k'}u_k\,\gamma_{-k',-\sigma'}\gamma_{k\sigma}\right)$$

$$\pm\left(-v_k v_{k'}\,\gamma^+_{k'\sigma'}\gamma_{k\sigma} + u_k u_{k'}\,\gamma^+_{-k-\sigma}\gamma_{-k'-\sigma'} + u_k v_{k'}\,\gamma^+_{k'\sigma'}\gamma^+_{-k-\sigma}\right.$$
$$\left. + v_k u_{k'}\,\gamma_{-k'-\sigma'}\gamma_{k\sigma}\right)$$

$$= B_{\substack{k'\sigma'\\k\sigma}}\left\{(u_{k'}u_k \mp v_k v_{k'})\,\gamma^+_{k'\sigma'}\gamma_{k\sigma} - (v_{k'}v_k \mp u_k u_{k'})\,\gamma^+_{-k-\sigma}\gamma_{-k'-\sigma'}\right.$$
$$\left. + (u_{k'}v_k \pm u_k v_{k'})\,\gamma^+_{k'\sigma'}\gamma^+_{-k-\sigma} + (v_{k'}u_k \pm v_k u_{k'})\,\gamma_{-k'-\sigma'}\gamma_{k\sigma}\right\}$$

$$= B_{\substack{k'\sigma'\\k\sigma}}\cdot\left\{(u_{k'}u_k \mp v_{k'}v_k)\left(\gamma^+_{k'\sigma'}\gamma_{k\sigma} \pm \gamma^+_{-k-\sigma}\gamma_{-k'-\sigma'}\right)\right.$$
$$\left. + (v_k u_{k'} \pm u_k v_{k'})\left(\gamma^+_{k'\sigma'}\gamma^+_{-k-\sigma} \pm \gamma_{-k'-\sigma'}\gamma_{k\sigma}\right)\right\}$$

Note: Magnitude가 아니라 phase에 영향

Transition Probability

$$= \left|B_{\substack{k'\sigma'\\k\sigma}}\right|^2 \times \text{Coherence factor}$$

Coherence factor

$(uu' \mp vv')^2$: Scattering of quasi particles

$(vu' \pm uv')^2$: Creation or annihilation of two

Now

$(uu' \mp vv')^2$: quasi-particle scatt:

. E, E' same sign.

$$= \frac{1}{4}\left[\left\{\left(1+\frac{\xi}{E}\right)\left(1+\frac{\xi'}{E'}\right)\right\}^{1/2} \mp \left\{\left(1-\frac{\xi}{E}\right)\left(1-\frac{\xi'}{E'}\right)\right\}^{1/2}\right]^2$$

$$= \frac{1}{4}\left\{\,1+\frac{\xi}{E}+\frac{\xi'}{E'}+\frac{\xi\xi'}{EE'}+1-\frac{\xi}{E}-\frac{\xi'}{E'}+\frac{\xi\xi'}{EE'}\right.$$
$$\left.\mp 2\left[\left(1-\frac{\xi^2}{E^2}\right)\left(1-\frac{\xi'^2}{E'^2}\right)\right]^{1/2}\right\}$$

$$= \frac{1}{2}\left(1+\frac{\xi\xi'}{EE'} \mp \frac{\Delta^2}{EE'}\right)$$

When summed over all values of ξ_k, which appear in pairs

Similarly

$(vu' \pm uv')^2$: Creation of two q-p :

$$= \frac{1}{4}\left[\left\{\left(1-\frac{\xi}{E}\right)\left(1+\frac{\xi'}{E'}\right)\right\}^{1/2} \pm \left\{\left(1+\frac{\xi}{E}\right)\left(1-\frac{\xi'}{E'}\right)\right\}^{\frac{1}{2}}\right]^2$$

$$= \frac{1}{4}\left[\,1-\frac{\xi}{E}+\frac{\xi'}{E'}-\frac{\xi\xi'}{EE'}+1+\frac{\xi}{E}-\frac{\xi'}{E'}-\frac{\xi\xi'}{EE'}\right.$$
$$\left.\pm 2\left\{\left(1-\frac{\xi^2}{E^2}\right)\left(1-\frac{\xi'^2}{E'^2}\right)\right\}^{1/2}\right.$$

$$= \frac{1}{2}\cdot\left(1-\frac{\xi\xi'}{EE'} \pm \frac{\Delta^2}{EE'}\right) \longrightarrow \frac{1}{2}\left(1 \pm \frac{\Delta^2}{EE'}\right)$$

In general

$$F = \frac{1}{2}\left(1 \mp \frac{\Delta^2}{EE'}\right) \qquad \begin{array}{l} \ominus \ \text{Case I} \\ \oplus \ \text{case II} \end{array}$$

The greatest effect of the coherence factors is for energies E and E' near the gap edge Δ, in which case $F \sim 0$ or 1

1) low energy Scattering Processes :

$\hbar\omega \ll \Delta$: No quasi particles Created

$E, E' \simeq \Delta$

Ⓘ $F = \frac{1}{2}\left(1 - \frac{\Delta^2}{EE'}\right) \ll 1$

Ⓘ $F = \frac{1}{2}\left(1 + \frac{\Delta^2}{EE'}\right) \simeq 1$

2) High Energy processes

$\hbar\omega \geq 2\Delta$: Creates pairs of quasi particle

Ⓘ $F = \frac{1}{2}\left(1 - \frac{\Delta^2}{EE'}\right) \simeq 1$

Ⓘ $F \ll 1$

3) if $E, E' \gg \Delta$: no difference between Ⓘ & Ⓘ.

Superconducting coherence becomes unimportant.

Then, we expect a net transition rate bet. energy levels E and $E' = E + \hbar\omega$ to be

$$\alpha_s = \int |M|^2 F(\Delta, E, E+\hbar\omega)\, N_s(E)\, N_s(E+\hbar\omega) \times [f(E) - f(E+\hbar\omega)]\, dE$$

$$= |M|^2 N^2(0) \int_{-\infty}^{\infty} \frac{1}{2}\left(1 \mp \frac{\Delta^2}{E(E+\hbar\omega)}\right) \frac{E}{\sqrt{E^2-\Delta^2}} \cdot \frac{E+\hbar\omega}{\sqrt{(E+\hbar\omega)^2-\Delta^2}}$$

$$\times [f(E) - f(E+\hbar\omega)]\, dE$$

OR

$$= \frac{1}{2}|M|^2 N^2(0) \int_{-\infty}^{\infty} \frac{(E(E+\hbar\omega) \mp \Delta^2)}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}} [f(E) - f(E+\hbar\omega)]\, dE$$

In the normal state, $\Delta = 0$

$$\alpha_n = \frac{1}{2}|M|^2 N^2(0) \int_{-\infty}^{\infty} [f(E) - f(E+\hbar\omega)]\, dE$$

$$= \frac{1}{2}|M|^2 N^2(0)\, \hbar\omega$$

$$\frac{\alpha_s}{\alpha_n} = \frac{1}{\hbar\omega} \int_{-\infty}^{\infty} \frac{(E(E+\hbar\omega) \mp \Delta^2)}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}} [f(E) - f(E+\hbar\omega)]\, dE$$

$\ominus$ Case I

Ultrasonic Attenuation : an example of case I.

Ultrasonic exp:

$$f \lesssim 10^9\ Hz \longrightarrow \hbar\omega \leq 10^{-2}\Delta(0)$$

$$\hbar\omega << k_BT$$

$\longrightarrow$ Case of low freq. limit

$$\frac{\alpha_s}{\alpha_n} \xrightarrow{\hbar\omega\to 0} \frac{1}{\hbar\omega}\int_{-\infty}^{\infty}\frac{E^2-\Delta^2}{E^2-\Delta^2}\left[f(E)-f(E+\hbar\omega)\right]dE$$

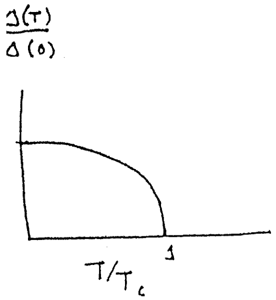
$$= -\int\frac{\partial f}{\partial E}\,dE$$

$$= -\int_{-\infty}^{-\Delta}\frac{\partial f}{\partial E}\,dE - \int_{\Delta}^{\infty}\frac{\partial f}{\partial E}\,dE$$

$$= -f(-\Delta)+f(-\infty) - f(\infty) + f(\Delta)$$

$$= 1 - f(-\Delta) + f(\Delta)$$

$$= 2\,f(\Delta) = \frac{2}{e^{\beta\Delta(T)}+1}$$



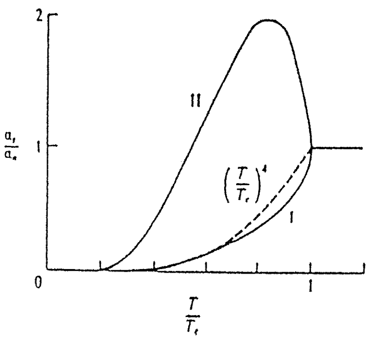


FIGURE 3.9
Temperature dependence of low-frequency absorption processes obeying case I and II coherence factors, compared with the $(T/T_c)^4$ dependence that might be expected for *all* processes from a simple two-fluid model. The curve for case I applies to ultrasonic attenuation, and it is a well-defined low-frequency limit. The curve for case II, which applies to nuclear relaxation or electromagnetic absorption, has no well-defined low-frequency limit unless gap anisotropy or level broadening is taken into account. The curve drawn here corresponds to a broadening of about $0.02\Delta(0)$.

i) For $T << T_c$, $\frac{\alpha_s}{\alpha_n} \sim e^{-\beta\Delta(0)}$

exponentially small as the # of thermally excited quasi-particles avaiable to absorb energy → 0

ii) $\Delta(T)$ can be inferred from ultrasonic exp. at low enough temperatures.

iii) Anisotropy of Δ w.r.t. a crystal axis ← an exp. with a single crystal.

Nuclear Spin Relaxation : An example of case (II)

- by interaction with quasi particles.
- low energy scattering process
- $\hbar\omega = \hbar\gamma H << \Delta(0)$ or k_BT

$$\frac{\alpha_s}{\alpha_n} = \int_{-\infty}^{\infty}\frac{E(E+\hbar\omega)+\Delta^2}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}\left(-\frac{\partial f}{\partial E}\right)dE$$

$$= \int_{\Delta}^{\infty} + \int_{-\infty}^{-\Delta-\hbar\omega}\underset{E+\hbar\omega\,\equiv\,-E'}{(\qquad)}\,dE$$

$$- \int_{\infty}^{\Delta}\frac{(-E'-\hbar\omega)(-E')+\Delta^2}{\sqrt{(E'+\hbar\omega)^2-\Delta^2}\sqrt{E'^2-\Delta^2}}\left(-\frac{\partial f(E')}{\partial E'}\right)dE'$$

$$= 2\int_{\Delta}^{\infty} \frac{E(E+\hbar\omega)+\Delta^2}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}\left(-\frac{\partial f(E)}{\partial E}\right) dE$$

$$\xrightarrow{\omega\to 0}\ 2\int_{\Delta}^{\infty} \frac{E^2+\Delta^2}{E^2-\Delta^2}\left(-\frac{\partial f}{\partial E}\right) dE$$

$$= 2\int_{\Delta}^{\infty} -\frac{\partial f}{\partial E}\, dE + 4\Delta^2\int_{\Delta}^{\infty}\frac{1}{E^2-\Delta^2}\left(-\frac{\partial f}{\partial E}\right) dE$$

$\omega \to 0$

$\sim \ln\left(\frac{\Delta}{\hbar\omega}\right)$ logarithmically diverge

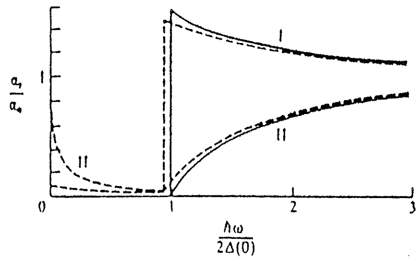


FIGURE 3.10
Frequency dependence of absorption processes obeying case I and II coherence factors at $T = 0$ (solid curves) and $T \approx \frac{1}{2} T_c$ (dashed curves).

이 방법으로 Gap을 잴수 있지 않을까?

propagating sound in different direction in single crystals. 어렵다.

① Morse and Coworkers

- Δ_K 를 방향에 따라 측정

그러나 이 방법은 무리.
$\hat{K}_s$ 방향으로의 sound propagation 은 Δ_K 의 average 측정. $v_{sound} << v_{electron}$
따라서 energy transfer 가 충분치 않다.

$2\Delta(0)$ 가 $3.3KT_c \sim 3.9KT_c$ 였다.

위의 방법은 too simplified

② λ 측정하자. - Fossheim

T_c 바로 근처에서는 λ 잴수있다.

electromagnetic Screening of transverse Sound wave — 이것이 없어지는 경우가 T_c 근처

③ Electronic damping of the motion of dislocations driven by the sound waves

dislocation pinning effect

→ Nonlinear responce

$\frac{\alpha_s}{\alpha_n}$ 이 작은 경우 제외하고 어렵다

이것 고려 Mason 이 lead 에서 $2\Delta(0)$

3.a.2.

Nuclear Relaxation

Case II 로 constructive interference in the relevant low energy scattering process.

$\frac{1}{T_1}$ 에서  보임

실험적으로 Hebel 과 Slichter에 의해 처음 확인

이들은 $\lambda^{-2} \sim \left(1-\left(\frac{T}{T_c}\right)^4\right)$ 측정

Two fluid Model

Normal - electron density

그러나 Coherent peak 설명하지는 못함

$\boxed{\text{실험주의}}$ $$\frac{\alpha_s}{\alpha_n} = 2\int_{\Delta}^{\infty} \frac{E^2+\Delta^2}{E^2-\Delta^2}\left(-\frac{\partial f}{\partial E}\right) dE$$

실험에서 α 값이 크게나옴 OK. $\alpha \sim \int N^2(E)\,dE$ 그래 sharp 하지 않나.

1. Anisotropy of the energy gap in real Crystals.

   $\frac{1}{T_1}$ 측정하니 Anderson의 dirty superconductor 이론과 일치.

   Masuda는 dirtier Sample 사용

2. finite lifetime of the quasi particle states against decay into phonons limits the sharpness of the peak.

고온초전도 : No Hebel - Slichter peak

Electromagnetic Absorption : Case ⓘⓘ

- One can carry over the results of the nuclear relaxation to describe the absorption of low frequency e-m radiation.

$$\frac{\alpha_s}{\alpha_n} \longleftrightarrow \frac{\sigma_{1s}}{\sigma_n} \qquad \text{where } \sigma(\omega) = \sigma_1(\omega) - i\sigma_2(\omega)$$

For a radiation with the large enough to Create pairs of quasi particles $(\hbar\omega \geq 2\Delta)$

initial energy $E < -\Delta$

final energy $E+\hbar\omega > \Delta$

$\Big\}$ $\sigma_1(\omega) = 0$ for $\hbar\omega < 2\Delta$.

"absorption edge"

Mattis and Bardeen 의 이론. $\hbar\omega$ 가 큰 경우에도 성립

T = 0 에서는 quasi particle 없어 pair 깨지면서

$$\left.\frac{\sigma_{1s}}{\sigma_n}\right|_{T=0} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta} \frac{\left|E(E+\hbar\omega)+\Delta^2\right|\left[f(E)-f(E+\hbar\omega)\right]}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}\, dE$$

<u>$\hbar\omega \geq 2\Delta$ 인 경우</u> <u>Initial state $E \leq -\Delta$, final state $E+\hbar\omega \geq \Delta$</u>

$$= \int_{\Delta-\hbar\omega}^{-\Delta} \frac{\left|E(E+\hbar\omega)+\Delta^2\right|}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}\, dE$$

$$= \left(1+\frac{2\Delta}{\omega}\right)E(k) - \frac{4\Delta}{\hbar\omega}K(k)$$

$$,\ k = \frac{\hbar\omega - 2\Delta}{\hbar\omega + 2\Delta}$$

existence and width of the energy gap

Glover and Tinkham 의 Far-Infrared Radiation.

Lead 에서 약간 BCS와 다르다

— strong-coupling effect

이것은 tunneling 실험에서의 strong-coupling 효과와 비슷하다.

Note 1. Sum Rule

$$\int_0^\infty \sigma_1(\omega)\,d\omega = \frac{\pi n e^2}{2m}$$

Missing area가 delta function으로 간다.

Kramers-Kronig Relations.

$$\sigma_1(\omega) = \frac{2}{\pi}\int_0^\infty \frac{\omega'\sigma_2(\omega')\,d\omega'}{\omega'^2-\omega^2} + \text{const}$$

$$\sigma_2(\omega) = -\frac{2\omega}{\pi}\int_0^\infty \frac{\sigma_1(\omega')\,d\omega'}{\omega'^2-\omega^2}$$

$\sigma_1 = A\delta(\omega)$ 일때 $\sigma_2 = \frac{1}{\Lambda\omega} = \frac{ne^2}{m\omega} = \frac{c^2}{4\pi\lambda^2\omega}$

penetration depth 는 missing Area 와 연관

$$\lambda^{-2} = \frac{8A}{c^2}$$

gap 에서의 missing area가 없으면

초전도 유리 — Semiconductor 와 다른점

gapless superconductor by magnetic

Electrodynamics

Response to electro-magnetic field.

dissipative process 뿐 아니라

nondissipative supercurrent effect.

total field including the effects of screening by Super Currents

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{p} = m\vec{v} + \frac{e}{c}\vec{A}$$

$$K.E = \frac{1}{2m}\left(\vec{p} - \frac{e\vec{A}}{c}\right)^2$$

$$= \frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}\vec{A}\right)^2$$

A linear response to a weak field,

$$H_1 = \frac{ie\hbar}{2mc}\sum_i\left(\nabla_i\cdot\vec{A} + \vec{A}\cdot\nabla_i\right)$$

runs over all particles

$$\vec{A} = \sum_q \vec{a}(\vec{q})\, e^{i\vec{q}\cdot\vec{r}}$$

expanded in Fourier Components

$$\psi(\vec{r}) = \sum_k C_k\, e^{i\vec{k}\cdot\vec{r}}$$

This can be expanded in terms of γ operator

For example, in the London theory

$$\vec{J}(\vec{r}) = -\frac{c}{4\pi\lambda_L^2}\vec{A}(\vec{r}) = -\frac{c}{4\pi\lambda_L^2}\sum_q \vec{a}(\vec{q})\, e^{i\vec{q}\cdot\vec{r}}$$

$$\vec{J}(\vec{q}) = -\frac{c}{4\pi\lambda_L^2}\vec{a}(\vec{q}) \Rightarrow K_L(q) = \frac{1}{\lambda_L^2} = K(0)$$

$$\vec{J}(\vec{q}) = \vec{J}_1(\vec{q}) + \vec{J}_2(\vec{q})$$

$$= -\frac{c}{4\pi}K_1(\vec{q})\vec{a}(\vec{q}) - \frac{c}{4\pi\lambda_L^2}\vec{a}(\vec{q})$$

$$= -\frac{c}{4\pi}\underbrace{\left(K_1(\vec{q}) + \frac{1}{\lambda_L^2}\right)}_{K(\vec{q})}\vec{a}(\vec{q})$$

$$K(\vec{q}) = \frac{1}{\lambda_L^2(0)} + K_1(\vec{q}) = \frac{1}{\lambda_L^2(q,T)}$$

where

$$\frac{e\hbar}{m}\sum_k \vec{k}\, C^+_{\vec{k}-\vec{q}} C_{\vec{k}} = -\frac{c}{4\pi}K_1(\vec{q})\vec{a}(\vec{q})$$

$$K_1(\vec{q}) = -\frac{4\pi e\hbar}{mc}\sum_k \frac{\vec{k}\cdot\vec{a}}{a^2} C^+_{\vec{k}-\vec{q}} C_{\vec{k}}$$

$$K(0,T) = \frac{1}{\lambda_L^2(T)}$$

For $q = 0$, $H_1 = -\frac{e\hbar}{mc}\sum_k \vec{k}\cdot\vec{a}(0)\left(\gamma^+_{k\uparrow}\gamma_{k\uparrow} - \gamma^+_{-k\downarrow}\gamma_{-k\downarrow}\right)$

$$E_{k\uparrow} \rightarrow E_{k\uparrow} - \frac{e\hbar}{mc}\vec{k}\cdot\vec{a}(0)$$

$$E_{-k\downarrow} \rightarrow E_{-k\downarrow} + \frac{e\hbar}{mc}\vec{k}\cdot\vec{a}(0)$$

The perturbation simply shifts the energies of the quasi particle excitations.

Similarly

$$\vec{J}_1(0) = \frac{e\hbar}{m}\sum_k \vec{k}\, C^+_{\vec{k}} C_{\vec{k}}$$

$$= \frac{e\hbar}{m}\sum_k \vec{k}\left(\gamma^+_{k\uparrow}\gamma_{k\uparrow} - \gamma_{-k\downarrow}\gamma_{k\downarrow}\right)$$

$$= \frac{e\hbar}{m}\sum_k \vec{k}\left(f_{k\uparrow} - f_{-k\downarrow}\right)$$

$$= \frac{e\hbar}{m}\sum_k \vec{k}\left\{f_0\left(E_{k\uparrow} - \frac{e\hbar}{mc}\vec{k}\cdot\vec{a}(0)\right\} - f_0\left(E_{-k\downarrow} + \frac{e\hbar}{mc}\vec{k}\cdot\vec{a}(\right.\right.$$

$$= \frac{2e^2\hbar^2}{m^2c}\sum_k \left(\vec{a}(0)\cdot\vec{k}\right)\vec{k}\left(-\frac{\partial f_0}{\partial E_k}\right)$$

$\vec{j}_1(0) /\!/ \vec{a}(0)$ by symmetry

$$.= \frac{2e^2\hbar^2}{m^2c}\frac{k_F^2}{3}\vec{a}(0)\sum_k -\frac{\partial f_0}{\partial E_k}$$

$$= -\frac{c}{4\pi}K_1(0,T)\,\vec{a}(0)$$

$$) = -\frac{4\pi}{c}\,\frac{2e^2\hbar^2}{m^2c}\,\frac{k_F^2}{3}\sum_k\left(-\frac{\ }{\partial E_k}\right)$$

$$= -\frac{2\cdot 4\pi e^2}{3mc^2}\cdot 2E_F\sum_k\left(-\frac{\partial f}{\partial E_k}\right)$$

$$= -\frac{4\pi m e^2}{mc^2}\,\frac{1}{N(0)}\sum_k\left(\,''\,\right)$$

$$= -\frac{1}{\lambda_L^2(0)}\int_{-\infty}^{\infty}-\left(\frac{\partial f}{\partial E}\right)d\xi$$

$$= -\frac{1}{\lambda_L^2(0)}\int_{-\infty}^{\infty}\left(-\frac{\partial f}{\partial E}\right)\left(\frac{d\xi}{dE}\right)dE$$

$$= -\frac{1}{\lambda_L^2(0)}\cdot 2\cdot\int_{\Delta}^{\infty}\left(-\frac{\partial f}{\partial E}\right)\frac{E}{\sqrt{E^2-\Delta^2}}\,dE$$

$$\therefore K(0,T) = \frac{1}{\lambda_L^2(0)}\left\{1+\lambda_L^2(0)\,K_1(0,T)\right\}$$

$$= \frac{1}{\lambda_L^2(0)}\left\{1-2\int_{\Delta}^{\infty}\left(-\frac{\partial f}{\partial E}\right)\frac{E}{\sqrt{E^2-\Delta^2}}\,dE\right\}$$

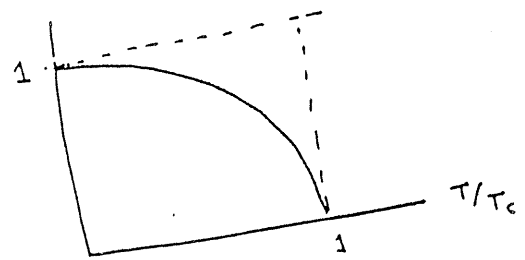
$$= \frac{1}{\lambda_L^2(T)}$$

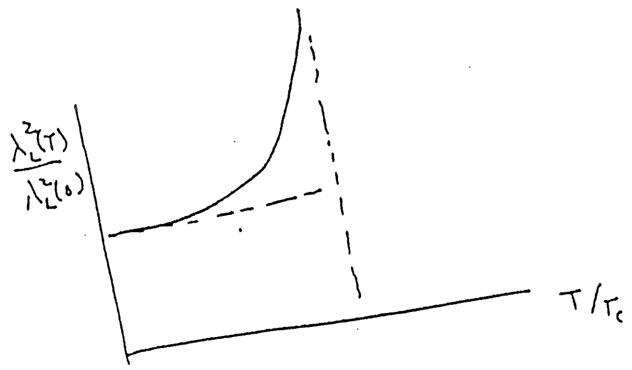
$$K(0,\ T\geq T_c) = 0$$

ii) At low enough temperatures $\left(\frac{\Delta(0)}{K_BT}\gg 1\right)$ the 2nd term becomes exponentially small.

$$K(0,T)\xrightarrow{T\to 0}\lambda_L^{-2}(0)$$

$$\frac{K(0,T)}{K(0,0)} = \frac{\lambda_L^2(0)}{\lambda_L^2(T)}$$





$K(q,T)$ at $T=0$:

the perturbed state in the presence of H_1,

$$|\Psi\rangle = |\Psi_G\rangle - \sum_n \frac{\langle\Psi_n|H_1|\Psi_G\rangle}{E_n}|\Psi_n\rangle$$

n: runs over the excited states with

Excitation energy E_n

$$\langle\Psi|\vec{J}_1(\vec{q})|\Psi\rangle$$

$$= \langle\Psi_G|\cancelto{0}{\vec{J}_1(\vec{q})}|\Psi_G\rangle - \sum_n \frac{\langle\Psi_n|H_1|\Psi_G\rangle\langle\Psi_G|\vec{J}_1(\vec{q})|\Psi_n\rangle}{E_n}$$

$$- \sum_n \frac{\langle\Psi_n|H_1|\Psi_G\rangle^*\langle\Psi_n|\vec{J}_1(\vec{q})|\Psi_G\rangle}{E_n}$$

$$= -2\,\mathrm{Re}\sum_n \frac{\langle\Psi_n|H_1|\Psi_G\rangle\langle\Psi_G|\vec{J}_1(\vec{q})|\Psi_n\rangle}{E_n}$$

Recall that

$$H_1 = -\frac{e\hbar}{mc}\sum_{k,q} \vec{k}\cdot\vec{a}(\vec{q})\; C^+_{k+q,\sigma}\, C_{k,\sigma}$$

$$= -\frac{e\hbar}{mc}\sum_{k,q} \vec{k}\cdot\vec{a}(q)\,\Big\{ (U_k U_{k+q} + V_k V_{k+q})(\gamma^+_{k+q\uparrow}\gamma_{k\uparrow} - \gamma^+_{-k\downarrow}\gamma_{-(k+q)\downarrow})$$

$$+ (V_k U_{k+q} - U_k V_{k+q})(\gamma^+_{k+q\uparrow}\gamma^+_{-k\downarrow} - \gamma_{-(k+q)\downarrow}\gamma_{k\uparrow})$$

Nonvanishing contribution:

terms with $\gamma^+_{k+q\uparrow}\gamma^+_{-k\downarrow} - \gamma_{-(k+q)\downarrow}\gamma_{k\uparrow}$

$$\langle\Psi_n|H_1|\Psi_G\rangle = V_k U_{k+q} - U_k V_{k+q}$$

with $E_n = E_k + E_{k+q}$

Similarly

$$\langle\Psi_G|\vec{J}_1(\vec{q})|\Psi_n\rangle = -(V_k U_{k+q} - U_k V_{k+q})$$

with E_n as above

Thus,

$$\langle\Psi|\vec{J}_1(\vec{q})|\Psi\rangle$$

$$= (-2)\left(-\frac{e\hbar}{mc}\right)\left(\frac{e\hbar}{m}\right)\sum_k \frac{(V_k U_{k+q} - U_k V_{k+q})^2}{E_k + E_{k+q}} \underbrace{(\vec{k}\cdot\vec{a}(\vec{q}))\,\vec{k}}_{\frac{k_F^2}{3}\vec{a}(\vec{q})}$$

$$\equiv -\frac{c}{4\pi} K_1(q,0)\,\vec{a}(\vec{q})$$

$$K_1(q,0) := -\frac{4\pi}{c}\frac{2e^2\hbar^2}{m^2c}\frac{k_F^2}{3}\sum_k \frac{(V_k U_{k+q} - U_k V_{k+q})^2}{E_k + E_{k+q}}$$

$$= -\frac{1}{\lambda_L^2(0)}\frac{1}{N(0)}\sum_k \Big(\qquad \Big)$$

OR

$$K_1(q,0) = -\frac{1}{\lambda_L^2(0)} \int_{-\infty}^{\infty} \frac{\left(V_k U_{k+q} - U_k V_{k+q}\right)^2}{E_k + E_{k+q}} \, d\xi$$

Thus

$$K(q,0) = \frac{1}{\lambda_L^2(0)} \left\{ 1 - \int_{-\infty}^{\infty} \left(\text{위식} \right) d\xi \right\}$$

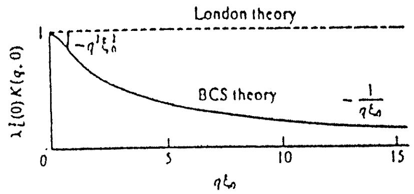


FIGURE 3.12
Comparison of q-dependent response of the nonlocal BCS theory with the q-independent response of the local London theory. In both cases, the curves are drawn for pure metals, with infinite mean free paths.

where $\xi_0 \equiv \frac{\hbar v_F}{\pi \Delta(0)}$

- $q_0 \xi_0 >> 1$, $K(q,0) \longrightarrow K(0,0) \frac{3\pi}{4 q \xi_0}$

- Normal state, at $T=0$

  $\Delta(0) \to 0$

  $\xi_0 \to \infty$

  then $K(q,0) \to 0$ for all q

Nonlocal Response

$$\vec{J}(\vec{q}) = -\frac{c}{4\pi} K(\vec{q}) \vec{a}(\vec{q})$$

$$\vec{J}(\vec{r}) = -\frac{3c}{4\pi^2 \xi_0 \lambda_L^2(T)} \int \frac{\vec{R}\,(\vec{R}\cdot\vec{A}(\vec{r}\,'))}{R^4} K_R(\vec{R},T)\, d^3\vec{r}\,'$$

K_R : real space Kernel

$\vec{R} = \vec{r} - \vec{r}\,'$

the relation between $K(q,T)$ & $K_R(\vec{R},T)$

$$K(q,T) = -\frac{4}{\xi_0 \lambda_L^2(T)} \int_0^{\infty} \frac{3}{qR} \underbrace{j_1(qR)}_{\text{Spherical Bessel fnc.}} K_R(R,T)\, dR$$

$K(R,T)$ very close to e^{-R/ξ_0}

$\longrightarrow$

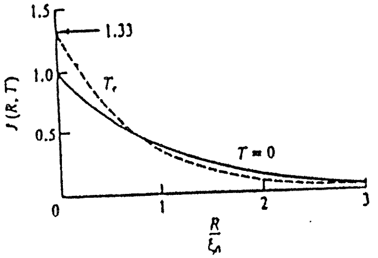


FIGURE 3.13
Schematic comparison of the BCS range function $J(R,T)$ at $T=0$ and T_c. Note that the range of nonlocality is reduced by a factor of about 0.75 on going from $T=0$ to T_c.

if $\vec{A}(\vec{r}\,')$ is constant

$$\vec{J}(\vec{r}) \Rightarrow -\frac{c}{4\pi \lambda^2(T)} \vec{A}(\vec{r}) \quad : \text{London form}$$

Impurity Effect

For a dirty metal, $K(R,T) \times e^{-R/\ell}$

to make the e-m response more local.

$\ell <<$ spatial variational length scale of $\vec{A}$

$$\frac{\lambda_L^2(T)}{\lambda_{eff}^2(T)} = \frac{K(0,T,\ell)}{K(0,T,\infty)}$$

$$= \frac{\int_0^\infty K_R(R,T)\, e^{-R/\ell} dR}{\underbrace{\boxed{\int_0^\infty K_R(R,T)\, dR}}_{\xi_0}}$$

dirty limit : $\ell << \xi_0$

$$\int_0^\infty K_R(R,T)\, e^{-R/\ell} dR \simeq K_R(0,T) \int_0^\infty e^{-R/\ell} dR$$

$$= K_R(0,T)\,\ell$$

$$\therefore\ \lambda_{eff}(T) = \lambda_L(T) \left(\frac{\xi_0}{\ell}\right)^{1/2} \underbrace{\left[K_R(0,T)\right]^{-\frac{1}{2}}}_{\text{small correction}}$$

$1 \le K_R(0,T) \doteq 1.33$

OR, if we use $K_R \rightarrow e^{-R/\xi_0}$

$$\lambda_{eff}(T) \doteq \lambda_L(T) \left(1 + \frac{\xi_0}{\ell}\right)^{1/2}$$

$$= \lambda_L(T) \left(\frac{\xi_0}{\xi}\right)^{\frac{1}{2}} \qquad \xi^{-1} = \xi_0^{-1} + \ell^{-1}$$

An improved form

$$\lambda_{eff}(T) = \begin{cases} \lambda_L(T) \left(1 + 0.75 \frac{\xi_0}{\ell}\right)^{1/2} & T \simeq T_c \\ \lambda_L(T) \left(1 + \frac{\xi_0}{\ell}\right)^{1/2} & T = 0 \end{cases}$$

3.10.5

Complex Conductivity

전에는 (2.5.1) Complex conductivity 는 $\omega << \omega_g$ 인 경우였다. 이 Section 에서는 $\omega \sim \omega_g$ 인 경우에도 적용.

$\boxed{\text{시작}}$

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\frac{i\omega}{c}\vec{A}$$

$$\vec{J}(q,\omega,T) = \sigma(q,\omega,T)\,\vec{E}(q,\omega) = -\frac{c}{4\pi} K(q,\omega,T)\,\vec{A}(q,\omega)$$

we have

$$\sigma(q,\omega,T) = \frac{ic^2}{4\pi\omega} K(q,\omega,T)$$

low freq. limit

$$\frac{\sigma_s}{\sigma_n} = \frac{\pi\Delta}{\hbar\omega} \tanh\frac{\Delta}{2kT} \qquad \hbar\omega << 2\Delta$$

limiting form

$$\frac{\sigma_2}{\sigma_1} \longrightarrow \begin{cases} \dfrac{\pi\Delta}{\hbar\omega} & T \ll T_c \\ \dfrac{\pi}{2}\cdot\dfrac{\Delta^2}{kT\hbar\omega} & T \simeq T_c \end{cases}$$

London eq. 에서는

$$\sigma_{2L} = \frac{n_s e^2}{m\omega}$$

Note 1. $n_s \sim \Delta$ $T \ll T_c$

$n_s \sim \Delta^2$ $T \simeq T_c$

그러므로 G.L eq. 에서는 $(T \simeq T_c)$

$$\psi \sim \Delta$$

More complete picture of a nonlocal or q-dependent response.

질문: 앞에서 구한 σ_{1s}/σ_n 은 어떤 경우인가?

$R=0$ 의 경우 (즉 nonlocal 에서 $R=0$ 인 경우)

이것은 dirty limit $l \ll \xi_0$ 인 경우다

$e^{-R/a}$ 이 $J(R,\omega,T)$ 가 많이 바뀌기전 끝난다

즉 $q\xi_0 \gg 1$ 인 $\overset{\text{extreme}}{\wedge}$ anomalous limit.

An explicit form for σ_2 at $T=0$

$$\frac{\sigma_{2s}}{\sigma_n} = \frac{1}{2}\left(1+\frac{2\Delta}{\hbar\omega}\right)E(k') - \frac{1}{2}\left(1-\frac{2\Delta}{\hbar\omega}\right)K(k')$$

where $k' = (1-k^2)^{1/2}$

$$k = \left|\frac{2\Delta-\hbar\omega}{2\Delta+\hbar\omega}\right|$$

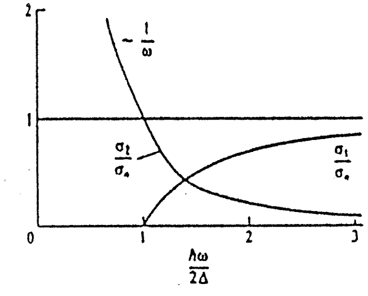


FIGURE 3.14
Complex conductivity of superconductors in extreme anomalous limit (or extreme dirty limit) at $T=0$. The rise of σ_2 as $1/\omega$ below the gap describes the accelerative supercurrent response. Its coefficient is proportional to the "missing area" under the $\sigma_1(\omega)$ curve at finite frequencies (see the discussion in Sec. 3.9.3).

중요 feature

1. $\sigma_2 \sim \frac{1}{\omega}$ for $\hbar\omega \ll 2\Delta$

즉 $K(q,\omega) \sim K(q)$ i.e indep. of ω

freq.-independent penetration depth와 같다.

실험에서 λ. thin film이 substrate 위에 있으면

$$T = \left[\left(1+\frac{\sigma_1 d Z_0}{n+1}\right)^2 + \left(\frac{\sigma_2 d Z_0}{n+1}\right)^2\right]^{-1}$$

n: index of refraction of the substrate

① $T_s \to 0$ as $\omega \to 0$

$\because$ $\sigma_{2s} \sim \frac{1}{\omega} \sim \infty$

② $\hbar\omega \gg 2\Delta$ 이면

$\sigma_{2s} \longrightarrow 0$, $\sigma_{1s} \to \sigma_n$

So $T_s \to T_n$

③ 이사이 에서는

There is a peak in transmission at which

$T_s > T_n$

observation by Glover and Tinkham

: strong support to an energy-gap model of superconductivity

$\Rightarrow$ Support BCS model.

3.11.1.

Preliminary Estimate of λ for Nonlocal case.

정식으로 풀기전 Pippard 의 초전도 분석하자

즉 $\xi_0 \gg \lambda_L$ i.e, nonlocal electrodynamics

exponential penetration in nonlocal case

$$h_y \approx B_0 e^{-z/\lambda}$$

$$A_x \approx \lambda B_0 e^{-z/\lambda}$$

$$\therefore \bar{A} = \lambda B_0$$

$$\| $$

$$-\frac{c}{4\pi\lambda_L^2}\left(\frac{\lambda}{\xi_0}\right)\bar{A} = \frac{c\lambda^2}{4\pi\lambda_L^2}\cdot\frac{B_0}{\xi_0}$$

Maxwell eq. to the surface layer

$$\frac{B_0}{\lambda} \approx |\nabla \times h|$$

$$= \frac{4\pi \tilde{J}}{c} \approx \frac{\lambda^2 B_0}{\lambda_L^2 \xi_0}$$

$$\therefore \lambda \approx (\lambda_L^2 \xi_0)^{1/3}$$

Pippard superconductor 에서 온도 의존성

① $\lambda_L(T) > \xi_0$ near T_c

$\therefore$ $\lambda(T) \approx \lambda_L(T) \sim (T_c - T)^{-\frac{1}{2}}$

② low temp. $\lambda_L(T) < \xi_0$

$\lambda(T) \sim (\lambda_L^2 \xi_0)^{1/3} \sim (T_c - T)^{-\frac{1}{3}}$

$\lambda_L(0)/\xi_0$ 가 초전도체에 따라 다르기에

universal 한 function은 없다.

3.11.2 Solution by Fourier Analysis

field penetration problem

J와 A의 Fourier analysis

$$J(q) = -\frac{c}{4\pi} K(q)\, a(q)$$

이후 detail은 Appendix.

표면문제는 Some care 필요

왜냐면 앞에서의 K(q)는 infinite medium

에서만 유효하기에

two limiting case

Completely diffuse

Completely Specular reflection of electron

at the surface

local approximation인 경우

London form, but with a modified

penetration depth.

$$\lambda_{eff}(l,T) = \lambda_L(T)\,(1+\xi_0'/l)^{1/2}$$

$$T=0 \qquad \lambda_{eff}(l,T) = \lambda_L(T)\,(1+\xi_0/l)^{1/2}$$

$$T\to T_c \qquad = \lambda_L(T)\,(1+0.75\,\xi_0/l)^{1/2}$$

Extreme anomalous case

K(q)에 의해 기술

$$K(q) \sim 1/q$$

diffused surface Scattering인 경우

$$\lambda_{\infty,diff} = 0.65\,(\lambda_L^2\,\xi_0')^{1/3}$$

Aluminuum $\lambda_{\infty,diff}$ 기술 잘됨

$\because\ \lambda_L \simeq 160\text{Å},\quad \xi_0 \simeq 15{,}000\text{Å}$

Tin $\lambda_L \sim 350\text{Å},\quad \xi_0 \approx 3000\text{Å}$

moderate approximation

lead $\lambda_L \approx \frac{\xi_0}{2}$

London approximation이 더

잘 맞는다.

HTSC : $\lambda_L \approx 1500\text{Å}$

$\xi_0 \approx 15\text{Å}$

Electrodynamics is completely local

Same is true of typical alloy superconductors,

in which the short mean free path assures

that $\xi \approx l \ll \lambda_L$

Nonlocal electrodynamics

1. High Fermi velocity
2. low T_c
3. long mean free path

Such as clean aluminum

대부분 Classical pure superconductor의 penetration depth ~ 500Å 임을 설명한다.

3.11.3. Temperature Dependence of λ

$\frac{\lambda(T)}{\lambda(0)}$ ① not universal temp. dependence ($\frac{T}{T_c}$)

② 또한 two fluid model과 많이 차이 나지는 않는다. 또한 T→0 에서는 sensitivity 때문에 측정 쉽지 않다.

③ HTSC

T→0 limit 에서 Classical pure Superconductor와 상당히 다르다.

Pure metal in the extreme anomalous limit

$\frac{\lambda(T)}{\lambda(0)} = \frac{\lambda_L(T)}{\lambda_L(0)}$ 인 관계 이용.

$$\frac{\lambda_\infty(T)}{\lambda(0)} = \left[\frac{\lambda_L^2(T)}{\lambda_L^2(0)\, J(0,T)}\right]^{1/3} = \left[\frac{\Delta(T)}{\Delta(0)} \tanh \beta \frac{\Delta(T)}{2}\right]^{-\frac{1}{3}}$$

Dirty local limit

$$\frac{\lambda_{eff}(T)}{\lambda_{eff}(0)} = , \frac{\lambda_L(T)}{\lambda_L(0)\, J^{1/2}(0,T)} = \left[\frac{\lambda_\infty(T)}{\lambda_\infty(0)}\right]^{1/2}$$

Two fluid model

$$\frac{\lambda(T)}{\lambda(0)} = \frac{1}{\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2}}$$

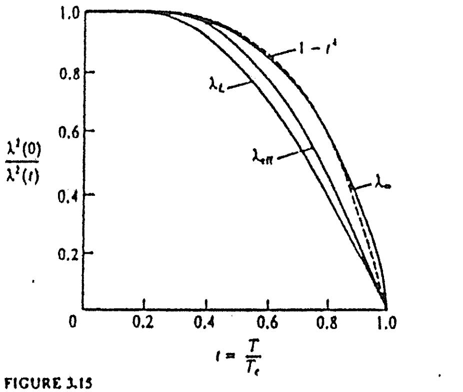


FIGURE 3.15
Comparison of the predicted temperature dependence for $1/\lambda^2$ in various limiting cases of the BCS theory. The dashed curve depicts the empirical approximation (3.135).

3.11.4 Penetration depth in thin film

λ_{eff} and $\lambda_\perp$

1) thin film 으로 하여 λ(T) 구하자

질문. 과연 bulk 의 λ(T) 인가?

2) 많은 실험이 되어 이 경우를 이해하자.

결론.

① Sufficiently thin film인 경우

film // B field

$\Rightarrow A_y = H_z x$ characteristic of the unscreened field.

Supercurrent response of a nonlocal superconductor is essentially equivalent to that of a local superconductor with

$$\lambda_{eff} \approx \lambda_L (\xi_0' / d)^{1/2} \quad \text{for } d \ll \xi_0'$$

② field ⊥ film $d \ll \lambda$

$\lambda_\perp \approx \lambda^2 / d$ Pearl 의 Paper.

$\lambda > \lambda_\perp$

current density 의 fall off

$\frac{1}{r^2}$ rather than exponential $e^{-x/\lambda}$

3.11.5

Measurement of λ

earliest exp

large number of colloidal particles

Thin film with a small dimension d comparable to λ.

그러나 major uncertainty 있다.

Casimir

ac susceptibility technique should be sufficiently sensitive to allow the temperature dependent change in field penetration $\lambda(T) - \lambda(0)$ at the surface of a single bulk sample to be measured.

mutual inductance bridge operating at 70 Hz, but their sensitivity did not allow .... .

Pippard - 10^{10} Hz

microwave techniques to measure λ(T) change

Resonance frequency of a cavity

Schawlow Delvin

λ(T) in 10^5 Hz

BCS not two fluid

absolute value 는 n_s 에 관한 정보

온도 의존성은 microscopic theory

BCS $\frac{\lambda(T)}{\lambda(0)} - 1 \sim T^{-\frac{1}{2}} e^{-\Delta(0)/kT}$

gap 에 Node가 있는 경우 $\left(\frac{T}{T_c}\right)^n$

Muon spin Resonance

: bulk sensitive 하다.

Determine the local magnetic fields in superconductor in the mixed state

λ(T) 측정

① Sensitivity – AC 보다는 작다.

② lattice 의 thermal motion 때문에 분석이 복잡하다.

This technique has been very useful in studying the high temperature superconductors.

결론 BASIC Features of the BCS theory

1. Cooper pairing due to a weak, phonon mediated attraction between electrons

2. Nature of the superconducting ground state, & condensation energy

3. excited quasi-particle states above the energy gap, and energy gap 의 온도 의존성

4. Electron tunneling

Confirming of the electron-phonon mechanism

5 Ultrasonic attenuation and nuclear relaxation processes – 온도 의존성이 다르더라

Coherent factor 로 설명

6. Electrodynamics of superconductors.

- absorption due to quasi particle process
- Computing the lossless supercurrent response in the presense of a Vector-potential

London limit

Pippard nonlocal generalization.

BCS

Microscopic foundation

$\updownarrow$

현상론적 London and Ginzburg-Landau theory of superconductivity

Ginzburg - Landau theory

BCS theory : when Δ constant in space

G-L theory :

- inhomogeneity present in n_c
- local theory
- $T \sim T_c$

$\Psi(r)$: Complex order parameter ($\sim \Delta(\vec{r})$

$|\Psi(\vec{r})|^2 = n_s(\vec{r})$

$$f(T) = f_{no}(T) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}\left|(-i\hbar\nabla - \frac{e^*}{c}\vec{A})\Psi\right|^2 + \frac{h^2}{8\pi}$$

$f_{no}(T) = f_{no}(0) - \frac{1}{2}\gamma T^2$ (without field)

- Ψ small at $T \sim T_c$
- Ψ varying slowly in space

i) powers of Ψ excluded ← f to be real

ii) " $Re(\Psi)$ excluded

← f not depends on the absolute phase of Ψ

i) if $\Psi = 0$

$f = f_{no} + \frac{h^2}{8\pi}$ normal state free energy

ii) if $\Psi \neq 0$ without field & without gradients

$$\delta f = f_s - f_n = \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 \quad (\beta > 0)$$

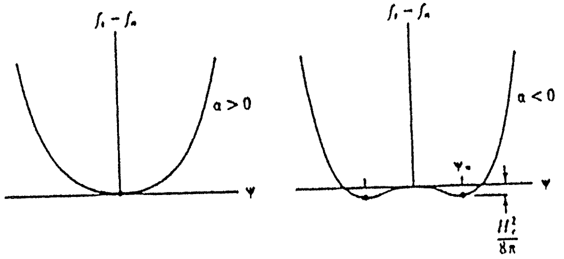
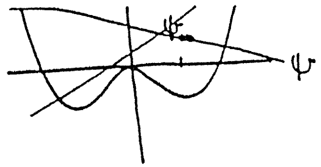


FIGURE 4.1
Ginzburg-Landau free-energy functions for $T > T_c (\alpha > 0)$ and for $T < T_c (\alpha < 0)$. Heavy dots indicate equilibrium positions. For simplicity, ψ has been taken to be real.



$$\frac{\partial}{\partial|\Psi|^2}\delta f = \alpha + \beta|\Psi|^2 = 0$$

$$f - f_n = -\frac{\alpha^2}{2\beta} = -H_c^2/8\pi$$

$\alpha(t) = \alpha'(t-1)$ with $\alpha' > 0$

$H_c(t) \sim 1-t \rightarrow 1-t^2$ more accurately

$|\Psi(t)|^2 \sim 1-t$ near T_c

$\sim n_s$

iii) fields & gradients present :

$$\psi = |\psi| e^{i\phi}$$

$$\frac{1}{2m^*} \left| \left(-i\hbar\nabla - \frac{e^*}{c}\vec{A} \right) \psi \right|^2$$

$$= \frac{1}{2m^*} \left(i\hbar\nabla\psi^* - \frac{e^*}{c}\vec{A}\psi^* \right)\left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right)$$

$$= \frac{1}{2m^*} \left(i\hbar e^{-i\phi}\nabla|\psi| + \hbar\psi^*\nabla\phi - \frac{e^*}{c}\vec{A}\psi^* \right) \cdot$$
$$\left(-i\hbar e^{i\phi}\nabla|\psi| + \hbar\psi\nabla\phi - \frac{e^*}{c}\vec{A}\psi \right)$$

$$= \frac{1}{2m^*} \Big[\hbar^2 (\nabla|\psi|)^2 + i\hbar\nabla|\psi| \cdot \left(\hbar|\psi|\nabla\phi - \frac{e^*}{c}\vec{A}|\psi| \right)$$
$$- i\hbar\nabla|\psi| \cdot \left(\hbar|\psi|\nabla\phi - \frac{e^*}{c}\vec{A}|\psi| \right) + \left(\hbar\nabla\phi - \frac{e^*}{c}\vec{A} \right)^2 |\psi|^2 \Big]$$

$$= \frac{1}{2m^*} \left[\hbar^2 (\nabla|\psi|)^2 + \left(\hbar\nabla\phi - \frac{e^*}{c}\vec{A} \right)^2 |\psi|^2 \right]$$

Energy associated with gradient in the magnitude of ψ

KE associated with supercurrent in a gauge-invariant form

$$= \frac{1}{2} m^* v_s^2 \cdot n_s$$

G-L differential Equations :

Without B.C.

→ F is minimized by having $\psi = \psi_\infty$ everywhere

With Boundary Condition , such as fields, Currents OR gradients

→ ψ adjust itself to minimize F

$$F = \int \left\{ f_{no} + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left| \left(-i\hbar\nabla - \frac{e^*}{c}\vec{A} \right)\psi \right|^2 + \frac{h^2}{8\pi} \right\} d^3\vec{r}$$

$$\delta F = 0$$

$$\int \left| \left(-i\hbar\nabla - \frac{e^*}{c}\vec{A} \right)\psi \right|^2 d^3\vec{r}$$

$$= \int \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right) \cdot \left(i\hbar\nabla\psi^* - \frac{e^*}{c}\vec{A}\psi^* \right) d^3\vec{r}$$

$$= \int \left\{ \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right) \cdot i\hbar\nabla\psi^* - \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right) \cdot \frac{e^*}{c}\vec{A}\psi^* \right\} d^3r$$

$$\nabla \cdot \left\{ \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right) i\hbar\psi^* \right\}$$

$$= \left\{ \nabla \cdot \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right) \right\} \times i\hbar\psi^*$$
$$+ \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi \right) \cdot i\hbar\nabla\psi^*$$

$$\therefore \int \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi\right)\cdot i\hbar\nabla\psi^* \, d^3\vec{r}$$

$$= \int \left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi\right) i\hbar\,\psi^* \cdot d\vec{s}$$

$$- \int \nabla\cdot\left(-i\hbar\nabla\psi - \frac{e^*}{c}\vec{A}\psi\right) \times i\hbar\,\psi^* \, d^3\vec{r}$$

$$\delta F = 0 \quad \longrightarrow \quad \frac{\partial F}{\partial\psi^*}\,\delta\psi^* = 0$$

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)^2\psi = 0$$

$$\vec{J} = n_s e^* v_s$$

$$= n_s \cdot \frac{e^*}{m^*}\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)$$

$$\longrightarrow \frac{1}{2}\frac{e^*}{m^*}\Big\{ \psi^*(-i\hbar\nabla\psi) - \frac{e^*}{c}\vec{A}\psi^*\psi$$

$$+ \psi(i\hbar\nabla\psi^*) - \frac{e^*}{c}\vec{A}\psi\psi^* \Big\}$$

$$= \frac{e^*\hbar}{2m^* i}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{e^{*2}}{m^* c}\psi^*\psi\vec{A}$$

OR

$$= \frac{e^*}{m^*}|\psi|^2\left(\hbar\nabla\phi - \frac{e}{c}\vec{A}\right)$$

GL eq.

i) Same form as Schrödinger eq. with eigenvalue $-\alpha$, potential energy $\beta|\psi|^2$

ii) the repulsive pot → favors uniform $\psi(\vec{r})$

B.C. to the G-L eq.

i) S-I interface : no Current through the interface

$$\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)\psi\Big|_n = 0$$

ii) S-N interface without current

$$\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)\psi\Big|_n = \frac{i}{b}\psi$$

b: real Constant.

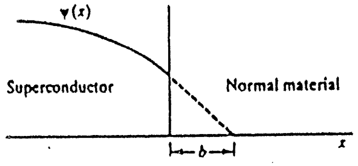


FIGURE 4.2
Schematic diagram illustrating the boundary condition (4.15a) at an interface characterized by an extrapolation length b.

Without field $h=0,\ \vec{A}=0$

Take ψ real, since the eq. has only real coefficient

introduce $f = \dfrac{\psi}{\psi_\infty}$ $\quad \left(\psi_\infty^2 = -\dfrac{\alpha}{\beta} > 0 \right)$

1D:

$$\alpha\psi + \beta|\psi|^2\psi - \frac{\hbar^2}{2m^*}\frac{d^2\psi}{dx^2} = 0$$

OR

$$\psi - \frac{|\psi|^2}{|\psi_\infty|^2}\psi + \frac{\hbar^2}{2m^*|\alpha|}\frac{d^2\psi}{dx^2} = 0$$

$$f - f^3 + \xi(T)\frac{d^2 f}{dx^2} = 0$$

$$\xi(T) \equiv \frac{\hbar^2}{2m^*|\alpha|} \propto \frac{1}{1-t}$$

the characteristic length for variation of ψ

linearized form : $f(x) = 1 + g(x)$, $g(x) \ll 1$

$$\xi^2 g''(x) + (1+g) - (1+3g+\cdots) = 0$$

$$\xi^2 g''(x) = 2g$$

$$\therefore\ g(x) \sim e^{\pm\sqrt{2}x/\xi}$$

한편

$$\frac{\alpha^2}{2\beta} = \frac{H_c^2}{8\pi}.$$

$$= \frac{\alpha}{2}\left(\frac{\alpha}{\beta}\right) = -\frac{\alpha}{2}|\psi_\infty|^2$$

$$\alpha(T) = -\frac{H_c^2}{4\pi|\psi_\infty|^2}$$

$$= -H_c^2 \cdot \frac{\lambda_{eff}^2 e^{*2}}{m^* c^2} \qquad \left(\lambda_{eff}^2 = \frac{m^* c^2}{4\pi|\psi_\infty|^2 e^{*2}} \right)$$

$$\beta(T) = -\frac{\alpha}{|\psi_\infty|^2} = \frac{4\pi e^{*4}}{m^{*2} c^4} H_c^2(T)\lambda_{eff}^4$$

$$\xi^2(T) = \frac{\hbar^2}{2m^*|\alpha|}$$

$$= \frac{c^2\hbar^2}{2e^{*2} H_c^2(T)\cdot\lambda_{eff}^2(T)} \rightarrow \frac{\Phi_0^2}{2(2\pi)^2}\cdot\frac{1}{H_c^2(T)\lambda_{eff}^2(T)}$$

OR

$$\xi(T) = \frac{\Phi_0}{\sqrt{2}\,2\pi H_c(T)\lambda_{eff}(T)}$$

OR

$$H_c(T) = \frac{\Phi_0}{\sqrt{2}\cdot 2\pi\,\xi(T)\lambda_{eff}(T)}$$

$$\text{cf.}\quad H_{c_2}(T) = \frac{\Phi_0}{2\pi\xi^2(T)}$$

$$N(0) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon_F^{1/2}$$

$$\xi_0 = \hbar v_F / \pi \Delta(0)$$

$$\frac{H_c^2(0)}{8\pi} = \frac{1}{2} N(0)\, \Delta^2(0)$$

$$H_c^2(0) = \frac{4\hbar^2}{\pi^3 m \xi_0^2}\,(3\pi^2 n)$$

$$= \frac{12\hbar^2}{\pi m}\,\frac{mc^2}{4\pi \lambda_L^2(0)\, e^2} \qquad \swarrow \quad \lambda_L^2(0) = \frac{mc^2}{4\pi n e^2}$$

$$\phi_0 \doteq \left(\frac{2}{3}\right)^{1/2} \pi^2 \xi_0\, \lambda_L(0)\, H_c(0)$$

$$\frac{\xi(T)}{\xi_0} = \frac{\pi}{2\sqrt{3}} \cdot \frac{H_c(0)\,\lambda_L(0)}{H_c(T)\,\lambda_{eff}(T)}$$

BCS results

$$H_c(T) = 1.73\; H_c(0)\;(1-t)$$

$$\lambda_L(t) = \lambda_L(0) \Big/ [2\,(1-t)]^{1/2}$$

$$\lambda_{eff}(t)\Big|_{dirty} = \lambda_L(t)\left(\frac{\xi_0}{1.33\,l}\right)^{1/2}$$

Using the BCS results, $T \simeq T_c$

$$\xi(T) = \begin{cases} 0.74 \cdot \dfrac{\xi_0}{\sqrt{1-t}} & \text{pure limit} \\ 0.855\; \dfrac{(\xi_0 l)^{1/2}}{\sqrt{1-t}} & \text{dirty limit} \end{cases}$$

Critical current of a thin wire or film

thin wire or film $(d \ll \xi(T))$:

$|\Psi| \neq \Psi_\infty$ but constant everywhere

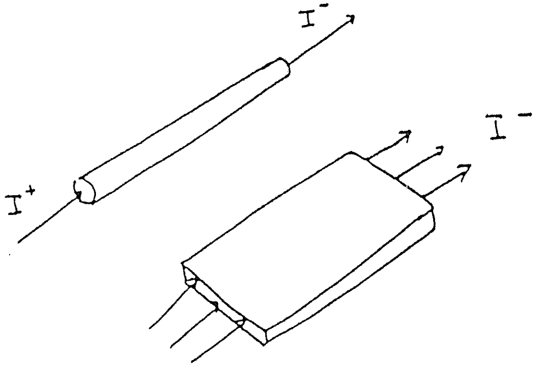
$\Psi(\vec{r}) \simeq |\Psi| e^{i\phi(\vec{r})}$

$|\Psi|$: r independent

Then

$$\vec{j}_s = \frac{2e}{m^*}|\Psi|^2\left(\hbar\nabla\phi - \frac{2e}{c}\vec{A}\right) = 2e|\Psi|^2\vec{v}_s$$

$$f = f_{no} + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \underbrace{|\Psi|^2\frac{1}{2}m^* v_s^2}_{\frac{1}{2m^*}\left|\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)\Psi\right|^2} + \frac{h^2}{8\pi}$$



가정: uniform current density through a thin film or wire.

$\frac{h^2}{8\pi}$ 을 무시.

$|\Psi|^2$ 의 optimal value를 구하자.

$$\alpha + \beta|\Psi|^2 + \frac{1}{2}m^* v_s^2 = 0$$

$$-1 + \frac{|\Psi|^2}{\Psi_\infty^2} + \frac{m^* v_s^2}{2|\alpha|} = 0$$

or

$$|\Psi|^2 = \Psi_\infty^2\left(1 - \frac{m^* v_s^2}{2|\alpha|}\right)$$

$$= \Psi_\infty^2\left\{1 - \left(\xi(T)m^* v_s/\hbar\right)^2\right\}$$

the corresponding current density

$$J_s = 2e|\Psi|^2 v_s$$

$$= 2e\Psi_\infty^2\left(1 - \frac{m^* v_s^2}{2|\alpha|}\right)v_s$$

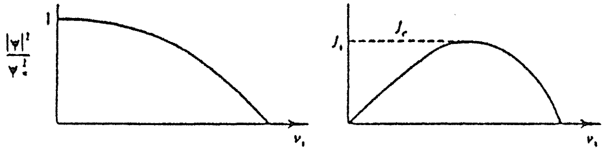


FIGURE 4.4
Variation of $|\Psi|^2$ and of J_s with the superfluid velocity v_s.

$$\frac{dj_s}{dv_s} = 0 \qquad \therefore \quad 1 - \frac{3}{2}\,\frac{m^* v_s^2}{2|\alpha|} = 0$$

$$\text{OR} \quad \frac{1}{2} m^* v_s^2 = \frac{|\alpha|}{3}$$

$$\therefore \left.\frac{|\psi|^2}{\psi_\infty^2}\right|_c = 1 - \frac{m^* v_s^2}{2|\alpha|} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$J_c = 2e\,\psi_\infty^2 \cdot \frac{2}{3} \cdot \left(\frac{2}{3}\,\frac{|\alpha|}{m^*}\right)^{1/2}$$

$$\lambda^2 = \frac{m^* c^2}{4\pi |\psi|^2 e^{*2}}$$

$$|\alpha| = \frac{e^{*2}}{m^* c^2} H_c^2(T)\,\lambda^2(T)$$

$$= \frac{H_c(T)\,c}{3\sqrt{6}\,\pi\,\lambda(T)}\;\underbrace{\frac{\psi_\infty^2}{|\psi|^2}}_{\frac{3}{2}}$$

$$= \frac{H_c(T)\,c}{2\sqrt{6}\,\pi\,\lambda(T)} \sim (1-t^2)(1-t^4)^{1/2}$$

$$\sim (1-t)^{3/2} \quad \text{near } T_c$$

$J_c \sim (1-t)^{3/2}$: for a thin film or a wire

" G-L result '

For measurements on films with Width ≥ λ

J_c turns out much smaller than expected.

1. Thickness non uniform
2. Supercurrent piling up at the edges of the film

⟨ Remedy ⟩

i) Make the strip narrow enough

$$wd \ll \lambda^2$$

ii) use a ground plane geometry

iii) use a cylindrical film

(without edges)

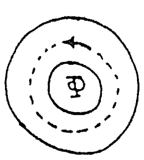
Flux Quantization

A multiply connected superconductor in the presence of a magnetic field.

Bohr - Sommerfeld quantization rule.

$$\oint \vec{p} \cdot d\vec{l} = nh$$

$$\oint \left(m^* \vec{v}_s + \frac{e^*}{c} \vec{A} \right) \cdot d\vec{l}$$



$$\xrightarrow[\text{S.C.}]{\text{inside the}} \frac{e^*}{c} \oint \vec{A} \cdot d\vec{l}$$

$$= \frac{e^*}{c} \Phi$$

$$= nh$$

$$\therefore \quad \Phi = n \frac{hc}{2e} = n \Phi_0 \quad \text{quantized}$$

an evidence of

pairing

$$\Phi_0 = 2.07 \times 10^{-7} \, G \cdot cm^2$$

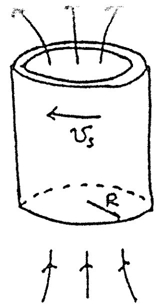
From the point of view of G.L theory $\psi = |\psi| e^{i\phi}$

Single-valuedness of complex s.c. order parameter

$$\oint \nabla \phi \cdot d\vec{l} = 2n\pi \, , \quad \text{Since} \quad m^* \vec{v}_s = \vec{p} - \frac{e^*}{c} \vec{A} = \hbar \nabla \phi - \frac{e^*}{c} \vec{A}$$

$$\oint \left(m^* \vec{v}_s + \frac{e^*}{c} \vec{A} \right) \cdot d\vec{l} = 2n\pi\hbar = nh$$

A shift in $T_c(H)$?



No distinction between H and the field inside the cylinder

$$\because \; |\psi| \to 0 \, , \quad J_s \to 0$$

$\Phi = \pi R^2 H$: flux

$\Phi' =$ fluxoid $= n \phi_0$

$$\oint m^* \vec{v}_s \cdot d\vec{l} + \frac{e^*}{c} \Phi = nh$$

$$m^* v_s \cdot 2\pi R$$

$$v_s = \frac{e^*}{m^* c} \frac{\Phi_0}{2\pi R} \left(n - \frac{\Phi}{\Phi_0} \right)$$

$$= \frac{\hbar}{m^* R} \left(n - \frac{\Phi}{\Phi_0} \right)$$

The energy of the currents minimum, for an integer n for which v_s is a minimum.

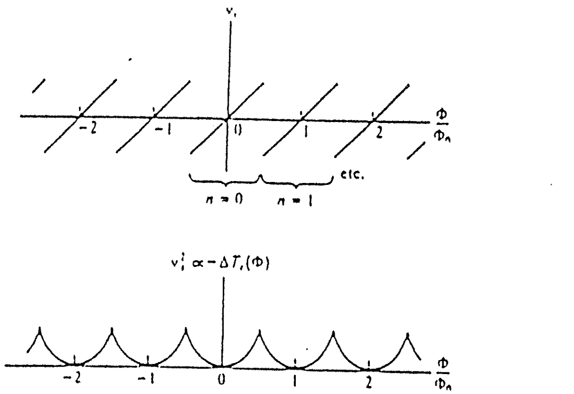


FIGURE 4.5
Variation of v_s and v_s^2 with flux threading the hollow cylinder in the Little-Parks experiment. The depression of T_c, and hence the increase in resistance in the actual experiment, is proportional to v_s^2 and thus displays the scalloped shape of the lower curve.

The previous result:

$$|\psi|^2 = \psi_\infty^2 \left\{ 1 - \left(\frac{\xi m^* v_s}{\hbar} \right)^2 \right\}$$

the transition at $|\psi|^2 = 0$

$$\frac{1}{\xi^2(T)} = \left(\frac{m^* v_s}{\hbar} \right)^2$$

$$= \frac{1}{R^2} \left(n - \frac{\Phi}{\Phi_0} \right)^2 \sim 1 - \frac{T_c(H)}{T_c} = \frac{\Delta T_c(H)}{T_c}$$

maximum depression of T_c at $n - \frac{\Phi}{\Phi_0} = \frac{1}{2}$

$$\frac{\Delta T_c(H)}{T_c} = \begin{cases} 0.55 \, \frac{\xi_0^2}{R^2} \left(n - \frac{\Phi}{\Phi_0} \right)^2 & : \text{clean} \\ 0.73 \, \frac{\xi_0 \ell}{R^2} \left(n - \frac{\Phi}{\Phi_0} \right)^2 & : \text{dirty} \end{cases}$$

$$(\Delta T_c)_{max} \simeq 0.8 \times 10^{-3} \, T_c \simeq 3 \, mK$$

$\xi_0 = 2 \times 10^{-5}$ cm, $\ell = 10^{-6}$ cm, $R = 7 \times 10^{-5}$ cm

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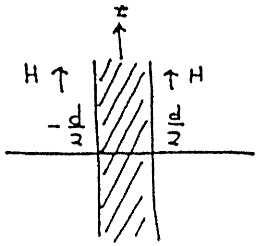
~~051-901 5906 [illegible]~~

$H_c^{\parallel}$ of thin Films:



$d < \lambda$: neglect screening

$|\psi|$ and φ constant

$$A_y(x) = \int_0^x h(x') \, dx' \simeq Hx$$

applied field

$$\vec{v}_s = \frac{1}{m^*} \left(-\frac{e^*}{c} \vec{A} \right) = -\frac{2e}{m^* c} Hx$$

Gibbs free energy per unit area of film

$$G = \int_{-d/2}^{d/2} \left(f - \frac{hH}{4\pi} \right) dx$$

$$= \int_{-d/2}^{d/2} \left\{ f_{no} + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{m^*}{2} \left(\frac{2eHx}{m^* c} \right)^2 |\psi|^2 + \frac{(h-H)^2}{8\pi} - \frac{H^2}{8\pi} \right.$$

$$= d \left(f_{no} + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 - \frac{H^2}{8\pi} \right) + \frac{e^2 d^3 H^2}{6 m^* c^2} |\psi|^2 + \int_{-d/2}^{d/2} \overset{0}{\cancel{\frac{(h-H)^2}{8\pi}}} \, dx$$

$h = H$ in weak screening limit

$$\frac{dG}{d|\psi|^2} = 0$$

$$\therefore \quad \alpha + \beta |\psi|^2 + \frac{e^2 d^3 H^2}{6 m^* c^2} = 0$$

$$|\Psi|^2 = \frac{|\alpha|}{\beta} - \frac{e^2 d^2 H^2}{6 m^* c^2 \beta}$$

$$= \Psi_\infty^2 \left(1 - \frac{e^2 d^2 H^2}{6 m^* c^2 \beta \, \Psi_\infty^2} \right)$$

$$= \Psi_\infty^2 \left(1 - \frac{e^2 d^2 H^2}{6 m^* c^2} \frac{m^{*2} c^4}{4\pi e^{*4} H_c^2 \lambda^4} \cdot \frac{4\pi e^{*2} \lambda^2}{m^* c^2} \right)$$

$$= \Psi_\infty^2 \left(1 - \frac{d^2 H^2}{24 H_c^2 \lambda^2} \right)$$

Critical field, $|\Psi|^2 = 0$

$$H_c'' = \left(\frac{24 H_c^2 \lambda^2}{d^2} \right)^{1/2} = 2\sqrt{6}\, H_c \lambda / d \gg H_c$$

$$\frac{|\Psi|^2}{\Psi_\infty^2} = 1 - \frac{H^2}{(H_c'')^2}$$: 2nd order transition

∴ Energy gap (or $|\Psi|$) 가 H 에 의해 연속적으로 줄어든다.

- Early electron tunneling experiment 에 의해 confirm 되다.

∘ 최근 실험

$H \approx 0.95 H_{c\parallel}$ 에서 gapless 가 된다.

gap 은 없는데 초전도 이다. Gapless superconductor

Abrikosov 와 Gorkov 가 처음 제안

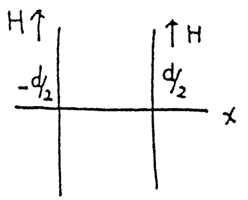
Time-reversal noninvariant perturbation (typically magnetic)

4.6.1. Thick films

2nd order phase transition이 어디서 깨지나?

∘ Screening present : $d \gtrsim \lambda$

∘ So long as $d \ll \xi$, $|\Psi| \sim$ Constant over the film



$$h = H \frac{\cosh\{x/\lambda_{eff}(H)\}}{\cosh\{d/2\lambda_{eff}(H)\}}$$

where $\lambda_{eff} = \dfrac{m^* c^2}{4\pi e^{*2} |\Psi|^2}$

$$A_y = \int h(x')\,dx'$$
$$= H \lambda_{eff}(H) \frac{\sinh\{x/\lambda_{eff}(H)\}}{\cosh\{d/2\lambda_{eff}(H)\}}$$

$$\frac{dG}{d|\Psi|^2} = 0$$

$$d(\alpha + \beta|\Psi|^2) + \frac{2e^2 H^2 \lambda_{eff}^2}{m^* c^2} \underbrace{\int_{-d/2}^{d/2} \frac{\sinh^2\{x/\lambda_{eff}(H)\}}{\cosh^2\{d/2\lambda_{eff}(H)\}}}_{\frac{\frac{\lambda_{eff}}{2}\sinh\left(\frac{d}{\lambda_{eff}}\right) - \frac{d}{2}}{\cosh^2\{d/2\lambda_{eff}(H)\}}} dx = 0$$

G 의 minimum 찾자.

2-dimension의 maximum

$$d_{max,\,2d\text{-}order} = \sqrt{5}\,\lambda$$

4.7. The Linearized GL equation.

Nucleation in Bulk Samples : H_{c2}

$$\Psi_\infty^2 = -\frac{\alpha}{\beta}.$$

$$\therefore \beta|\Psi|^2\Psi \sim \alpha\Psi \quad \text{when } \Psi \simeq \Psi_\infty$$

linearized GL eq. :

Ψ reduced much from Ψ_∞ due to magnetic field.

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)^2\Psi = 0$$

OR

$$\left(-i\nabla - \frac{2\pi\vec{A}}{\Phi_0}\right)^2\Psi = -\frac{2m^*\alpha}{\hbar^2}\Psi$$
$$= \frac{\Psi}{\xi^2(T)}$$

Schrödinger eq. of a free particle of mass m^*,

Charge e^* in a field $\vec{h} = \nabla\times\vec{A}$, with

$|\alpha|$, the energy eigenvalue.

Put $\vec{A} = \vec{A}_{ext}$, Since screening effect $\sim |\Psi|^2$

$$\vec{J} = e^*|\Psi|^2\vec{U}_s$$

decoupled from the 1st eq.

which governs Ψ

Consider a bulk (infinite) S.C. :

$$\vec{H} = H\hat{z}$$

$$A_y = Hx \quad , \quad A_x = A_z = 0$$

$$\left(-i\nabla - \frac{2\pi\vec{A}}{\Phi_0}\right)^2\Psi = \frac{\Psi}{\xi^2(T)}$$

$$-\frac{\partial^2\Psi}{\partial x^2} - \frac{\partial^2\Psi}{\partial z^2} + \left(-i\frac{\partial}{\partial y} - \frac{2\pi Hx}{\Phi_0}\right)^2\Psi = \frac{\Psi}{\xi^2(T)}$$

OR

$$\left\{-\nabla^2 + \frac{4\pi i}{\Phi_0}Hx\frac{\partial}{\partial y} + \left(\frac{2\pi H}{\Phi_0}\right)^2x^2\right\}\Psi = \frac{\Psi}{\xi^2(T)}$$

Since the effective Pot. depends only on x,

$$\Psi = e^{ik_y y}\cdot e^{ik_0 z} f(x)$$

then

$$-f''(x) + k_z^2 f(x) + \left(k_y - \frac{2\pi Hx}{\Phi_0}\right)^2 f(x) = \frac{f(x)}{\xi^2}$$

OR

$$-f''(x) + \left(\frac{2\pi H}{\Phi_0}\right)^2\left(x - \frac{\Phi_0}{2\pi H}k_y\right)^2 f(x) = \left(\frac{1}{\xi^2} - k_z^2\right)f(x)$$

$$-f''(x) + \left(\frac{2\pi H}{\Phi_0}\right)^2 (x - \underbrace{\frac{\Phi_0}{2\pi H} k_y}_{x_0})^2 f(x) = \left(\frac{1}{\xi^2} - k_z^2\right) f(x)$$

$\frac{\hbar^2}{2m}\ \frac{7}{\upsilon}$. : Schrödinger eq. of charged particle of mass m^*
in a magnetic field $\rightarrow$ Landau levels.

cf. Schrödinger eq. for a harmonic Osc.

$$-\frac{\hbar^2}{2m^*}\nabla^2\psi + \frac{1}{2}k x^2\psi = \varepsilon\psi$$

$$\frac{\hbar^2}{2m^*}\left(\frac{2\pi H}{\Phi_0}\right)^2 \longleftrightarrow \frac{1}{2}k = \frac{1}{2}m^*\omega_c^2$$

$$\omega_c = \frac{2\pi\hbar H}{\Phi_0 m^*} = \frac{e^* H}{m^* c}$$

the eigenvalue

$$\varepsilon_n = (n+\tfrac{1}{2})\hbar\omega_c$$
$$= (n+\tfrac{1}{2})\hbar\left(\frac{e^* H}{m^* c}\right)$$
$$= \left(\frac{1}{\xi^2} - k_z^2\right)\frac{\hbar^2}{2m^*}$$

Therefore

$$H = \frac{\hbar c}{2(2e)(n+\frac{1}{2})}\left(\frac{1}{\xi^2} - k_z^2\right)$$
$$= \frac{\Phi_0}{4\pi(n+\frac{1}{2})}\left(\frac{1}{\xi^2} - k_z^2\right)$$

$$H_{c_2} \longrightarrow \left|\begin{array}{l} k_z = 0 \\ n = 0 \end{array}\right.$$

H_{max} allowed for a superconducting State.

$$H_{c_2} = \frac{\Phi_0}{2\pi\xi^2}$$
$$= \sqrt{2}\kappa H_c$$

The highest field at which S.C. can nucleate in the interior of a large sample in a decreasing external field.

$$H_c = \frac{\Phi_0}{\sqrt{2}\,2\pi\xi\lambda} \qquad \kappa = \frac{\lambda}{\xi}$$

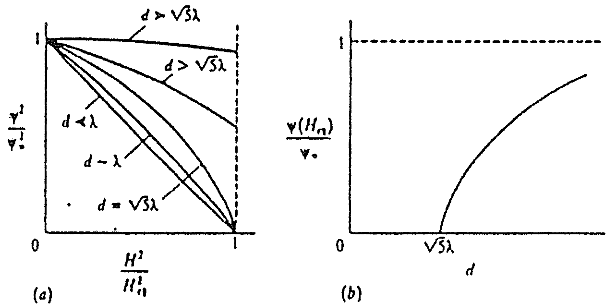


FIGURE 4.6
Dependence of ψ on the magnetic field for various film thicknesses. The size of the discontinuity of ψ at the first-order transition for thickness $d > \sqrt{3}\lambda$ is shown in (b). It is assumed that $d \ll \xi(T)$ throughout.

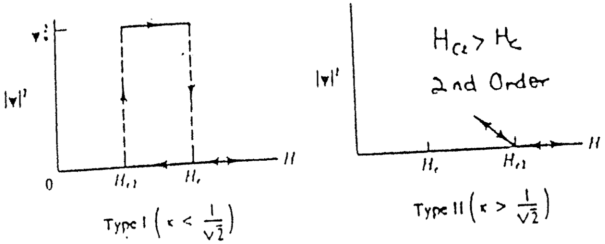


FIGURE 4.7
Contrast of behavior of order parameter at H_{c2} in type I and type II superconductors. Note hysteretic behavior with type I and reversible behavior with type II.

type I. $\quad H_{c_2} < H_c \quad (\kappa < \frac{1}{\sqrt{2}})$

Supercooling below H_c

1st order phase transition.

Corresponding eigen function (with $k_z = 0$)

$$- f''(x) + \left(\frac{2\pi H}{\phi_0}\right)^2 (x - x_0)^2 f(x) = \frac{1}{\xi^2} f(x)$$

$$H = \frac{\phi_0}{2\pi \xi^2}$$

$$- f''(x) + \frac{(x - x_0)^2}{\xi^4} f(x) = \frac{1}{\xi^2} f(x)$$

Solution

$$f(x) = \exp\left\{ - \frac{(x - x_0)^2}{2\, \xi^2(T)} \right\}$$

$$f'(x) = - \frac{(x - x_0)^2}{\xi^2(T)} f(x)$$

$$f'' = - \frac{1}{\xi^2} f(x) + \frac{(x - x_0)^2}{\xi^4} f(x)$$

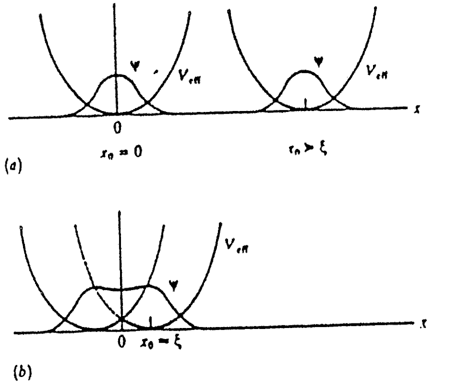
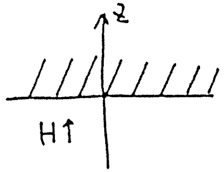


FIGURE 4.8
(a) Surface and interior nucleation at H_{c2}. (b) Surface nucleation at H_{c3}.

Surface Superconductivity

linearized GL eq: $\quad \left(-i\nabla - \frac{2\pi \vec{A}}{\phi_0}\right)^2 \psi = \frac{\psi}{\xi^2}$

B.C. $\quad \left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)\psi\Big|_n = 0$



1) $H \perp$ surface of S.C.

$A_y = Hx, \quad A_x = A_z = 0$

$$- \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} + \left(-i\frac{\partial}{\partial y} - \frac{2\pi H x}{\phi_0}\right)^2 \psi = \frac{\psi}{\xi^2}$$

b.c. $\frac{\partial \psi}{\partial z} = 0 \quad$ at $\quad z = 0$

Solution $\psi \sim f(x)\, e \quad \cos k_z z$

$$-f''(x) + k_z^2 f(x) + \left(k_y - \frac{2\pi H x}{\Phi_0}\right)^2 f(x) = \frac{f}{\xi^2}$$

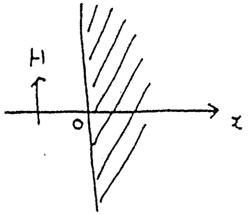
the same form as the previous one.

$$H = \frac{\Phi_0}{4\pi(n+\frac{1}{2})}\left(\frac{1}{\xi^2} - k_z^2\right)$$

$$H_{c_3} = H\,(k_z = 0,\ n = 0) = \frac{\Phi_c}{2\pi\xi^2} = H_{c_2}$$

Hence, for a field ⊥ the boundary, the presence of the surface does not modify the nucleation field.

2) H ∥ Surface of S.C.



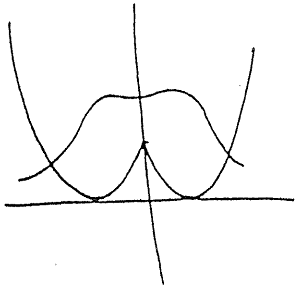
$A_y = Hx, \quad A_x = A_z = 0$

Sol. $\psi = e^{ik_y y} \cdot e^{ik_z z} f(x)$

b.c. $\frac{\partial\psi}{\partial x} = 0$ at $x = 0$

$$f''(x) + \left(\frac{2\pi H}{\Phi_0}\right)^2 (x - x_0)^2 f(x) = \left(\frac{1}{\xi^2} - k_z^2\right) f$$

$$= \frac{1}{\xi^2} f \qquad k_z = 0 \text{ for lowest eigenvalue}$$



- V_{eff} : broader lowest eigenvalue
- $H_{c_3} = 1.695\ H_{c_2}$

Remarks.

i) normal metal plating the S.C. surface

→ Suppress the surface s.c.

→

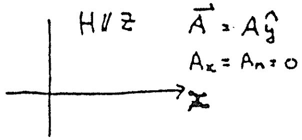
pairs formed at the surface diffuse into the normal layer

ii) Type I w/o metal plating

H_{c_3} → Supercooling field

Type II mat. without metal plating

H_{c_3} → Condensation field

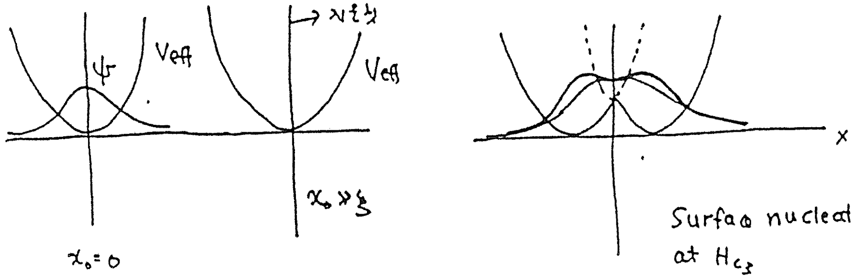


만약 Insulator interface 가 있다면

H_{c_3}는 어떻게 되나:

$$\left(\frac{\nabla}{i} - \frac{2\pi\vec{A}}{\Phi_0}\right)\psi\Big|_n = 0$$

$A_x = A_n = 0 \qquad \therefore \frac{\partial\psi}{\partial x}\Big|_{Surface} = \frac{df}{dx}\Big|_{Surface} = 0$



Surface nucleation at H_{c_3}

정확한 계산 $H_{c_3} = \frac{1.}{0.59} H_{c_2} = 1.695 H_{c_2} = 1.695(\sqrt{2}\kappa H_c)$

Variational Approach to find H_{c3} : Variation으로 구해보자.

trial func. $\psi \doteq e^{-ax^2} e^{ik_y y}$, $k_z = 0$

B.C. is automatically saticfies at $x = 0$

a and k_y to be determined to minimize G

$$G - G_n = \int_0^\infty \left\{ \alpha |\psi|^2 + \frac{1}{2m^*} \left| \left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)\psi \right|^2 \right\} dx$$

$$= \frac{\hbar^2}{2m^*} \int_0^\infty \left\{ -\frac{|\psi|^2}{\xi^2} + \left| \left(-i\nabla - \frac{2\pi}{\phi_0}\vec{A}\right)\psi \right|^2 \right\} dx$$

$A_x = A_z = 0$, $A_y = Hx$

$$\left| \left(-i\nabla - \frac{2\pi}{\phi_0}\vec{A}\right)\psi \right|^2$$

$$= \left|\frac{\partial\psi}{\partial x}\right|^2 + \left|\frac{\partial\psi}{\partial z}\right|^2 + \left| i\frac{\partial\psi}{\partial y} + \frac{2\pi A_y}{\phi_0}\psi \right|^2$$

$$G - G_n = \frac{\hbar^2}{2m^*} \int_0^\infty \left\{ -\frac{1}{\xi^2} + (2ax)^2 + \underbrace{\left(k_y - \frac{2\pi Hx}{\phi_0}\right)^2}_{\left(\frac{2\pi H}{\phi_0}\right)^2 \left(x - \frac{\phi_0 k_y}{2\pi H}\right)^2 = \left(\frac{2\pi H}{\phi_0}\right)^2 (x - x_0)^2} \right\} e^{-2ax^2} dx$$

$$= \frac{\hbar^2}{2m^*} \left[\left(\frac{\pi}{2a}\right)^{1/2} \left\{ -\frac{1}{2\xi^2} + \frac{a}{2} + \left(\frac{2\pi H}{\phi_0}\right)^2 \left(\frac{1}{8a} + \frac{x_0^2}{2}\right) \right\} + \left(\frac{2\pi H}{\phi_0}\right)^2 \left(-\frac{x_0}{2a}\right) \right]$$

$$\delta(G - G_n) = \frac{\partial}{\partial a}(G - G_n)\,\delta a = 0$$

$$\left(\frac{\pi}{2}\right)^{1/2}\left(-\frac{1}{2}\right) a^{-\frac{3}{2}} \left\{ -\frac{1}{2\xi^2} + \frac{a}{2} + \left(\frac{2\pi H}{\phi_0}\right)^2 \left(\frac{1}{8a} + \frac{x_0^2}{2}\right) \right\}$$

$$+ \left(\frac{\pi}{2a}\right)^{\frac{1}{2}} \left\{ \frac{1}{2} + \left(\frac{2\pi H}{\phi_0}\right)^2 \left(-\frac{1}{8a^2}\right) \right\} + \left(\frac{2\pi H}{\phi_0}\right)^2 \cdot \frac{x_0}{2a^2} = 0$$

oR

$$\frac{1}{\xi^2} + a - \left(\frac{\pi H}{\phi_0}\right)^2 \left\{ \frac{3}{a} + 4x_0^2 - 8\cdot\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{x_0}{a^{1/2}} \right\} = 0 \quad \cdots ①$$

한편 H_{c3} 에서 $G - G_n = 0$

$$-\frac{1}{\xi^2} + a + \left(\frac{\pi H}{\phi_0}\right)^2 \left(\frac{1}{a} + 4x_0^2 - \frac{4x_0}{a^{1/2}}\left(\frac{2}{\pi}\right)^{1/2} \right) = 0 \quad \cdots ②$$

From ① and ②, we obtain

$$a = \frac{1}{2\xi^2}$$

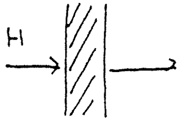
$$x_0 = \frac{\xi}{\sqrt{\pi}} \qquad \longrightarrow H_{c3} = \left(\frac{\pi}{\pi - 2}\right)^{1/2} \frac{\phi_0}{2\pi\xi^2}$$

$$= 1.16 H_{c2}$$

Angular Dependence of Nucleation Critical Field in thin film:

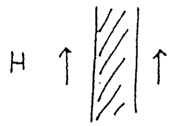
$$H_{c\perp} = H_{c_2}$$

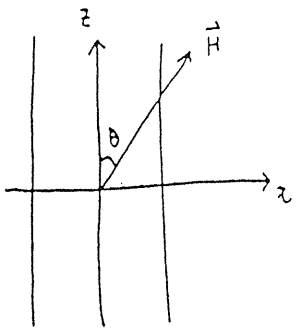
$$(= \sqrt{2}\, K(d)\, H_c)$$



$$H_{c\parallel} = 2\sqrt{6}\, H_c \cdot \frac{\lambda}{d}$$

$\gg H_{c\perp}$, Very thin films





$$A_y = H\,(x\cos\theta - z\sin\theta)$$

linearized GL eq.:

$$\left(-i\nabla - \frac{2\pi\vec{A}}{\phi_0} \right)^2 \psi = \frac{\psi}{\xi^2}$$

B.C. $\left(-i\nabla - \frac{2\pi\vec{A}}{\phi_0} \right) \psi \Big|_n = 0$

ψ : independent of y ($\because$ y not in the diff. eq.)

ψ is independent of x for $d \ll \xi$, the b.c. is atomatically satisfied.

$\psi = \psi(z)$, then

$$-\frac{1}{\xi^2}\psi - \frac{d^2\psi}{dz^2} + \left(\frac{2\pi H}{\phi_0}\right)^2 (x\cos\theta - z\sin\theta)^2 \psi = 0$$

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = d^2/12$$

$$-\frac{d^2\psi}{dz^2} + \left(\frac{2\pi H\sin\theta}{\phi_0}\right)^2 z^2 \psi = \left\{ \frac{1}{\xi^2} + \left(\frac{2\pi H}{\phi_0}\right)^2 \left(2\langle x\rangle \sin\theta\cos\theta - \langle x^2\rangle \cos^2\theta \right\} \psi$$

$$= \left\{ \frac{1}{\xi^2} - \left(\frac{2\pi H}{\phi_0}\right)^2 \cdot \frac{d^2}{12} \cos^2\theta \right\} \psi \quad \cdots ①$$

$$\omega_c = \frac{2\pi\hbar H\sin\theta}{m^*\phi_0}$$

$$\varepsilon_n = (n+\tfrac{1}{2})\hbar\omega_c = (n+\tfrac{1}{2}) \frac{2\pi\hbar^2 H\sin\theta}{m^*\phi_0}$$

$$= \left\{ \frac{1}{\xi^2} - \left(\frac{\pi H d\cos\theta}{\sqrt{3}\,\phi_0} \right)^2 \right\} \frac{\hbar^2}{2m^*}$$

Critical field $\longleftrightarrow$ $n=0$

$$\frac{1}{\xi^2} - \left(\frac{\pi H d\cos\theta}{\sqrt{3}\,\phi_0} \right)^2 = \frac{2\pi H\sin\theta}{\phi_0} \quad \cdots\cdots ②$$

(2) → (1)

$$-\frac{d^2\psi}{dz^2} + \left(\frac{2\pi H \sin\theta}{\phi_0}\right)^2 z^2 \psi = \left(\frac{2\pi H \sin\theta}{\phi_0}\right)\psi$$

$$\psi \sim \exp\left(-\frac{\pi H z^2 \sin\theta}{\phi_0}\right)$$

$$\frac{d\psi}{dz} \sim -\frac{2\pi H z \sin\theta}{\phi_0}\psi$$

$$\frac{d^2\psi}{dz^2} \sim \frac{-2\pi H \sin\theta}{\phi_0}\psi + \left(\frac{2\pi H z \sin\theta}{\phi_0}\right)^2\psi$$

In eq (2),

using $H_{c_2} = H_{c\perp} = \phi_0/2\pi\xi^2$

$$\frac{2\pi H_{c\perp}}{\phi_0} - \left(\frac{\pi H d \cos\theta}{\sqrt{3}\,\phi_0}\right)^2 = \frac{2\pi H \sin\theta}{\phi_0}$$

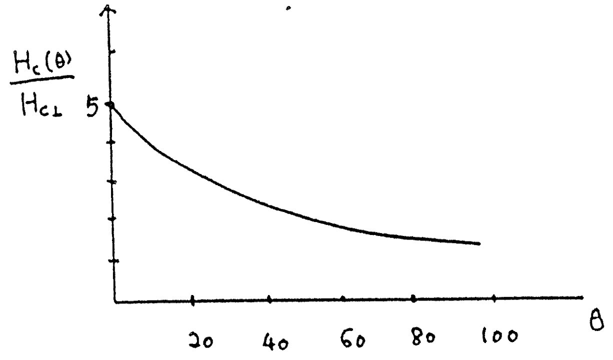
OR $$H_{c\perp} - \frac{\pi H^2 d^2 \cos^2\theta}{6\phi_0} = H \sin\theta$$

$\therefore$ $$H_{c\perp} - \frac{\pi H_{c\parallel}^2 d^2}{6\phi_0} = 0 \quad \rightarrow \quad \phi_0 = \frac{\pi d^2 H_{c\parallel}^2}{6 H_{c\perp}}$$

$$H_{c\perp} - H_c^2 \cos^2\theta \cdot \frac{H_{c\perp}}{H_{c\parallel}^2} = H_c \sin\theta$$

OR $$\frac{H_c(\theta)\sin\theta}{H_{c\perp}} + \left(\frac{H_c(\theta)\cos\theta}{H_{c\parallel}}\right)^2 = 1$$

Suppose $\frac{H_{c\parallel}}{H_{c\perp}} = 5$



$$\frac{H_c(\theta)\sin\theta}{H_{c\perp}} + \left(\frac{H_c(\theta)\cos\theta}{H_{c\parallel}}\right)^2 = 1$$

Nucleation in Thicker Films :

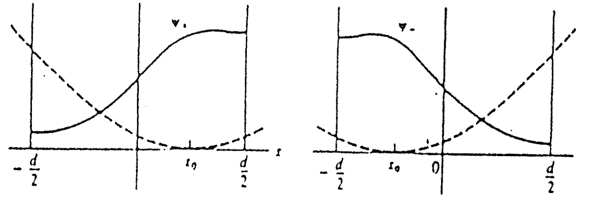


FIGURE 4.9
The two equivalent asymmetric solutions for nucleation in a film of intermediate thickness. The dashed curves centred at $\pm x_0$ represent the effective potential due to the magnetic field.

만약 $d \ll \xi$ 이면 ψ는 x에 무관하다.

그러나 만약 두꺼워 지면 x 의존성을 생각해야 한다.

$$1 + c \cdot \cos \frac{2\pi x}{d}$$

$$\frac{d\psi}{dx} = 0 \quad \text{at} \quad x = \pm \frac{d}{2}$$

이경우 variational method로 $H_{c\parallel}$ 구해보면

$$H_{c\parallel} = \frac{2\sqrt{6}\, H_c \lambda}{d} \left(1 + \frac{9d^2}{\pi^6 \xi^2} \right)$$

$d < d_c$ 이면 lowest solution은 $x_0 = 0$ $(k_y = 0)$ 에서 나타난다

$d > d_c$ numerical calculation으로 구한다.

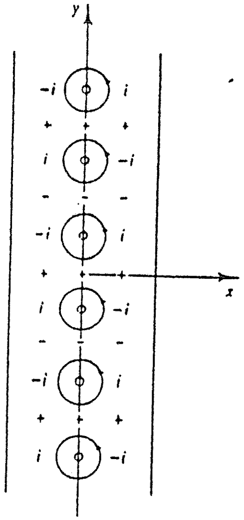


FIGURE 4.10
Vortex pattern in superconducting film of intermediate thickness set up by superposition of the two asymmetric solutions of Fig. 4.9. Notations $\pm$, $\pm i$ denote phase factor $e^{i\varphi}$ of ψ. Arrows indicate $\nabla\varphi$, to which J_s is proportional.

$d \geq d_c$, for $H \leq H_{c3}$

$$\psi = \psi_+ + \psi_- \quad : \text{due to interference of two solution}$$

$$= e^{ik_y y} f(x) + e^{-ik_y y} f(-x)$$

$$= \cos k_y y \, \{ f(x) + f(-x) \} + i \sin k_y y \, \{ f(x) - f(-x) \}$$

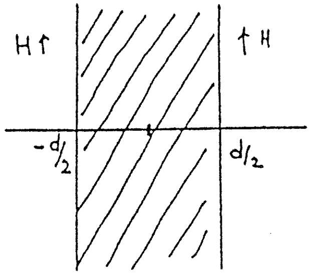
nodes along the midplane $(x=0)$, at intervals

$$\Delta y = \frac{\pi}{k_y} = \frac{\phi_0}{2x_0 H} \approx \frac{\Phi_0}{H(d-d_c)}$$

Thicker Films :

- Screening Present : $d \gtrsim \lambda$
- So long as $d \ll \xi$, $|\psi| \sim$ Constant over the film

$$h = H \frac{\cosh\{x/\lambda_{eff}(H)\}}{\cosh\{d/2\lambda_{eff}(H)\}}$$



$$\text{where } \lambda_{eff} = \frac{m^{*}c^{2}}{4\pi e^{*2}|\psi|^{2}}$$

$$A_y = \int h(x')\,dx'$$

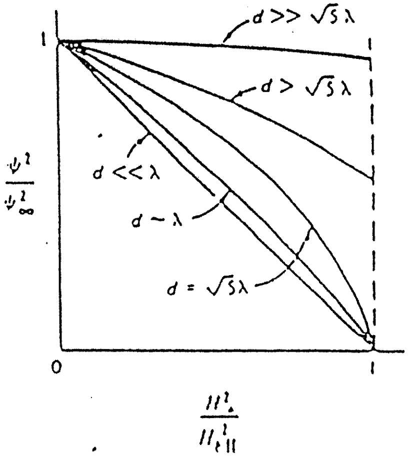
$$= H\lambda_{eff}(H)\frac{\sinh\{x/\lambda_{eff}(H)\}}{\cosh\{d/2\lambda_{eff}(H)\}}$$

$$\frac{dG}{d|\psi|^{2}} = 0 \qquad \text{then } (4\text{-}44)$$

$$G = \int_{-d/2}^{d/2}(f - Hh/4\pi)\,dx$$

$$= d\,[f_{no} + \alpha|\psi|^{2} + \frac{\beta}{2}|\psi|^{4} - \frac{H^{2}}{8\pi}] + \frac{e^{2}d^{3}H^{2}}{6m^{*}c^{2}}|\psi|^{2} + \nearrow$$

$$0 = d(\alpha + \beta|\psi|^{2}) + \frac{2e^{2}H^{2}\lambda_{eff}^{2}}{m^{*}c^{2}}\underbrace{\int_{-d/2}^{d/2}\frac{\sinh^{2}\{x/\lambda_{eff}(H)\}}{\cosh^{2}\{d/2\lambda_{eff}(H)\}}\,dx}_{\frac{\frac{\lambda_{eff}}{2}\sinh(\frac{d}{\lambda_{eff}}) - d/2}{\cosh^{2}\{d/2\lambda_{eff}(H)\}}}$$



$d > \sqrt{5}\lambda$
1st order transition

$d < \sqrt{5}\lambda$
2nd order transition

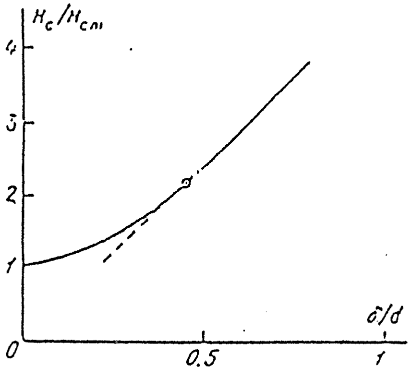
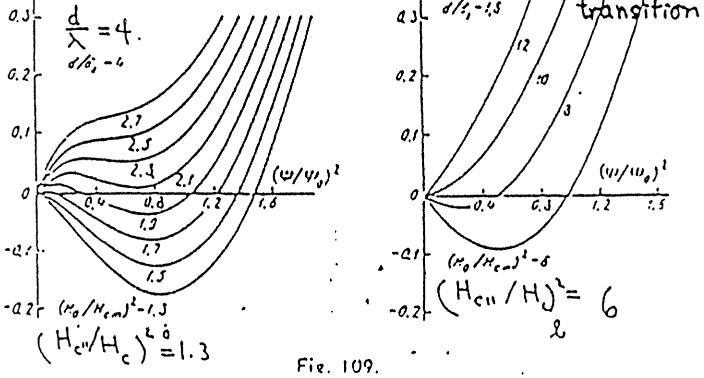


Fig. 108.

1st order phase transition. $d/\lambda = 1.6$ 2nd order phase transition.



$(H_{c\parallel}/H_c)^2 \doteq 1.3$

$(H_{c\parallel}/H)_l^2 = 6$

Fig. 109.

Abrikosov Vortex state at H_{c2}.

$\exists$ infinite # of interior solutions at H_{c2}, of the form

$$\Psi_k = e^{ik_y y} f(x)$$

$$= e^{ik_y y} e^{-(x-x_k)^2/2\xi^2}$$

with $x_k = \frac{k\phi_0}{2\pi H}$

- Each representing a gaussian slice of S.C. at the plane $x = x_k$
- all giving the same H_{c2}.
- expect a crystalline array of vortices.

$$K_n = ng \longrightarrow \Delta y = \frac{2\pi}{g}$$

$$X_n = \frac{k_n\phi_0}{2\pi H} = \frac{ng\phi_0}{2\pi H} \longrightarrow \Delta x = \frac{g\phi_0}{2\pi H}$$

$\therefore$ $\Delta x \cdot \Delta y \cdot H = \phi_0$ at $H = H_{c2}$

also true that

$\Delta x \cdot \Delta y \cdot B = \phi_0$ at $H < H_{c2}$

A general solution (to GL. eq. at H_{c2})

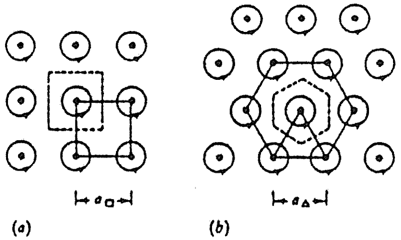
$$\Psi = \sum C_n \Psi_n$$

$$= \sum_n C_n e^{ingy} e^{-(x-x_n)^2/2\xi^2}$$

. periodic in y

. periodic in x if $C_{n+\nu} = C_n$

ν : integer



energetically favorable

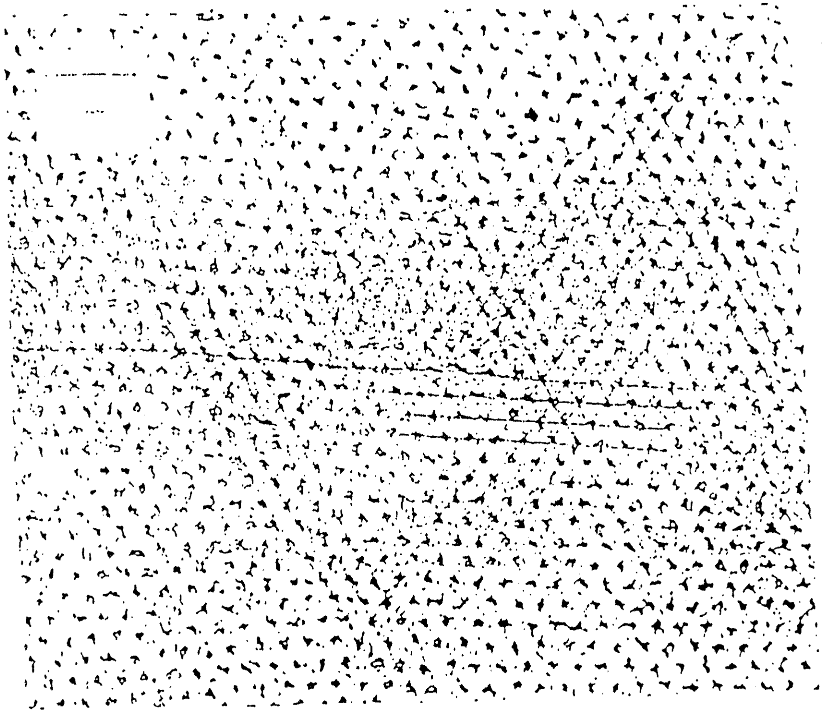
FIGURE 4.11
Schematic diagram of square and triangular vortex arrays. The dashed lines outline the basic unit cell.

$$\frac{\sqrt{3}}{2} a_\triangle^2 B = \phi_0$$

$$a_\triangle = \left(\frac{4}{3}\right)^{1/4} \left(\frac{\phi_0}{B}\right)^{1/2} = 1.075 \left(\frac{\phi_0}{B}\right)^{1/2}$$

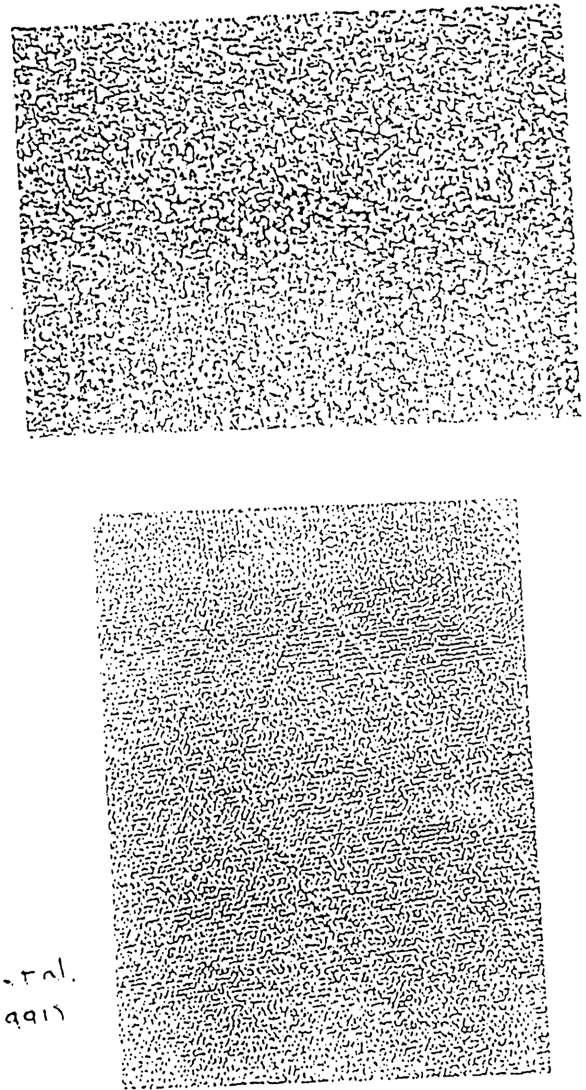
$$= 1.075\ a_\square$$

. Actual Vortex-lattice structure determined by int. with the underlying mat. lattice



PATTERN OF INDIVIDUAL FLUXONS IN A TYPE-II SUPERCONDUCTOR

This photograph shows the triangular pattern of fluxons in a type-II superconductor (see Chapter 12). The pattern is revealed by allowing very small (300 Å) ferromagnetic particles to settle on the surface of a magnetized specimen (lead–indium alloy). The particles locate themselves where the magnetic flux intersects the surface. The photograph was obtained by electron microscopy of the deposited particles. (Photograph by courtesy of U. Essmann and H. Träuble, Max Planck Institut für Metallforschung.)



Bishop et al.
(1991)

Fig. [illegible] Shown are two [illegible] different [illegible]

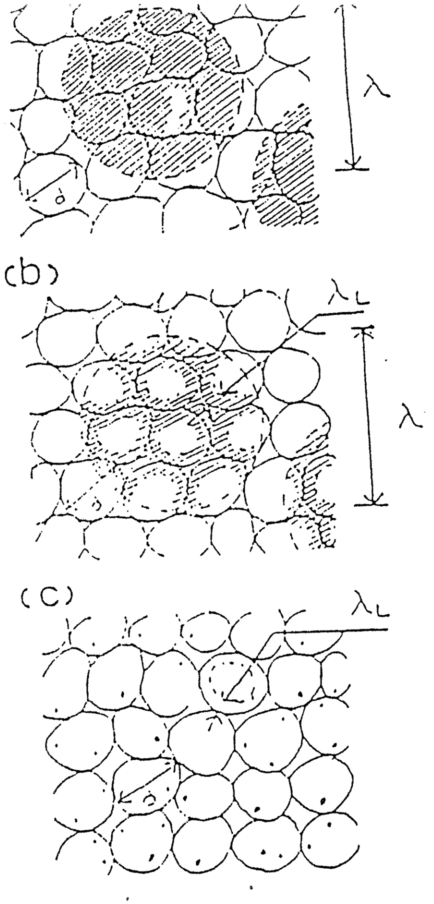


Fig. 4-5 A schematic picture of different low field vortices in Josephson medium for different conditions: (a) continuous hypervortex, (b) net-like hypervortex, and (c) Abrikosov vortex.

Chapter 5.

Magnetic Properties of classic type II Superconductors

previous chapter

$\kappa > \frac{1}{\sqrt{2}}$ 일때의 GL equation의 Sol.

$|\Psi| > 0$ until $H_{c2} (> H_c)$

1. Abrikosov 의 Solution 얻다.

Regular array of vortices of current surrounding nodal lines of Ψ

Each unit cell of the array carried total flux equal to $\Phi_0 = hc/2e$.

이번 장

type II 초전도체에서 H field $0 - H_{c2}$

Effect of the Lorentz force due to a transport current on the flux in a type II Superconductor.

electrical resistance associated with flux creep or flow.

Fluctuation - induced resistance to the case of relatively weak fluctuations

5.1. Behavior near H_{c_1} : The Structure of an isolated vortex.

H_{c_1}을 구해보자

$$G_s\Big|_{\text{no flux}} = G_s\Big|_{\text{first flux}}$$

$$G = F - \frac{H}{4\pi}\int h\, d^3r$$

$$F_s = F_s + \underset{\substack{\uparrow \\ \text{Vortex creation energy per unit length}}}{\epsilon_1 L} - \frac{H_{c_1}}{4\pi}\int h\, d^3r$$

$$= F_s + \epsilon_1 L - \frac{H_{c_1}}{4\pi}\phi_0 L$$

L : Length of the vortex line

$$H_{c_1} = \frac{4\pi\epsilon_1}{\phi_0}$$ ϵ_1을 알아야 한다.

Vortex wave function : $\psi = \psi_\infty f(r) e^{i\theta}$

$$\vec{A} = A(r)\hat{\theta} \qquad \nabla\times\vec{A} = \hat{e}_z \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)$$
$$= \vec{h}$$

$$A(r) = \frac{1}{r}\int_0^r r' h(r')\, dr'$$

near the vortex center : $A(r) = \frac{1}{r}h(0)\int_0^r r' dr'$

$$= r h(0)/2$$

far from the vortex,

$$\int \vec{h}\cdot d\vec{a} = \int \nabla\times\vec{A}\cdot d\vec{a}$$
$$= \oint \vec{A}\cdot d\vec{l} = A_\infty \cdot 2\pi r = \phi_0$$
$$A_\infty = \frac{\phi_0}{2\pi r}$$

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\underbrace{\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)\left(-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right)}_{-\hbar^2\nabla^2 + \frac{i\hbar e^*}{c}\psi\nabla\cdot\vec{A} + i\cdot\frac{2\hbar e^*\vec{A}}{c}\nabla\psi + \left(\frac{e^*}{c}\vec{A}\right)^2\psi}\psi = 0$$

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left\{-\frac{\hbar^2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) - \frac{\hbar^2}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + i\cdot\frac{2e^*\hbar}{c}\frac{A}{r}\frac{\partial\psi}{\partial\theta}\right.$$
$$\left. + \left(\frac{e^*A}{c}\right)^2\psi\right\} = 0$$

$$\alpha f + \beta\psi_\infty^2 f^3 + \frac{1}{2m^*}\left\{-\frac{\hbar^2}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) + \left(\frac{\hbar}{r} - \frac{e^*A}{c}\right)^2 f\right\} = 0$$

$$f - f^3 - \underbrace{\frac{\hbar^2}{2m^*|\alpha|}}_{\xi^2}\left\{-\frac{\hbar^2}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) + \left(\frac{\hbar}{r} - \frac{e^*A}{c}\right)^2 f\right\} = 0 \qquad \cdots \text{①}$$

$$j \quad - \quad 4\pi$$

$$= \frac{c}{4\pi}\,\hat{\theta}\,.\left(-\frac{\partial h_z}{\partial r}\right)$$

$$= -\frac{c}{4\pi}\,\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\,rA(r)\right)\hat{\theta}$$

한편

$$\vec{j} = \frac{e^*}{m^*}\,|\psi|^2\,\left(\hbar\nabla\theta - \frac{e^*}{c}\vec{A}\right)$$

$$= \frac{e^*}{m^*}\,\psi_\infty^2 f^2\,\left(\frac{h}{r}\hat{\theta} - \frac{e^*}{c}A\,\hat{\theta}\right)$$

$$\therefore\ j_\theta = -\frac{c}{4\pi}\,\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\,rA(r)\right) = \frac{e^*\hbar}{m^*}\,\psi_\infty^2 f^2\left(\frac{1}{r} - \frac{2\pi A}{\phi_0}\right)$$

$$\cdots\ ②$$

Simultaneous solutions of eqs (1) and (2).

→ $f(r)$, $A(r)$: numerical method.

---

limiting cases : Solve analytically

i) $r \to 0$

$$f - f^3 - \xi^2\left\{\left(\frac{1}{r} - \frac{\pi h(0)}{\phi_0}r\right)^2 f - \frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right)\right\} = 0$$

assume : $f = cr^n \quad (n \geq 0)$

$$cr^n - c^3r^{3n} - \xi^2\left\{\left(\frac{1}{r} - \frac{\pi h(0)}{\phi_0}r\right)^2 cr^n - n^2cr^{n-2}\right\} = 0$$

leading term as $r \to 0$

$$:\ r^{n-2}(1-n^2) = 0$$

$$\therefore\ n = 1$$

Including the higher-order term,

$$f(r) = cr + dr^3$$

$$(cr + dr^3) - c^3r^3 - \xi^2\left\{\left(\frac{1}{r} - \frac{\pi h(0)}{\phi_0}r\right)^2(cr + dr^3) - \frac{c}{r} - 9dr\right\} = 0$$

OR

$$(cr + dr^3) - c^3r^3 - \xi^2\left\{\frac{c}{r} + dr - \frac{2\pi h(0)}{\phi_0}(cr + dr^3) + \left(\frac{\pi h(0)}{\phi_0}\right)^2 cr^3 - \frac{c}{r} - 9dr\right\} = 0$$

Coefficient of the leading term = 0

$$c + 2\pi\,\xi^2 h(0)\,c/\phi_0 + 8\,\xi^2 d = 0$$

OR

$$c + \frac{h(0)\,c}{H_{c_2}} + 8\,\xi^2 d = 0$$

$$\therefore\ d = -\frac{c}{8\xi^2}\left(1 + \frac{h(0)}{H_{c_2}}\right)$$

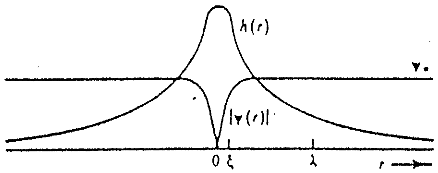


FIGURE 5.1
Structure of an isolated Abrikosov vortex in a material with $\kappa \approx 8$. The maximum value of $h(r)$ is approximately $2H_{c1}$.

$$\therefore\ f(r) = cr + dr^3$$

$$= cr\left\{1 - \frac{r^2}{8\xi^2}\left(1 + \frac{h(0)}{H_{c2}}\right)\right\}$$

$$\simeq \tanh\frac{\nu r}{\xi}$$

$f(r)$ starts to saturate at $r \approx 2\xi$, as might be expected.

$r \approx 2\xi$ 정도에서 saturate 된다

따라서

$C \sim \frac{1}{2\xi}$ 정도 여야 $f \to 1$ 로 saturate

Reasonable approximation to f over the entire range is

$$f \approx \tanh\frac{\nu r}{\xi}$$

where ν is a constant ~ 1

만약 $f = Cr + dr^2$ 이라 가정하면

$\underline{d=0}$

$$\because\ 0 = Cr + dr^2 - \xi^2\left\{\frac{c}{r} + d - \frac{2\pi h(0)}{\Phi_0}(cr + dr^2) - \frac{c}{r} - 4d\right\}$$

$\therefore\ d=0$

Actually only odd terms are retaines in the expansion of f.

5.1.1. The high K approximation

$\lambda >> \xi$ 즉 $\kappa >> 1$ 인 경우

Core를 넘어서면 $(r > \xi)$ 이면 $f \simeq 1$, in which case the London equations govern the field and Currents.

Thus outside the core

$$\frac{4\pi\lambda^2}{c}\nabla\times\vec{J}_s + \vec{h}_s = 0$$

Include core

$$\frac{4\pi\lambda^2}{c}\nabla\times\vec{J}_s + \vec{h} = \hat{z}\,\Phi_0\,\delta_2(\vec{r})$$

Maxwell eq. $\nabla\times\vec{h} = \frac{4\pi}{c}\vec{J}$

$$\lambda \quad \nabla\times\nabla\times h \quad + \quad h = \hat{z}\,\Phi_0 \delta_2(r)$$

Since $\text{div}\,\vec{h} = 0$, this can be written

$$\nabla^2\vec{h} - \frac{\vec{h}}{\lambda^2} = -\frac{\Phi_0}{\lambda^2}\,\hat{z}\,\delta_2(\vec{r})$$

This equation has the exact solution

$$h(\vec{r}) = \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right)$$

$$K_0\left(\frac{r}{\lambda}\right) \longrightarrow e^{-r/\lambda} \qquad r \to \infty$$

$$\ln(\lambda/r) \qquad r \to 0$$

그러나 실제로는 ξ 에 의해
제한을 받게 된다.

$$h(r) \rightarrow \frac{\Phi_0}{2\pi\lambda^2}\left(\frac{\pi}{2}\,\frac{\lambda}{r}\right)^{1/2} e^{-r/\lambda} \qquad r \to \infty$$

$$h(r) \rightarrow \frac{\Phi_0}{2\pi\lambda^2}\left[\ln\frac{\lambda}{r} + 0.12\right] \qquad \xi \ll r \ll \lambda$$

Finding H_{c_1} :

ϵ_1 = free energy of unit length of a vortex line neglecting the core.

$$= \frac{1}{8\pi}\int\left(h^2 + \lambda^2(\nabla\times\vec{h})^2\right)ds$$

$$\text{OR} \quad \frac{1}{2}m^* v_s^2$$

field energy, Kinetic energy of the Current

$$\frac{1}{2}m^* v_s^2 = \frac{1}{2}m^*\left(\frac{J_s}{e^*|\psi|}\right)^2$$

$$= \frac{1}{2}m^*\left(\frac{\frac{c}{4\pi}\,\nabla\times\vec{h}}{e^*|\psi|}\right)^2$$

$$= \frac{m^* c^2}{2\,(16\pi^2 e^{*2}|\psi|^2)}\,|\nabla\times\vec{h}|^2$$

$$= \frac{1}{8\pi}\lambda^2\,|\nabla\times\vec{h}|^2$$

$$= \frac{1}{8\pi}\int\left(h^2 + \lambda^2|\nabla\times\vec{h}|^2\right)ds$$

$$|\nabla\times\vec{h}|^2 = (\nabla\times\vec{h})\cdot(\nabla\times\vec{h})$$

$$= \vec{\alpha}\cdot(\nabla\times\vec{h})$$

$$\nabla\cdot(\vec{\alpha}\times\vec{h}) = \vec{h}\cdot(\nabla\times\vec{\alpha}) - \vec{\alpha}\cdot(\nabla\times\vec{h})$$

$$\therefore\ |\nabla\times\vec{h}|^2 = \vec{h}\cdot(\nabla\times\vec{\alpha}) - \nabla\cdot(\vec{\alpha}\times\vec{h})$$

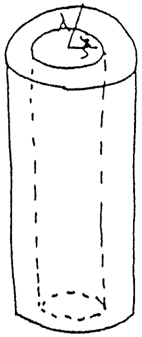
$$= \vec{h}\cdot\{\nabla\times(\nabla\times\vec{h})\} + \nabla\cdot\{\vec{h}\times(\nabla\times\vec{h})\}$$

$$\therefore\ \epsilon_1 = \frac{1}{8\pi}\int \left(h^2 + \lambda^2 |\nabla\times\vec{h}|^2\right) ds$$

$$= \frac{1}{8\pi}\int \{\vec{h} + \lambda^2 \nabla\times(\nabla\times\vec{h})\}\cdot\vec{h}\, ds$$

$$+ \frac{\lambda^2}{8\pi}\int \nabla\cdot\{\vec{h}\times(\nabla\times\vec{h})\}\, ds$$

$$= \underbrace{\overset{0}{\cancel{\frac{1}{8\pi}\int |h|\, \phi_0\, \delta_2(\vec{r})\, ds}}}_{\text{Since the core is excluded}} + \frac{\lambda^2}{8\pi}\oint \vec{h}\times(\nabla\times\vec{h})\cdot d\vec{s}$$



The surface integration is over the unit length of inner and outer surface.

$$\epsilon_1 = \frac{\lambda^2}{8\pi}\int h(r)\,\hat{e}_z \times \left(-\frac{dh}{dr}\right)\hat{e}_\theta \cdot d\vec{s}$$

$$= \frac{\lambda^2}{8\pi}\, h\,\frac{dh}{dr}\int \hat{e}_r\cdot d\vec{s}$$

$$= \frac{\lambda^2}{8\pi}\left[h\,\frac{dh}{dr}\cdot 2\pi r\right]_\xi$$

Now using $h(r) \simeq \frac{\phi_0}{2\pi\lambda^2}\ln\left(\frac{\lambda}{r}\right)$ $\quad \xi < r < \lambda$

$$\epsilon_1 = \frac{\lambda^2}{8\pi}\left(\frac{\phi_0}{2\pi\lambda^2}\right)^2\cdot 2\pi\ln\frac{\lambda}{\xi}$$

$$= \left(\frac{\phi_0}{4\pi\lambda}\right)^2\ln\frac{\lambda}{\xi} = \left(\frac{\phi_0}{4\pi\lambda}\right)^2\ln\kappa = \frac{\phi_0}{8\pi}\,h(r=\xi)$$

line energy is of the same order of magnitude as the condensation energy lost in the core, but it is larger by a factor of order $4\ln\kappa$.

OR using $H_c = \frac{\phi_0}{\sqrt{2}\,2\pi\xi\lambda}$

$$\epsilon_1 = \left(\frac{\phi_0}{4\pi\lambda}\right)^2\ln\kappa$$

$$= \frac{H_c^2}{8\pi}\,4\pi\xi^2\ln\kappa$$

~ Same order as the condensation energy

flux - penetration field H_{c_1}

" lower critical field "

$$H_{c_1} = \frac{4\pi}{\phi_0}\,\epsilon_1$$

$$= \frac{\phi_0}{4\pi\lambda^2}\ln\kappa$$

OR $= \frac{H_c}{\sqrt{2}\kappa}\ln\kappa \sim H_c/\kappa$

Interaction between Vortex Lines:

Energy of two vortex lines,

$$\vec{h} + \lambda^2 \nabla \times (\nabla \times \vec{h}) = \phi_0 \hat{e}_z \sum_i \delta_2(\vec{r} - \vec{r}_i)$$

$$|\vec{r}_1 - \vec{r}_2| \gg \xi(T)$$

$$\vec{h}(\vec{r}) = \vec{h}_1(\vec{r}) + \vec{h}_2(\vec{r})$$

$$h_1(\vec{r}) = \frac{\phi_0}{2\pi\lambda^2(T)} K_0\left(\frac{\vec{r} - \vec{r}_1}{\lambda(T)}\right)$$

$$\epsilon = \frac{1}{8\pi} \int (h^2 + \lambda^2 |\nabla \times \vec{h}|^2)\, ds$$

$$= \frac{\lambda^2(T)}{8\pi} \int \vec{h} \times (\nabla \times \vec{h}) \cdot d\vec{s}$$

The surface integral over the two surfaces of the

Core $\quad |\vec{r} - \vec{r}_1| \approx \xi(T)$ and $|\vec{r} - \vec{r}_2| \approx \xi(T)$

$$= \frac{\lambda^2}{8\pi} \int (d\vec{A}_1 + d\vec{A}_2) \cdot \{ (\vec{h}_1 + \vec{h}_2) \times (\nabla \times \vec{h}_1 + \nabla \times \vec{h}_2) \}$$

$$dA_1 \cdot \{ \vec{h}_1 \times (\nabla \times \vec{h}_1) + \vec{h}_1 \times (\nabla \times \vec{h}_2) + \vec{h}_2 \times (\nabla \times \vec{h}_1)$$
$$+ \vec{h}_2 \times (\nabla \times \vec{h}_2) \}$$

$$+ dA_2 \cdot \{ \qquad " \qquad \}$$

$$\epsilon = \frac{\lambda^2}{8\pi} \left\{ \int d\vec{s}_1 \cdot (\vec{h}_1 \times (\nabla \times \vec{h}_1)) + \int d\vec{s}_2 \cdot (\vec{h}_2 \times (\nabla \times \vec{h}_2)) \right\}$$
$$+ \frac{\lambda^2}{8\pi} \left\{ \int d\vec{s}_1 \cdot (\vec{h}_2 \times (\nabla \times \vec{h}_1)) + \int d\vec{s}_2 \cdot (\vec{h}_1 \times (\nabla \times \vec{h}_2)) \right\}$$
$$+ \frac{\lambda^2}{8\pi} \left\{ \underbrace{\int d\vec{s}_1 \cdot (\vec{h}_1 + \vec{h}_2) \times (\nabla \times \vec{h}_2) + \int d\vec{s}_2 \cdot (\vec{h}_1 + \vec{h}_2) \times (\nabla \times \vec{h}_1)}_{(\vec{h}_1 + \vec{h}_2) \cdot \{ (\nabla \times \vec{h}_2) \times d\vec{A}_1 + (\nabla \times \vec{h}_1) \times d\vec{A}_2 \}} \right\}$$

$$= \epsilon_{\substack{\text{Vortex} \\ \text{lines}}} + \epsilon_{\text{int}} + \epsilon_3$$

Order of magnitude 어림 $\qquad \nabla \times \vec{h} = -\hat{e}_\theta \frac{d\hat{h}(r)}{dr}$

$$\vec{h} = h(r)\, \hat{e}_z$$

$$\epsilon_{\text{v. lines}} \propto \ln(\lambda/\xi)$$

$$\epsilon_{\text{int}} \propto \ln \frac{\lambda}{|\vec{r}_1 - \vec{r}_2|} \qquad |\vec{r}_1 - \vec{r}_2| < \lambda$$

$$\epsilon_3 \sim \ln\left(\frac{\lambda}{\xi}\right) \frac{\xi}{|\vec{r}_1 - \vec{r}_2|} \ll \epsilon_{\text{int}}$$

$$\therefore \epsilon' = 2\epsilon_1 + 2\epsilon_{\text{int}}$$

$$= 2\frac{\phi_0}{8\pi} h_1(\vec{r}_1) + 2\frac{\phi_0}{8\pi} h_1(\vec{r}_2)$$

$$\epsilon = 2 \cdot \frac{\Phi_0}{8\pi} h_1(\vec{r}_1) + 2 \cdot \frac{\Phi_0}{8\pi} h_1(\vec{r}_2)$$

$$\frac{\lambda^2}{8\pi} \int d\vec{A}_1 \cdot \vec{h}_2 \times (\nabla \times \vec{h}_1)$$

$$= \frac{\lambda^2}{8\pi} h(|\vec{r}_1 - \vec{r}_2|) \underbrace{\frac{dh}{dr}\bigg|_{\xi}}_{\Phi_0/2\pi\lambda^2\xi} 2\pi\xi$$

$$= \frac{\Phi_0}{8\pi} \underbrace{h(|\vec{r}_1 - \vec{r}_2|)}_{h_1(\vec{r}_2) = h_2(\vec{r}_1)}$$

E_{12} ≡ total interaction energy

$$= \frac{\Phi_0}{4\pi} h_1(\vec{r}_2)$$

$$= \frac{\Phi_0}{4\pi} \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r_{12}}{\lambda}\right)$$: repulsive between vortices of the same sense.

$$\rightarrow \begin{cases} \dfrac{e^{-r_{12}/\lambda}}{\sqrt{r_{12}}} & r_{12} \gg \lambda \\ \ln\left(\dfrac{\lambda}{r_{12}}\right) & \xi \ll r_{12} \ll \lambda \end{cases}$$

Cf. In 2D thin films : $d \ll \lambda$

J. Pearl Appl. Physics letter 5, 65 (1964)

$$E_{12} = -\frac{\Phi_0^2}{8\pi\lambda_\perp} \left\{ \underset{\substack{\uparrow \\ \text{a Struve fnc.}}}{H_0\left(\frac{r}{\lambda_\perp}\right)} - \underset{\substack{\uparrow \\ \text{a Neuman fnc.}}}{Y_0\left(\frac{r}{\lambda_\perp}\right)} \right\}$$

$$= \begin{cases} -\dfrac{\Phi_0^2}{4\pi^2} \dfrac{1}{r} & r \gg \lambda_\perp \\ \dfrac{\Phi_0^2}{4\pi^2\lambda_\perp} \ln\left(\dfrac{r}{\xi}\right) & \xi \ll r \ll \lambda_\perp \end{cases}$$

where $\lambda_\perp$ = 2D penetration depth

$$= \frac{2\lambda^2}{d} \gg \lambda \quad \text{for } d \ll \lambda$$

Force between vortices

$$f_{2x} = -\frac{\partial E_{12}}{\partial x_2}$$

$$= \frac{-\phi_0}{4\pi} \frac{\partial}{\partial x_2} h_1(\vec{r}_2)$$

한편

$$\nabla \times \vec{h} = \frac{4\pi}{c}\vec{J}$$

$$-\hat{e}_y \frac{\partial h_{1z}(\vec{r}_2)}{\partial x_2} = \frac{4\pi}{c}\vec{J}$$

$$\therefore f_{2x} = -\frac{\phi_0}{4\pi}\left(-\frac{4\pi}{c} J_{1y}(\vec{r}_2)\right)$$

$$= \frac{\phi_0}{c} J_{1y}(\vec{r}_2)$$

$\vec{\Phi}_0 \parallel$ the direction of flux density

OR in a vector form

$\vec{f}_2 = \vec{J}_1(\vec{r}_2) \times \frac{\vec{\phi}_0}{c}$: force on a unit length of a Vortex line by a current density J, external or internal.

$\downarrow$

"Lorentz force"

Magnetization - Magnetization Curves

범위 : H_{c_1} 부터 H_{c_2}

i) $\frac{\phi_0}{B} \gg \lambda^2$ 즉 H_{c_1} 근처

Vortex는 λ 보다 훨씬 더 떨어져있다.

few nearest neighbors are important

ii) $\xi^2 \ll \frac{\phi_0}{B} \ll \lambda^2$

many vortices are within the interaction range. - More elaborated summing procedures are required.

It is still a good approximation to neglect details of the core.

3. $\xi^2 \simeq \frac{\Phi_0}{B}$, near H_{c_2}

Cores are almost overlapping.

Simple superposition technique is no longer accurate.

Abrikosov Solution to the linearized GL equation at H_{c2} is a helpful approximation.

Gibbs free energy per unit volume

$$G - G_{s0} = \underbrace{\frac{B}{\phi_0}}_{} \epsilon_1 + \sum_{i>j} E_{ij} - \frac{BH}{4\pi}$$

# of vortices per unit area ($\perp \vec{H}$)

i) if $H < H_{c1} = \frac{4\pi \epsilon_1}{\phi_0}$

$$\frac{BH}{4\pi} < \frac{B H_{c1}}{4\pi} = \frac{B}{\phi_0} \epsilon_1$$

$$\therefore \frac{B\epsilon_1}{\phi_0} - \frac{BH}{4\pi} > 0$$

$\sum_{i>j} E_{ij} \; (>0)$ $\therefore$ ~~Minim G~~

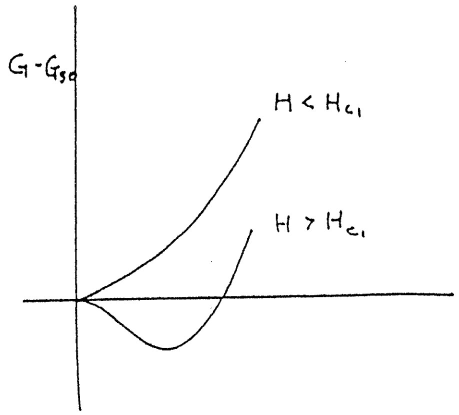
Minimum G 는 B=0 에서

ii) $H \geq H_{c1}$

$$\frac{\partial G}{\partial B} = 0 = \frac{\epsilon_1}{\phi_0} - \frac{H}{4\pi} + \frac{\partial}{\partial B} \sum_{i>j} E_{ij}$$

or

$$\frac{\partial}{\partial B} \sum_{ij} E_{ij} = \frac{H - H_{c1}}{4\pi}$$



1. low flux density

$$\frac{\phi_0}{B} \gg \lambda^2 \quad (\text{or } H \gtrsim H_{c1})$$

E_{ij} decrease exponentially as $r \uparrow$

nearest neighbor interaction only

$$a = c \left(\frac{\phi_0}{B} \right)^{1/2}$$

n.n. distance

$$c = \begin{cases} 1 & : \square \text{ array} \\ 1.075 & : \Delta \text{ array} \end{cases}$$

Z = Coordination #

$$\sum_{i>j} E_{ij} = \left(\frac{B}{\phi_0} \right) \frac{Z}{2} \frac{\phi_0^2}{8\pi^2\lambda^2} K_0\left(\frac{a}{\lambda} \right) \quad \text{per unit volume}$$

$$= \frac{BZ\phi_0}{16\pi^2\lambda^2} \sqrt{\frac{\pi}{2}\frac{\lambda}{a}} \; e^{-a/\lambda} \qquad a \gg \lambda$$

$a = a(B)$

$$\frac{\partial}{\partial B} \sum_{i>j} E_{ij} = \frac{Z\phi_0}{16\pi^2\lambda^2} \sqrt{\frac{\pi}{2}\frac{\lambda}{a}} \; e^{-\frac{a}{\lambda}} \left(1 - \frac{B}{2a}\frac{\partial a}{\partial B} - \frac{B}{\lambda}\frac{\partial a}{\partial B} \right)$$

where $\frac{\partial a}{\partial B} = c\sqrt{\phi_0} \left(-\frac{1}{2} B^{-\frac{3}{2}} \right) = -\frac{a}{2B}$

$$\therefore \frac{\partial}{\partial B}\sum_{i>j} E_{ij} = \left(\qquad \right)\left(1+\frac{1}{4}+\frac{a}{2\lambda}\right)$$

Hence

$$\frac{Z\phi_0}{16\pi^2\lambda^2}\left(\frac{\pi}{2}\frac{\lambda}{a}\right)^{1/2} e^{-a/\lambda}\left(\cancel{\frac{5}{4}}+\frac{a}{2\lambda}\right) = \frac{H-H_{c_1}}{4\pi}$$

negligible

Δ array 경우

$$\frac{6\phi_0}{4\pi\lambda^2}\left(\frac{\pi}{2}\frac{\lambda}{a}\right)^{1/2} e^{-a/\lambda}\left(\frac{a}{2\lambda}\right) = H-H_{c_1}$$

OR

$$\frac{3\phi_0}{4\pi\lambda^2}\left(\frac{\pi}{2}\frac{a}{\lambda}\right)^{1/2} e^{-a/\lambda} = H-H_{c_1}$$

$$\therefore \ln\left\{\frac{3\phi_0}{4\pi\lambda^2(H-H_{c_1})}\right\} = \frac{a}{\lambda}-\frac{1}{2}\ln\left(\frac{\pi}{2}\frac{a}{\lambda}\right)$$

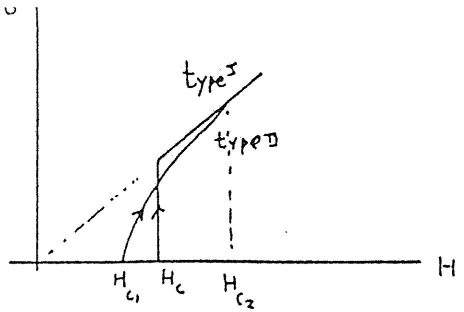
$$\simeq \frac{a}{\lambda}$$

$$= \frac{c}{\lambda}\sqrt{\frac{\phi_0}{B}}$$

OR

$$B = \frac{2\phi_0}{\sqrt{3}\lambda^2}\left[\ln\left\{\frac{3\phi_0}{4\pi\lambda^2(H-H_{c_1})}\right\}\right]^{-2}$$

$$= \frac{8\pi H_{c_1}}{\sqrt{3}\ln K}\frac{1}{\ln\left[\frac{3H_{c_1}}{(H-H\ )\ln K}\right]}$$



B : continuous at H_{c_1} : 2nd order phase transitions.

infinite slope at H_{c_1}.

※ type I Superconductor :

1st order transition at H_c.

Intermediate flux density

$\xi^2 \le \phi_0/B \ll \lambda^2$ $\qquad$ (or $H_{c_1} \le H \ll H_{c_2}$

Modified London eq.

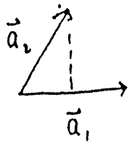
$$\vec{h} + \lambda^2 \; \nabla \times (\nabla \times \vec{h}) = \phi_0 \, \hat{e}_z \sum_i \delta_2 (\vec{r} - \vec{r}_i)$$

$$h_z(\vec{r}) = \sum_Q h_Q \, e^{i \vec{Q}\cdot\vec{r}} \qquad (\because \text{ array + Periodic})$$

$\vec{Q}$ on the 2D reciprocal lattice of the array.

In a □-array, $\vec{Q}_{mn} = \frac{2\pi}{a_0} (m \hat{x} + n \hat{y})$

In a △-array,



$$\left.\begin{aligned} \vec{a}_1 &= a_\Delta \hat{x} \\ \vec{a}_2 &= \frac{a_\Delta}{2} (\hat{x} + \sqrt{3} \hat{y}) \end{aligned}\right\} \rightarrow \begin{aligned} \vec{Q}_1 &= \frac{2\pi}{a_\Delta} (\hat{x} - \frac{\hat{y}}{\sqrt{3}}) \\ \vec{Q}_2 &= \frac{2\pi}{a_\Delta} \frac{2}{\sqrt{3}} \hat{y} \end{aligned}$$

$$(\text{c.f}, \;\; \vec{a}_i \cdot \vec{Q}_j = 2\pi \, \delta_{ij})$$

$$\nabla \times (\nabla \times \vec{h}) = - \nabla^2 \vec{h}$$

$$h_z - \lambda^2 \nabla^2 h_z = \phi_0 \sum_i \delta_2 (\vec{r} - \vec{r}_i)$$

$$\sum_Q (1 + \lambda^2 Q^2) \, h_Q \, e^{i\vec{Q}\cdot\vec{r}} = \phi_0 \sum_i \delta_2 (\vec{r} - \vec{r}_i)$$

$$\sum_Q (1 + \lambda^2 Q^2) \, h_Q \int \underbrace{e^{i\vec{Q}\cdot\vec{r}} \, e^{i\vec{Q}'\cdot\vec{r}}}_{S\,\delta(\vec{Q}\cdot\vec{Q}')} \, da$$

$$= \phi_0 \sum_i \int \delta_2 (\vec{r} - \vec{r}_i) \, e^{i\vec{Q}'\cdot\vec{r}} \, da$$

$$\therefore \; S \, (1 + \lambda^2 Q'^2) \, h_{Q'} = \phi_0 \underbrace{\sum_i e^{i\vec{Q}'\cdot\vec{r}_i}}_{N} \qquad \frac{\phi_0 N}{S} = B$$

$$\therefore \; h_Q = \frac{B}{1 + \lambda^2 Q^2}$$

$$h_z(\vec{r}) = B \sum_Q \frac{e^{i\vec{Q}\cdot\vec{r}}}{1 + \lambda^2 Q^2}$$

The increase in free energy per unit length

(neglecting the core effect)

$$F - F_{s_0} = \frac{1}{8\pi} \int (h^2 + \lambda^2 | \nabla \times \vec{h} |^2) \, dr$$

$$= \frac{\phi_0}{8\pi} \sum_i h(\vec{r}_i)$$

OR the increase per unit volume

$$F - F_{s_0} = \frac{B}{\phi_0} (F - F_{s_0}) \Big|_{h(r_i) \simeq h(0)}$$

$$= \frac{B}{\phi_0} \frac{\phi_0}{8\pi} h(0)$$

$$= \frac{B^2}{8\pi} \sum \frac{1}{1 + \lambda^2 Q^2} \; . \qquad Q_{max} \simeq \frac{1}{\xi}$$

$$G = F - \frac{BH}{4\pi}$$

$$G - G_{s0} = G - F_{s0}$$

$$= (F - F_{s0}) - \frac{BH}{4\pi}$$

$$= \frac{Bh(0)}{8\pi} - \frac{BH}{4\pi}$$

$$\frac{\partial G}{\partial B} = 0 = \frac{h(0)}{8\pi} + \frac{B}{8\pi}\frac{dh(0)}{dB} - \frac{H}{4\pi}$$

$$\therefore \quad H = \frac{1}{2}\left(h(0) + B\frac{dh(0)}{dB} \right) \qquad h(0) = B\sum_{Q}\frac{1}{1+\lambda^2 Q^2}$$

$$= h(0) + \frac{B^2}{2}\sum_{Q}\frac{-2\lambda^2 Q}{(1+\lambda^2 Q^2)^2}\frac{dQ}{dB}$$

$$Q \sim \frac{1}{a} \sim B^{1/2}, \quad \frac{dQ}{dB} = \frac{Q}{2B}$$

$$= B + B\sum_{Q>0}{}' \frac{1}{1+\lambda^2 Q^2} + \frac{B}{2}\sum_{Q>0}{}' \frac{-\lambda^2 Q^2}{(1+\lambda^2 Q^2)^2}$$

「Q=0 제외」

$$\therefore \quad H = B\left\{ 1 + \frac{1}{2}\sum_{Q>0}{}' \left[\frac{1}{1+\lambda^2 Q^2} + \frac{1}{(1+\lambda^2 Q^2)^2} \right] \right\}$$

Convergence 위해. $Q_{max} \approx \frac{1}{\xi}$

Remarks

i) if $B = 0 \longrightarrow H = \frac{1}{2}h(0) \simeq H_{c_1}$

ii) if $B \gg H_{c_1} \rightarrow$ Vortices highly overlapping

2nd term negligible in { }.

$B \approx H$

($\because \lambda a \gg 1$ in this case)

$$\sum_{Q>0} \frac{1}{1+\lambda^2 Q^2} \quad ? \qquad (\text{for } \lambda Q \gg 1)$$

$$\simeq \frac{A}{(2\pi)^2}\int_{Q_{min}}^{Q_{max}} \frac{2\pi Q\, dQ}{1+\lambda^2 Q^2}, \quad Q>0$$

$$= N \cdot \frac{\phi_0}{2\pi B}\int_{Q_{min}}^{Q_{max}} \frac{Q\, dQ}{1+\lambda^2 Q^2} \qquad \frac{\phi_0 N}{A} = B$$

Put $N = 1$

$$Q_{max} \simeq \frac{1}{\xi}$$

$$Q_{min}^2 \simeq |Q_2|^2 = \left(\frac{2\pi}{a_\Delta}\frac{2}{\sqrt{3}} \right)^2 \sim 4\pi\frac{B}{\phi_0}$$

$$\cong \frac{\phi_0}{4\pi B}\int_{Q_{min}^2}^{Q_{max}^2} \frac{dx}{1+\lambda^2 x} \qquad x \equiv Q^2$$

$$= \underbrace{\frac{\phi_0}{4\pi B}\frac{1}{\lambda^2}}_{\frac{H_{c_1}}{B}\frac{1}{\ln K}} \ln\underbrace{\left(\frac{1+\lambda^2/\xi^2}{1+4\pi\lambda^2\frac{B}{\phi_0}} \right)}_{-\ln\left(\frac{\lambda^2}{\xi^2}\frac{\phi_0}{4\pi\lambda^2 B}\right) \cong \ln(H_{c2}/B)}$$

$$H \cong B + H_{c_1}\frac{\ln(H_{c2}/B)}{\ln K}$$

$$H \doteqdot B + H_{c_1} \frac{\ln(H_{c2}/B)}{\ln K} \quad : \quad (H_{c_1} \le H \le H_{c_2})$$

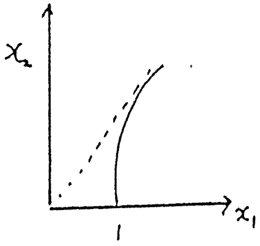
$$\frac{H}{H_{c_1}} \equiv x_1 \, , \qquad \frac{B}{H_{c_1}} \equiv x_2$$

$$\frac{B}{H_{c_2}} = \frac{B}{H_{c_1}} \underbrace{\frac{H_{c_1}}{6 H_{c_2}}} = x_2/x \qquad x \equiv \frac{H_{c2}}{H_{c_1}}$$

$$x_1 \doteqdot x_2 + \frac{\ln(x/x_2)}{\ln K}$$

$$B \doteqdot \frac{8\pi}{\sqrt{3}} \frac{H_{c_1}}{\ln K} \Big/ \left\{ \ln\left[\frac{3H_{c_1}}{(H - H_{c_1}) \ln K} \right] \right\}^2 \quad : \quad H \ll H_{c_1}$$

$$x_2 \doteqdot \frac{8\pi}{\sqrt{3} \ln K} \Big/ \left\{ \ln\left[\frac{B}{(x_1 - 1) \ln K} \right] \right\}^2$$



ex. Nb - 44 wt.% Ti

$H_{c_2}(0) = 96$ KA/cm
$H_{c_1}(0) = .112$ KA/cm
$x = 866$

$K = 24$

x_1	x_2
3.13	1
3.91	2
4.78	3
6.62	4

x_1	x_2
11.4	10
21.1	20
50.9	50
100.6	100
500.2	500

x_1	x_2
1	0
1.01	0.22
1.02	0.31
1.05	0.53
1.07	0.67
1.1	0.91
1.15	1.35

3. Regime near H_{c_2}

The vortex cores occupy much of the volume.

→ full GL treatment

linearized eq. $\because \frac{\Psi}{\Psi_\infty} \to 0$ near H_{c_2}

Abrikosov

$$M = \frac{B - H}{4\pi} \propto \langle \Psi^2(\vec{r}) \rangle$$

$$\propto H_{c_2} - H \qquad \text{near } H_{c_2}$$

more precisely

$$B = H + 4\pi M$$

$$= H - \frac{H_{c_2} - H}{(2\kappa^2 - 1)\beta_A} \, , \qquad \beta_A = \frac{\langle \Psi^4 \rangle}{\langle \Psi^2 \rangle^2} = \begin{matrix} 1.16 & \Delta\text{-arra} \\ 1.18 & \square - " \end{matrix}$$

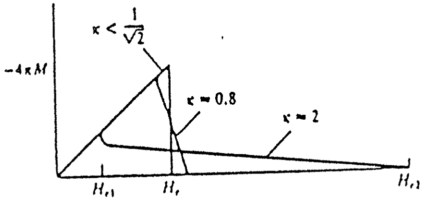


FIGURE 5.2
Comparison of magnetization curves for three superconductors with the same value of thermodynamic critical field H_c, but with different values of κ. For $\kappa < 1/\sqrt{2}$, the superconductor is of type I and exhibits a first-order transition at H_c. For $\kappa > 1/\sqrt{2}$, the superconductor is type II and shows second-order transitions at H_{c1} and H_{c2} (for clarity, marked only for the highest κ case). In all cases, the area under the curve is the condensation energy $H_c^2/8\pi$.

$$\left(\frac{\partial G}{\partial H}\right)_T = -\frac{B}{4\pi}$$

the area under the curve

$$= -\int M\,dH$$

$$= \frac{1}{4\pi}\int_0^{H_{c_2}} (H - B)\,dH$$

$$= \frac{H_{c_2}^2}{8\pi} + \int_0^{H_{c_2}} \frac{\partial G_s}{\partial H}\,dH$$

$$= \frac{H_{c_2}^2}{8\pi} + G_s(H_{c_2}) - G_s(0)$$

$$= F_n(0) - F_s(0)$$

$$= \frac{H_c^2}{8\pi}$$

$$G_{s_0}(0) = F_s(0)$$

$$G_n(H_{c_2}) = \underbrace{F_n(0) + \frac{H_{c_2}^2}{8\pi}} - \frac{H_{c_2}^2}{4\pi}$$

$$= F_n(H_{c_2}) - \frac{H_{c_2}^2}{4\pi}$$

Areas are all Same.

K 를 어떻게 구하나?

- Magnetization curves 에서 구한다.

T_c 근처에서 Simple G.L. 이론 OK

low temp 에서 Small discrepancies

- Maki 가 두장 K_1, K_2, K_3 정의

$$H_{c_2} = \sqrt{2}K_1 H_c, \quad 4\pi \frac{dM}{dH}\bigg|_{H_{c_2}} = (2K_2^2 - 1)^{-1}\beta_A^{-1}$$

$$H_{c_1} = H_c \cdot \frac{\ln K_3}{}$$

$$H_{c_1} = H_c \frac{\ln K_3}{\sqrt{2}K_3}$$

$K_3 \to K$ 로 되는경우는 K 가 큰 경우에 해당한다

이 Maki의 일은 Eilenberger에 의해 더 연구.

K_i 는 ℓ/ξ_0 뿐 아니라 degree of Anisotropy in the impurity Scattering.

degree of nonlocality 의 문제이다

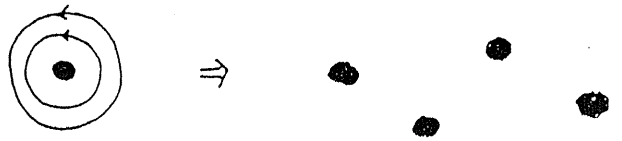
$T \to T_c$ 에서는 모든것이 local 하게 된다.

All K_i approaches common limiting Values.

이 K_i 값들은 약 20% 정도씩 틀리다.

따라서 복잡한 문제를 간단히 생각해도 된다

Flux pinning



Vortices prefer locating themselves at a defect site (Position of weakened or destroyed s.c.) in a s.c.

i) Weak field

Vortex flow under a Lorentz force dampening

→ Viscous flow of Vortices.

→ flux flow

ii) Strong pinning

thermally - assisted hopping between pinning sites.

→ flux Creep.

< Remark >

Suppose making a high field magnet.

Need a s.c. material,

i) with high H_{c_2}

ii) with high J_c in a high H ($< H_{c_2}$) field.

i) and ii) are separate requirements

ii) → need Pinning.

Origin of dissipation → flux motion due to Lorentz force.

$\vec{f} = \frac{1}{c} \vec{J} \times \vec{\phi}_0$: ~~Lowren~~ Lorentz force for a vortex line per unit length.

Lorentz force density ≡ LF per unit volume

$$\vec{F} = n_f \vec{f} = \frac{1}{c} \vec{J} \times (n_f \vec{\phi}_0)$$

n_f: Vortexal Areal density.

$$= \frac{1}{c} \vec{J} \times \vec{B}$$

$$= \frac{1}{c} \vec{J}_{ext} \times \vec{B}$$

$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J}_{ext}$

$$= \frac{1}{4\pi} (\nabla \times \vec{H}) \times \vec{B}$$

* Induced (equilibrium) currents do not contribute to the Lorentz force.

Moving vortex induces an electric field

$\vec{E} = \frac{1}{c} \vec{B} \times \vec{v}$ ⟶ leads to dissipation.

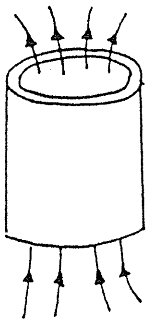
to the approx. $B \simeq H$

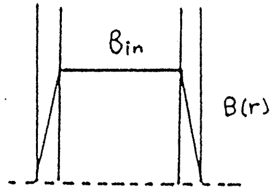
$$\alpha = \frac{1}{4\pi} \left| (\nabla \times \vec{H}) \times \vec{B} \right|$$

$$= \frac{B}{4\pi} \left| \frac{dB}{dx} \right|$$

$$= \frac{d}{dx} \left(\frac{B^2}{8\pi} \right)$$

= a gradient of magnetic pressure





$$\mathcal{E} = 2\pi R E$$

$$= -\frac{1}{c}\frac{d}{dt}\phi \qquad \phi = \pi R^2 B$$

$$= \frac{1}{c} 2\pi R B v$$

$\uparrow$ outward velocity of the flux density

$E = \frac{1}{c} B v$ as obtained above

Flux Flow:

$F = j \frac{\phi_0}{c} = \eta v_L$: force per unit length

Vortex viscosity

$$\frac{E}{J} = \rho_f = \left(\frac{1}{c} B v_L \right) \left(\frac{\phi_0}{c \eta v_L} \right) = \frac{B \phi_0}{\eta c^2}$$

질문. Vortex core의 Size는 얼마나 되나?

$$\frac{|\psi|^2}{\psi_\infty^2} = 1 - \frac{m^{*2} \xi^2 v_s^2}{\hbar^2}$$

v_s를 계산하기 위해

$$\phi_0 = \int \vec{B} \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

$$= \oint \frac{c}{e^*} (\hbar \nabla \phi - m^* \vec{v}_s) \cdot d\vec{l}$$

$$= \frac{m^* c}{e^*} v_s \, 2\pi r$$

$$\therefore v_s = \phi_0 \cdot \frac{e^*}{m^* c \, 2\pi r} = \frac{hc}{e^*} \frac{e^*}{m^* c \, 2\pi r} = \frac{\hbar}{m^* r}$$

$$\therefore \frac{|\psi|^2}{\psi_\infty^2} = 1 - \left(\frac{\xi}{r} \right)^2 \quad \rightarrow \quad r_{core} \simeq \xi$$

Bardeen Stefan의 법칙.

Assuming a hard core : $r_c \sim \xi$

$\vec{e} \equiv$ microscopic field outside the Core

$$= \frac{4\pi}{c^2} \frac{\partial}{\partial t} \left(\lambda_L^2 \vec{J}_s \right) \quad : \text{London eq.}$$

$$= \frac{\partial}{\partial t} \left(\frac{m^* \vec{v}_s}{e^*} \right)$$

$$= -\left(\vec{v}_L \cdot \nabla \right) \left(\frac{m^*}{e^*} \vec{v}_s \right) \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_L \cdot \nabla = 0$$

$$= -\vec{v}_L \cdot \nabla \left(\frac{m^*}{e^*} \frac{\hbar}{m^*} \frac{\hat{\theta}}{r} \right)$$

along the x-direction

$$\vec{e} = -v_{Lx} \frac{\hbar}{e^*} \frac{\partial}{\partial x} \left(\frac{\hat{\theta}}{r} \right)$$

$$\frac{\partial}{\partial x} \left(\frac{1}{r} \right) \hat{\theta} + \frac{1}{r} \left(\frac{\partial \hat{\theta}}{\partial x} \right) = -\frac{x}{r^3} \hat{\theta} + \frac{\sin\theta}{r^2} \hat{r}$$

$$= \frac{1}{r^2} \left(-\cos\theta \, \hat{\theta} + \sin\theta \, \hat{r} \right)$$

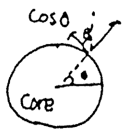
$$\therefore \vec{e} = \frac{v_{Lx} \phi_0}{2\pi c} \cdot \frac{1}{r^2} \left(\cos\theta \, \hat{\theta} - \sin\theta \, \hat{r} \right)$$

Inside

Continuity of the tangential Component of $\vec{e}$.

$$\left(e_{out} \right)_\theta = \left(e_{core} \right)_\theta$$

$$= \frac{v_{Lx} \phi_0}{2\pi c a^2} \cos\theta$$



$$\vec{e}_{core} = \frac{v_{Lx} \phi_0}{2\pi a^2 c} \hat{y}$$

dissipation per unit length of the core

$$W_{core} = \int \vec{J} \cdot \vec{e} \, da$$

$$= \pi a^2 \left(\sigma_n e_{core} \right) e_{core}$$

$$= \frac{\pi a^2}{\rho_n} \left(\frac{v_{Lx} \phi_0}{2\pi a^2 c} \right)^2$$

$$= \frac{v_{Lx}^2 \phi_0^2}{4\pi a^2 c^2 \rho_n}$$

Equal amount from outside the core

$$\frac{1}{\rho_n} \int_a^\infty \int_0^{2\pi} e^2(r) \, r \, dr \, d\theta = \frac{2\pi}{\rho_n} \int_a^\infty \left(\frac{v_{Lx} \phi_0}{2\pi c r^2} \right)^2 r \, dr = \frac{v_{Lx}^2 \phi_0^2}{4\pi a^2 c^2 \rho_n}$$

$$= W_{core}$$

$$w - \frac{\quad}{2\pi a^2 c^2 \rho_n} = \eta v_L^2$$

$$\eta = \frac{\phi_0^2}{2\pi a^2 c^2 \rho_n} \longrightarrow \frac{\phi_0^2}{2\pi \xi^2 c^2 \rho_n} = \frac{\phi_0 H_{c2}}{c^2 \rho_n}$$

$$\therefore \rho_f = \frac{B\phi_0}{\eta c^2} = \frac{B\phi_0}{c^2} \cdot \frac{2\pi \xi^2 c^2 \rho_n}{\phi_0^2} = 2\pi \xi^2 \rho_n \left(\frac{B}{\phi_0} \right)$$

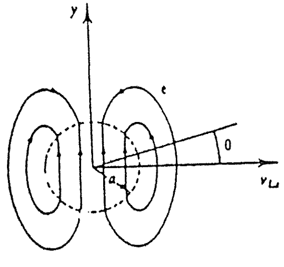


FIGURE 5.4
Schematic diagram of local electric field e near a moving vortex line. Dashed circle of radius a marks perimeter of core. A suitable surface charge is required at r = a to be consistent with the discontinuity in the normal component of e. In a more exact model, the discontinuity would be smeared out.

$$\text{OR} \quad \rho_f = \frac{B\phi_0}{\eta c^2} = \frac{B\phi_0}{c^2} \cdot \frac{2\pi \xi^2 c^2 \rho_n}{\phi_0^2} = 2\pi \xi^2 \rho_n \left(\frac{B}{\phi_0} \right)$$

$$\text{OR} \quad = 2\pi \xi^2 n_f \rho_n \qquad \text{areal flux density}$$

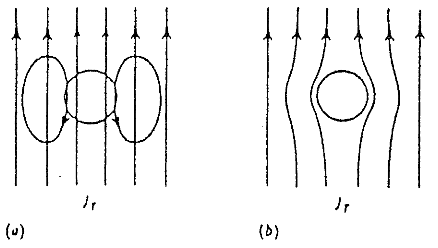
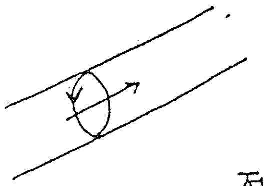


FIGURE 5.5
Backflow current pattern at pinned vortex. (a) Uniform transport current J_r and backflow current separately. (b) Superimposed current pattern, with zero current in core.



 Vortex가 생기나 중앙으로 들어가면서 없어진다.

저항이 생기나는 Critical current는 ?

$$I_{c1} = \frac{1}{2} H_{c1} c a$$

어떻게 Vortex ring이 shrink 되나?

$$\frac{d}{dt}\left(2\pi r \varepsilon_1\right) = 2\pi r \eta \left(\frac{dr}{dt}\right)^2$$

$$\therefore r\frac{dr}{dt} = \frac{\varepsilon_1}{\eta}$$

Each vortex ring shrink to annihilation

$$T = \frac{\eta a^2}{2\varepsilon_1} = \frac{H_{c2}}{H_{c1}} \frac{a^2}{\rho_n c^2}$$

$$\sim 10^{-5} \text{ sec.}$$

Valid for $I \sim I_{c1}$

일반적인 경우

$$\frac{J\Phi_0}{c} = \eta v \qquad J = \frac{c}{4\pi} \nabla \times H$$

$$v = \frac{cE}{B} \qquad \text{B는 local value of } \vec{h}$$

$$\frac{\Phi_0}{c} \frac{c}{4\pi} \nabla \times H = \eta \cdot \frac{cE}{B}$$

$$\frac{B}{r} \frac{\partial}{\partial r}(rH) = \frac{4\pi \eta c E}{\Phi_0}$$

여기서 B와 H는 circumferential,

E는 longitudinal 하다

limit case

$I \simeq I_{c1} \Rightarrow H \simeq H_{c1}$,

$$\frac{dB}{dH} \rightarrow \infty \text{ at } H_{c1}$$

B의 flux density 매우 작다.

$$\frac{\partial}{\partial r}(rH) \cong H_{c1}$$

따라서 $B(r) = \dfrac{4\pi \eta c E r}{\Phi_0 H_{c1}} = \dfrac{4\pi \cancel{c} E r}{\cancel{\Phi_0} H_{c1}} \cdot \dfrac{\cancel{\Phi_0} H_{c2}}{\rho_n c^{\cancel{2}}}$

5.58 에서 $\eta \cong \dfrac{\Phi_0 H_{c2}}{\rho_n c^2}$ $= \dfrac{4\pi E r}{c \cdot \rho_n} \dfrac{H_{c2}}{H_{c1}}$

$$\therefore \frac{E}{E_n} = \frac{R}{R_n} = \frac{B(H_s)}{2H_{c2}}$$

Other limit $B = H$

$$\frac{B}{r}\frac{\partial}{\partial r}(rH) = \frac{4\pi \eta c E}{\Phi_0}$$

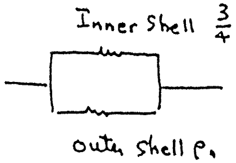
적분하면 $H^2 = B^2 = \dfrac{8\pi \eta c E r}{3\Phi_0}$

B와 H는 $r^{1/2}$에 비례

$$\frac{E}{E_n} = \frac{R}{R_n} = \frac{3H_s}{4H_{c2}} = \frac{3}{4}\frac{I}{I_{c2}}$$

$I > I_{c2}$ 이고 $H_s > H_{c2}$

outershell은 completely normal, while inner core has the effective resistivity $\frac{3}{4}\rho_n$,



$H(r_1) = H_{c2}$,

Effective average conductivity

$$= (1 + r_1^2/3a^2)\,\rho_n^{-1}$$

r_1 is the radius of the inner core

$$\frac{r_1}{a} = \frac{2I}{I_{c2}}\left[1 - \left(1 - \frac{3I_{c2}^2}{4I^2}\right)^{1/2}\right]$$

$\rightarrow \dfrac{3I_{c2}}{4I}$ $\qquad I \gg I_{c2}$

$\dfrac{R}{R_n} = 1 - \dfrac{3}{16}\left(\dfrac{I_{c2}}{I}\right)^2$ $\qquad I \gg I_{c2}$

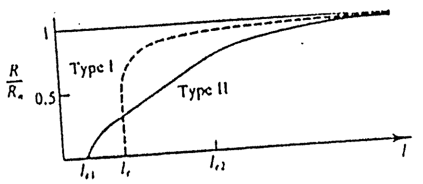
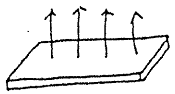


FIGURE 5.6
Onset of resistance in a wire of ideal type II superconductor with no pinning and κ ≈ 1.7. For comparison, the dashed curve shows the corresponding behavior of a type I superconductor with the same H_c.

5.5.3. Experimental

- Basic experiment

Kim, Hempstead



flux flow theory

$\rho/\rho_n \approx B/H_{c2}$

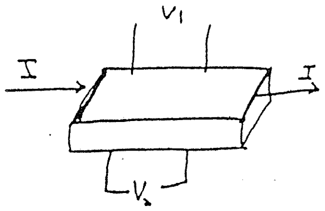
- Intermediate State of type I Superconductor

domain 이 충분히 커서 magneto-optic technique으로 움직임을 볼수있다.

- Electron holography by the group of Tonomura

직접 눈으로 보다

- DC transformer



- Giaever 가 처음으로 고안하다.

2nd Induced Voltage

만약 Slippage가 생겨나면 Induced Voltage 없다.

Tinkham, Deltour

coupling strength가 작아진다

magnetic field, temperature, bias current 증가하면 작아짐

Ooijen, Van Garp 1965. Phys. lett

Voltage = $\sum$ large # of flat top pulses of duration $\tau = W/v_L$

Measure of the amount of flux moving in each independent, discrete entity

Φ_0 부터 $1000\,\Phi_0$ 까지 bundle로 움직인다.

결론: Concept of flux motion is generally correct.

B w defects in real material samples Considerably complicate the idealized picture.

5.5.4. Concluding Remarks on Flux Flow

Lorentz force 에 의해 transverse motion을 한다. 그러나 liquid 에서는 처음에 drift 방향으로 간다.

Hall effect 를 보인다.

$$\vec{B} \times \vec{v}_L / c$$

보통은 이것이 매우 작다. Hall angle is small.

Bardeen - Stephen model에 의하면

Hall effect 가 normal core 수에 비례하게 크다.

대부분 type II alloy, intermetallic compound

— rather short electronic mean free path.

Hall angle 너무 작다

- Another problem

pinning에 의해 Intrinsic property 결정 어렵다.

Hall sign anomaly problem

HTSC

Note : Bardeen Stephen model

Hall effect stems from the quasi-normal core

따라서 normal state와 같은 Sign을 갖는다.

실험 : 반대 결과

Vortices starts drifting upstream

⇒ Contrary to the universal behavior of vortices in ordinary fluids.

설명 :
- Pinning
- quasi-particle back flow
- Anisotropic scaling in the time-dependent GL eq

아직 몰라

Entropy transport

: Quasi normal core가 entropy 옮긴다.

Ettinghasen effect — Solomon and Otter
Vidal

$\boxed{T_1} \xrightarrow[\text{entropy}]{} \boxed{T_1}+\Delta T$

이해가 된다.
quantitative result를 예측하는것 같다.

- Ideal flux flow resistance

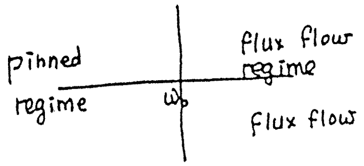
in the presence of pinning by making measurement at microwave frequencies.

Drag force $\eta v = \eta \omega dx$

pinning force $= - k \delta x$

Gittleman and Rosenblum

$$\omega_0 = k/\eta$$



flux flow resistance : ρ_f

$\omega_0 \sim 10^7$ Hz

5.6. The critical - state Model.

- Pinning strong enough to prevent any substantial vortex motion : " hard S.C."

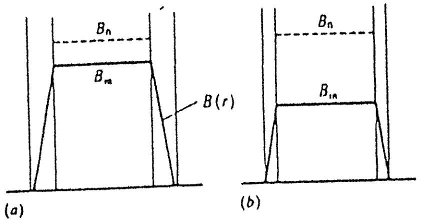
- Lorentz force per unit volume.

$$\vec{\alpha} = \frac{1}{c} \vec{J}_{ext} \times \vec{B}$$

α_c = pinning force per unit volume.

- Critical state : $\alpha \leq \alpha_c$

#1: hollow cylinder



(a) (b)

FIGURE 5.7
The critical state in a hollow superconducting cylinder. In (a), the wall thickness is sufficient to trap all the initial flux. In (b), the walls are too thin to do so. For simplicity, field profiles have been drawn using the Bean model, in which $J_c \propto dB/dr$ is constant. The same value of J_c has been taken in both (a) and (b).

$B_o > H_{c_1}$: fluxes leak out of the cylinder, reducing the field inside.

induced current : $j_\theta = \frac{c}{4\pi}\frac{dH}{dr}$

Critical state :

$$\alpha = \frac{1}{c} j_{ext} B$$
$$= \frac{B}{4\pi}\frac{dH}{dr}$$
$$= \frac{d}{dr}\left(\frac{B^2}{8\pi}\right) \le \alpha_c$$

$$\frac{B_{max}}{8\pi} = \int_{R_{in}} \alpha_c (B(r))\, dr$$

Suppose α_c = const

$B^2_{max} \quad \propto \quad \alpha_c (R_{out} - R_{in})$

maximum B that can be held inside the cylinder

$= \alpha_c d$

d: wall thickness

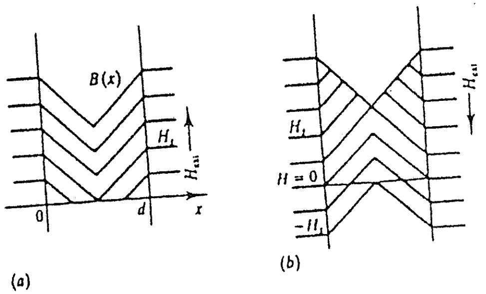
α_c cannot be const. all the way down to B = 0

$\alpha_c = j_c \cdot \frac{B}{c} \longrightarrow$ const $(B \to 0)$

$\Rightarrow J_c \to \infty$ impractical

Bean's model

J_c = const $= \frac{c}{4\pi}\frac{dH}{dr}$ $\Big\}$ $\alpha_c \propto \frac{dB}{dr}$



(a) (b)

FIGURE 5.8
Internal flux-density profiles in a slab subjected to (a) increasing and (b) decreasing external field. H_1 is the maximum applied field that can be screened at the midplane. Note the occurrence of canceling flux densities in (b) when $H_{ext} = -\frac{1}{2}H_1$.

Thermally activated Flux Creep - type II.

Thermal energy (T ≠ 0) → hopping of the flux lines.

i) Slow changes in trapped magnetic fields

Creep rate getting slower as the creep progresses → logarithmic in time.

ii) Resistive voltages ∝ average creep velocity

※ Anomalous to flux flow case with

$$v_{creep} \ll v_{flow}$$

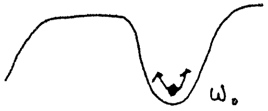
Anderson - Kim flux creep theory

- Assume that the flux creeps by bundles
- $\lambda > a \longrightarrow$ Collective motion

a: distance between lines

hopping rate

$$R = \omega_0 e^{-F_0/kT}$$



ω_0: attempt freq in a pinning potential

$10^5 - 10^{11}$ sec^{-1}

F_0: activation free energy

i) no flux density gradient : (1 d)

equal hopping rate in both directions.

ii) If flux density gradient exists :

tilting the spatial energy dependence

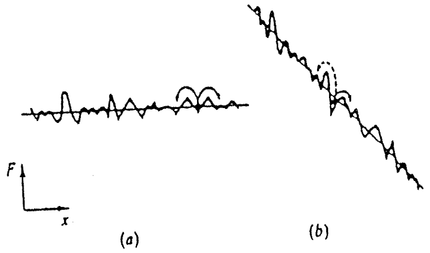


FIGURE 5.9
Schematic representation of flux bundles jumping over barriers to adjacent pinning sites. The ordinate represents the relative value of the total free energy as a function of the position of the center of the flux bundle. (a) Zero driving force. (b) Driving force due to current (or dB/dx) favoring jumps in a "downhill" direction.

$\alpha = \frac{1}{c} JB$: force density

V_d : Volume of a flux bundle

L : average hopping distance

$\Delta F \equiv$ shift in the barrier height

= work done by the driving force in going over the barrier $= \alpha V_d L$

R = net hopping rate $= R_+ - R_-$

$$= \omega_0 e^{-F_0/k_BT} \left(e^{\Delta F/k_BT} - e^{-\Delta F/k_BT} \right)$$

$$= 2\omega_0 e^{-F_0/kT} \sinh\left(\alpha V_d L\right.$$

a net flux Creep velocity

$$v = 2v_0 \, e^{-\bar{F}_0/k_BT} \sinh\left(\frac{\alpha V_d L}{k_BT}\right)$$

$v_0 = \omega_0 L$ = the creep velocity w/o barrier

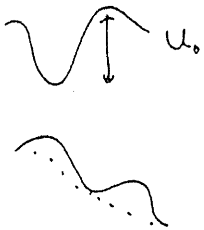
$\sim 10^{3\pm3}$ cm/sec

oR

$$R = 2R_0 \, e^{-U_0/k_BT} \sinh\left(\frac{jBV_dL}{c\,k_BT}\right)$$

$\uparrow$

resistance, $U_0 \equiv \bar{F}$: intrinsic pinning Potential



$$\frac{J_0 B V_d L}{c} = U_0$$

$$R = 2R_0 \, e^{-U_0/k_BT} \sinh\left(\frac{U_0}{k_BT}\frac{I}{I_0}\right)$$

$U_0 = P\left(\frac{H_c^2}{8\pi}\right) V_d$: arising from the spatial variation of condensation energy

$P \sim 10^{-3}$

① strong pinning centers (such as void) are rare

② small pinning caused by extended Pinning Center.

$H_c \sim 2000G$

$U_0/k_B \sim 1200°K$

$$v_1 = v_0 \, e^{-U_0/k_BT} \sim 10^{3\pm3} \, e^{-1200/T}$$

$$\sim 10^{3\pm3 - 500/T}$$

$\sim 10^{-50}$ cm/sec for T=10K

$\therefore \sinh\left(\frac{\alpha V_d L}{k_BT}\right) \gg 1$ to get an approprciable flux creep

$$\rightarrow \exp\left(\frac{\alpha V_d L}{k_BT}\right)$$

$$v = v_0 \, e^{-U_0/k_BT} \, e^{\alpha V_d L/k_BT}$$

$T=0$, $v=0$ unless $\alpha V_d L \geq U_0$

$\alpha_c(0) = \frac{U_0(0)}{\alpha V_d}$ Critical force density parameter at T=0

$$v(T) = v_0 \exp\left[-\frac{U_0(T) - U_0(0)\frac{\alpha(T)}{\alpha_c(0)}}{k_BT}\right]$$

$$v_{min} = v_0 \exp\left[-\frac{U_0(T) - U_0(0)\frac{\alpha_c(T)}{\alpha_c(0)}}{k_BT}\right]$$

$\uparrow$

detectable minimum Creep velocity

then

$$\frac{\alpha_c(T)}{\alpha_c(0)} = \underbrace{\frac{U_0(T)}{U_0(0)}}_{\text{pinning strength}} - \underbrace{\frac{k_B T}{U_0(0)} \ln\left(\frac{v_{min}}{v_0}\right)}_{\text{Creep rate}}$$

$T \ll T_c$

$U_0(T) \simeq U_0(0)\,(1-\beta t^2) \qquad \beta \sim 1$

$$\frac{\alpha_c(T)}{\alpha_c(0)} \simeq 1-\beta t^2 - \gamma t \qquad t \ll 1$$

where $\gamma \equiv \frac{k_B T_c}{U_0(0)} \ln\left(\frac{v}{v_{min}}\right) \sim 0.1$

i) $t \lesssim \frac{\gamma}{\beta} \simeq 0.1$ low temp. limit,

the linear term dominates.

이래서 linear term이 들어간다.

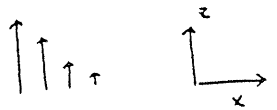
$\frac{d\alpha}{dT}$ 의 실험값 (Kim) 이 reasonable.

except $T \lesssim 1K$, βt^2이 dominant 하다.

Time dependence of vortex creep.

Conservation of flux lines

$$\frac{\partial \vec{B}}{\partial t} + \nabla\cdot(B\vec{v}) = 0$$



$\vec{v} \parallel \hat{x}$

OR $\frac{\partial B}{\partial t} = -\frac{\partial}{\partial x} Bv \;:\; 1d.$

We found

$$v = v_0\, e^{-U_0/k_BT}\; e^{\alpha V_d L/k_BT}$$

$$= v_1\, e^{\alpha/\alpha_1} \qquad \alpha_1 \equiv \frac{k_BT}{V_dL} = \frac{k_BT}{U_0(0)}\alpha_c(0)$$

$$\sim \alpha_c(0)/300$$

and

$$\alpha = -\frac{\partial}{\partial x}\left(\frac{B^2}{8\pi}\right)$$

then

$$\frac{\partial\alpha}{\partial t} = -\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right)$$

$$= -\frac{\partial}{\partial x}\left(\frac{B}{4\pi}\frac{\partial B}{\partial t}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{B}{4\pi}\frac{\partial}{\partial x}(Bv)\right)$$

$$\simeq \frac{B^2}{4\pi}\frac{\partial^2 v}{\partial x^2}$$

$$= \frac{B^2}{4\pi} v_1 \frac{\partial^2}{\partial x^2} e^{\alpha/\alpha_1}$$

$$\frac{\partial v}{\partial x} \propto \frac{\partial}{\partial x} e^{JBV_dL/cK_BT}$$

$$= \frac{JV_dL}{cK_BT}\frac{\partial B}{\partial x} e^{(\;\;)}$$

$$\frac{1}{v}\frac{\partial v}{\partial x} = \frac{JBV_dL}{cK_BT}\left(\frac{1}{B}\frac{\partial B}{\partial x}\right)$$

$$= \alpha\frac{V_dL}{K_BT}\frac{1}{B}\frac{\partial B}{\partial x}$$

$$= (\alpha/\alpha_1)\frac{1}{B}\frac{\partial B}{\partial x}$$

$$\hat{=}\; 300\;\frac{1}{B}\frac{\partial B}{\partial x}$$

Trial function

$$Ce^{\alpha/\alpha_1} = (ax^2+bx+c)\,g(t) \quad \text{代入} \quad ⊛$$

$$\frac{dg}{dt} = \frac{2ag^2}{\alpha_1}$$

Integrate this,

$$g(t) = -\frac{\alpha_1}{2at}$$

이것을 ⊛에 代入

$$\therefore \alpha = F(x) - \alpha_1 \ln t$$

where F(x) is a function only of x.

그런데 $\alpha \simeq \alpha_c$ 근처에서만 Creep이 일어난다

$$\therefore F(x) \simeq \alpha_c$$

$$\therefore \alpha = \alpha_c - \alpha_1 \ln t$$

For flux trapped in a hollow S.C. Cylinder

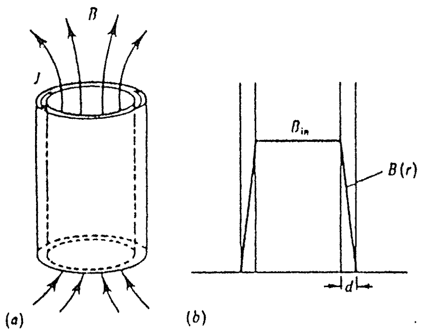


FIGURE 5.3
Flux trapped in a hollow cylinder of type II superconductor. (a) Sketch of overall geometry. (b) Local flux-density profile.

$$\alpha \simeq \frac{d}{dx}\left(\frac{B^2}{8\pi}\right)$$

$$\simeq \frac{B_{in}^2}{8\pi d} \leftarrow \text{wall thickness}$$

$$B_{in} \simeq (8\pi\alpha d)^{1/2}$$

$$= (8\pi\alpha_c d)^{1/2}\left(1 - \frac{\alpha_1}{\alpha_c}\ln t\right)^{1/2}$$

$$\simeq B_c\left(1 - \frac{\alpha_1}{2\alpha_c}\ln t\right)$$

holds only for small fractional change in B.

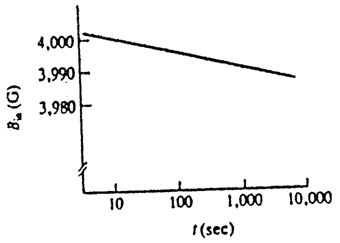
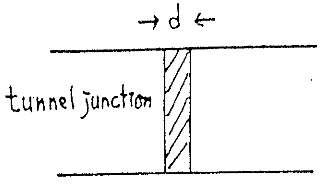


FIGURE 5.10
Evidence for logarithmic decay of "persistent" current in a hollow cylinder of type II superconductor. [After Kim, Hempstead, and Strnad, Phys. Rev. Lett. 9, 306 (1962).]

6장.

Josephson Effect.

Various weak links showing Josephson Effect.



SIS
SNS

SIS :

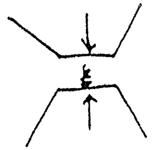
$d \lesssim 10\,\mathring{A}$ conventional S.C.

$\lesssim 1\,\mathring{A}$ high-T_c S.C.

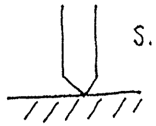
SNS :

$d \leq 2\xi_N$

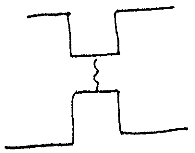
$$\xi_N = \begin{cases} \left(\dfrac{\hbar D}{2\pi K_B T}\right)^{1/2} & \text{clean limit} \\ \left(\dfrac{\hbar v_{FN} l_N}{6\pi K_B T}\right)^{1/2} & \text{diffusion limit} \end{cases}$$



microbridge



Point Contact

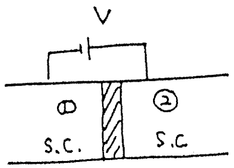


mostly high T_c material

Josephson Relations :

$J = J_0 \sin\gamma$: approx.

$\dfrac{d\gamma}{dt} = \dfrac{2eV}{\hbar}$: exact



Ψ_1 : amplitude (macroscopic) of finding a pair in ①

Ψ_2 : 〃 ②

Assume : ①, ② of the same material

$H = 0$

$i\hbar \dfrac{\partial \Psi_1}{\partial t} = eV\Psi_1 + K\Psi_2$

$i\hbar \dfrac{\partial \Psi_2}{\partial t} = -eV\Psi_2 + K\Psi_1$

K : coupling const.

Trial Sol.

$\Psi_1 = \sqrt{n_1}\, e^{i\varphi_1}$

$\Psi_2 = \sqrt{n_2}\, e^{i\varphi_2}$

n's : pair density
φ's : phases

$\dot{n}_1 = \dfrac{2}{\hbar} K \sqrt{n_1 n_2} \sin\gamma$

$\dot{n}_2 = -\dfrac{2}{\hbar} K \sqrt{n_1 n_2} \sin\gamma$

$\rightarrow \dot{n}_1 = -\dot{n}_2$

$J = J_0 \sin\gamma$

$\dot{\varphi}_1 = \dfrac{K}{\hbar}\sqrt{\dfrac{n_2}{n_1}} \cos\gamma - \dfrac{eV}{\hbar}$

$\dot{\varphi}_2 = \dfrac{K}{\hbar}\sqrt{\dfrac{n_1}{n_2}} \cos\gamma + \dfrac{eV}{\hbar}$

$\dot{\gamma} = \dot{\varphi}_2 - \dot{\varphi}_1$

$= 2eV/\hbar$

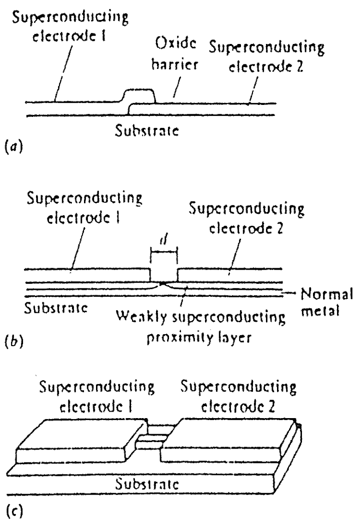
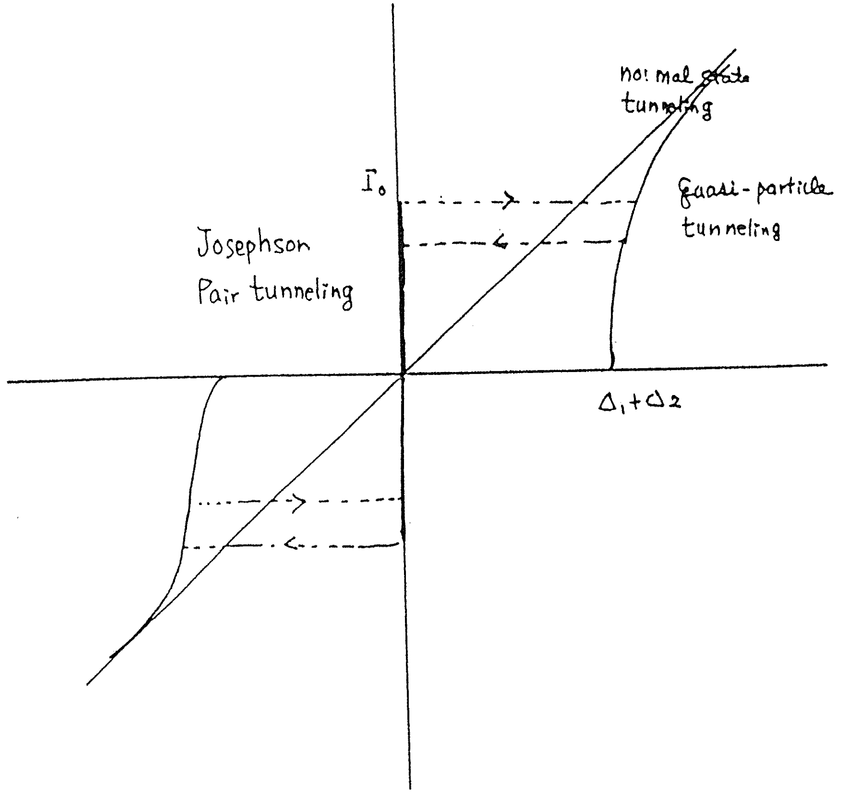


FIGURE 6.1
Three types of Josephson junction: (a) S-I-S. (b) S-N-S, and (c) S-c-S.



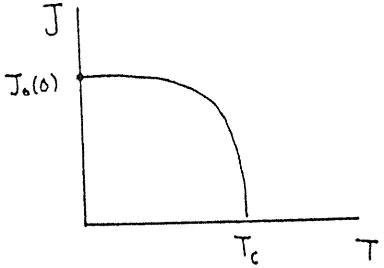
Remark

1) $J_0 = \frac{\pi \Delta(T)}{2eR_N} \tanh\left(\frac{\Delta(T)}{2K_BT}\right)$

Ambegaokar & Baratoff

$\frac{\pi\Delta(0)}{2eR_N}$ $T \to 0$

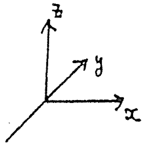
$\frac{\Delta^2(T)}{T} \propto T_c - T$ $T \to T_c$

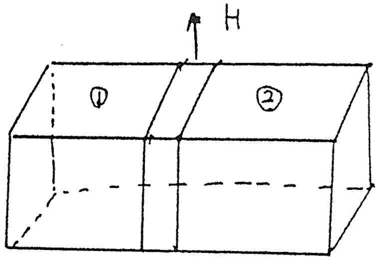


2) $V \neq 0$

$I = I_0 \sin\gamma + G_0V + G_{int}(\cos\gamma)V$

Josephson Junction in a Magnetic Field





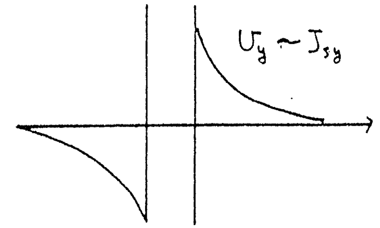
Cross sectional area of the barrier $\longrightarrow$ YZ

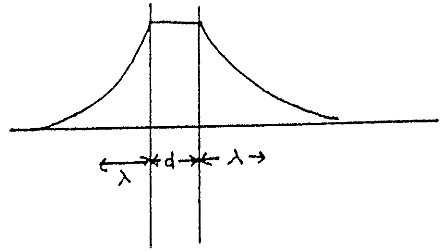
Y, Z, length of S.C. $\gg \lambda$

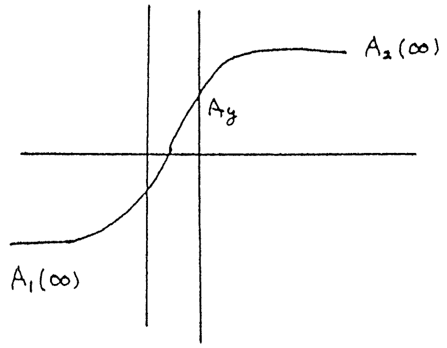
tunneling current $\ll$ screening current

$\vec{A} = A_y(x)\,\hat{y}$

$h_z = \dfrac{\partial A_y}{\partial x}$







$$\vec{v}_s = \frac{\hbar}{m^*}\left(\nabla\varphi - \frac{2\pi\vec{A}}{\phi_0}\right)$$

OR $$\nabla\varphi = \frac{m^*\vec{v}_s}{\hbar} + \frac{2\pi}{\phi_0}\vec{A}$$

$$\left.\begin{aligned}\frac{\partial\varphi}{\partial y} &= \frac{m^* v_{sy}}{\hbar} + \frac{2\pi}{\phi_0}A_y \\ \frac{\partial\varphi}{\partial x} &= 0\end{aligned}\right\} \quad \varphi = \varphi(y)$$

φ : independent of x $\longrightarrow$ φ can be calculated for any x.

For convenience, deep in the S.C.

$$\left.\begin{aligned}v_s &= 0 \\ h_z &= 0 \\ A_y &= A_y(\infty)\end{aligned}\right\} \rightarrow \quad \frac{\partial\varphi}{\partial y} = \frac{2\pi}{\phi_0}A_y(\infty)$$

$$\varphi(y) = \varphi(0) + \frac{2\pi}{\phi_0}A_y(\infty)\,y$$

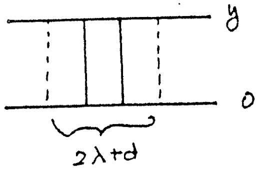
$$\gamma(y) = \varphi_2(y) - \varphi_1(y)$$

$$= \gamma(0) + \frac{2\pi}{\phi_0}\left[A_2(\infty) - A_1(\infty)\right] y$$

한편

$$\oint \vec{A}\cdot d\vec{s} = \left(A_2(\infty) - A_1(\infty)\right) y = \Phi(y)$$

= flux enclosed in the junction area $(2\lambda+d)\,y$

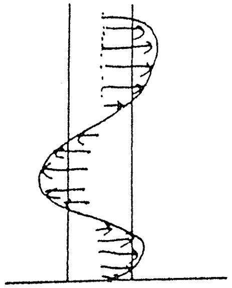


$$\therefore \quad \gamma(y) - \gamma(0) = \frac{2\pi\,\Phi(y)}{\phi_0}$$

$$= \frac{2\pi\, H\,(2\lambda + d)\, y}{\phi_0}$$

$$J_x = J_0 \sin\gamma(y)$$

Currents in various parts of the junction area tend to cancel.



$$I = Z \int_{-Y/2}^{Y/2} J(y)\, dy$$

$$= Z J_0 \int_{-Y/2}^{Y/2} \sin\left[\frac{2\pi H (2\lambda+d) y}{\phi_0} + \gamma(0) \right] dy$$

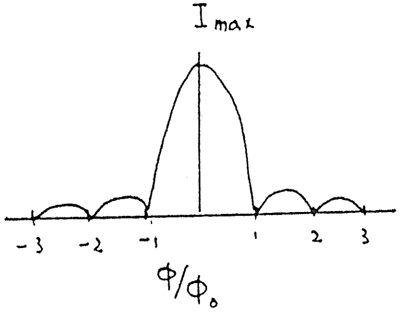
$$= - Z J_0 \frac{\phi_0}{2\pi H (2\lambda + d)} \cos\left(\frac{2\pi H (2\lambda+d) y}{\phi_0} + \gamma_0 \right) \Bigg|_{-Y/2}^{Y/2}$$

$$= \frac{Z J_0 \phi_0 Y}{\pi \Phi} \sin\left(\frac{\pi \Phi}{\Phi_0} \right) \sin\gamma(0) , \qquad \Phi = H (2\lambda + d) Y$$

$$= Y Z J_0 \sin\gamma(0) \cdot \frac{\sin(\pi \Phi / \Phi_0)}{\pi \Phi / \Phi_0}$$

OR

$$I_{max} = Y Z J_0 \left| \frac{\sin(\pi \phi / \phi_0)}{\pi \phi / \phi_0} \right|$$



If the tunneling current is not negligible,

→ Josephson current screens the field in the junction region with a weak Meissner effect.

$$\frac{\partial h}{\partial y} = \frac{4\pi}{c} J_x = \frac{4\pi}{c} J_0 \sin\gamma$$

$$\Phi(y) = (2\lambda + d) \int_0^y h(y')\, dy' = \frac{\phi_0}{2\pi} \left[\gamma(y) - \gamma(0) \right]$$

OR $$\frac{d\gamma}{dy} = \frac{2\pi}{\phi_0} (2\lambda + d)\, h$$

$$\therefore \quad \frac{d^2\gamma}{dy^2} = \frac{2\pi}{\phi_0} (2\lambda + d) \frac{dh}{dy}$$

$$= \frac{2\pi}{\phi_0} (2\lambda + d) \frac{4\pi}{c} J_0 \sin\gamma$$

$$= \frac{1}{\lambda_J^2} \sin\gamma$$

$$\lambda_J = \left[\frac{c\, \phi_0}{8\pi^2 J_0 (2\lambda + d)} \right]^{1/2}$$ Josephson penetration depth

$\simeq$ 1 mm typically

$$\frac{d^2\gamma}{dy^2} = \frac{1}{\lambda_J^2} \sin\gamma$$

i) $\gamma \ll 1$. $(H \ll 4\pi J_0 \lambda_J / c)$

$$\frac{d^2\gamma}{dy^2} = \frac{\gamma}{\lambda_J^2} \longrightarrow \gamma(y) \sim e^{\pm y/\lambda_J}$$

h and J_x also decay exp.

ii) γ not small:

Comparing to the eq. of motion of a pendulum.

$y \longleftrightarrow t$

$\gamma \longleftrightarrow \theta$

$\lambda_J^{-2} \longleftrightarrow \omega_0^2 = \frac{g}{L}$

weak screening limit $\rightarrow$ $\Delta\gamma(y) \propto y$

OR $\Delta\theta \propto t$

a pendulum with high kinetic energy

Effect of gravitation negligible.

$g \rightarrow 0$

$\omega_0^2 \rightarrow 0$ & $\lambda_J \rightarrow \infty$

$\therefore$ $\frac{d^2\gamma}{dy^2} = 0$ $\frac{d\gamma}{dy} = \text{Const} \longrightarrow$ Sinusoidal current pattern along y direction (previous case)

Intermediate Screening due to junction current

→ A pendulum with marginal Kinetic energy at the top of the circular motion

→ Nonsinusoidal, yet periodically reversing Current distribution.

$$mgh = mgh_0 + \frac{1}{2} m v_0^2$$

$$mg(h-h_0) = \frac{1}{2} m v_0^2$$

$$gL(1-\cos\theta_0) = \frac{1}{2} L^2 \left(\frac{d\theta_0}{dt}\right)^2$$

$$\left(\frac{d\theta}{dt}\right)_0^2 = \frac{2g}{L}(1-\cos\theta_0)$$

$$= 2\omega_0^2 (1-\cos\theta_0)$$

$$\longleftrightarrow \left(\frac{d\gamma}{dy}\right)_0^2 = \frac{2}{\lambda_J^2}(1-\cos\gamma_0)$$

$$= \left(\frac{2\pi H}{\Phi_0}\right)^2 (2\lambda+d)^2$$

Taking initial condition $h_0 = H$ at the edges of the Jnc.

$$\cos.\gamma_0 = 1 - \frac{1}{2}\left(\frac{2\pi H}{\Phi_0}\lambda_J(2\lambda+d)\right)^2$$

$$= 1 - \frac{1}{2}\left(\frac{CH}{4\pi J_0 \lambda_J}\right)^2$$

$\therefore$ In the weak field limit, $\gamma_0 = \frac{CH}{4\pi J_0 \lambda_J}$

γ_0 : value of $|\gamma|$ at the edges of the junction the strongest field which can be screened

$\gamma_0 = \pi$

$$-1 = 1 - \frac{1}{2}\left(\frac{CH_1}{4\pi J_0 \lambda_J}\right)^2$$

OR

$$H_1 = 8\pi J_0 \lambda_J / c \quad \sim \quad 1 \text{ Gauss}$$

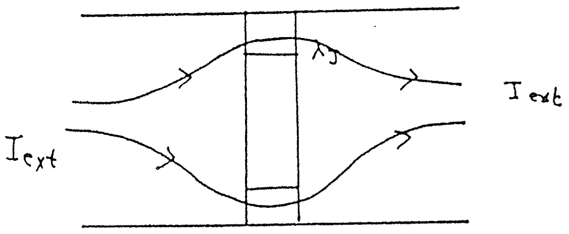
For $I > J_0 \lambda_J$ in a wide Jnc.

$h > H_1 \longrightarrow$ Vortex (Periodic)

State established

$\longrightarrow$ dissipative in the presence of

I ext.



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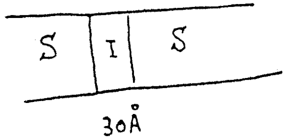
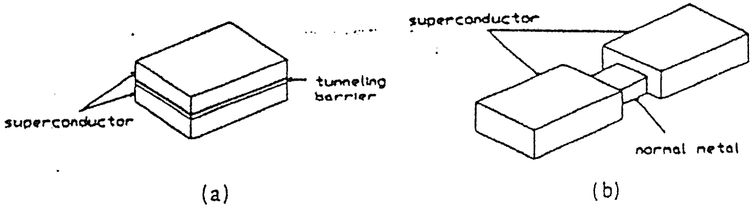
dc SQUID의 작동원리 1996.10.23 이성익교수

SQUID : Superconducting Quantum Interference Devices

Josephson junction.





(그림 3-1) 죠셉슨 접합의. 종류

a) 터널접합 : 이 접합은 초전도체 사이에 30Å 정도의 부도체를 삽입함으로써 죠셉슨 접합의 메카니즘이 얇은 부도체를 터널하여 일어난다. 주로 SIS접합을 이룬다.

b) Microbridge접합 : 초전도 전자들이 약 1000Å 정도의 금속 사이를 확산 메카니즘에 의해 터널하여 일어난다. 주로 SNS 접합을 이룬다.

Microbridge : 가운데는 Normal metal.

초전도 전자들의 확산에 의한 Weak link 형성

장점 : Coherence length 가 1000Å 정도.

Tunnel bridge : 만들기 어렵다.

종래의 고온 초전도체인 A15

. 온도를 약 900°C 이상으로 상승

. 초전도체의 제조는 가능.

죠셉슨 터널 접합의 제조는 매우 어렵다.

Josephson Junction

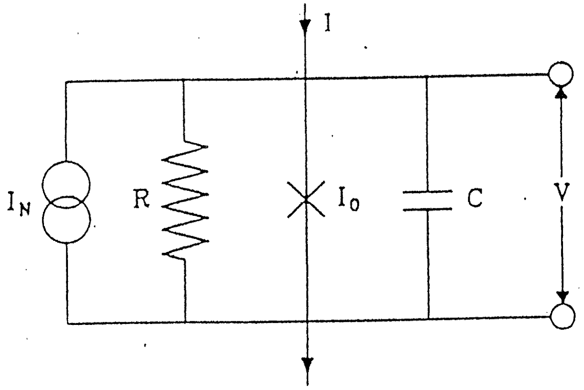
$$J = J_0 \sin(\theta_1 - \theta_2)$$

$$J = J_0 \sin\left(\frac{2eV}{\hbar}t + \theta\right)$$

$$\boxed{\frac{d\phi}{dt} = \frac{2eV}{\hbar}}$$

Josephson 접합의 전류 - 전압 특성

RSJ model : 1968년 Stewart 가 처음 제시



(그림 3-2) RSJ 모델

$$I = I_0 \sin\gamma + \frac{V}{R} + C\frac{dV}{dt}$$

만약 C 가 작다면

$$I = I_0 \sin\gamma + \frac{V}{R}$$

$$\frac{d\gamma}{dt} = \frac{2eV}{\hbar} \qquad \phi_0 = \frac{h}{2e}$$

$$I = I_0 \sin\gamma + \frac{V}{R}$$

$$= I_0 \sin\gamma + \frac{1}{R}\cdot\frac{\hbar}{2e}\frac{d\gamma}{dt}$$

$$= I_0 \sin\gamma + \frac{2\pi\hbar}{2eR\cdot 2\pi}\frac{d\gamma}{dt}$$

$$= I_0 \sin\gamma + \frac{\phi_0}{2\pi R}\frac{d\gamma}{dt}$$

$$I_0\left(\frac{I}{I_0} - \sin\gamma\right) = \frac{\phi_0}{2\pi R}\cdot\frac{d\gamma}{dt}$$

$$\frac{2\pi R I_0}{\phi_0}dt = \frac{d\gamma}{\frac{I}{I_0} - \sin\gamma}$$

$$\frac{2\pi R I_0 t}{\phi_0} = \frac{2}{\left[\left(\frac{I}{I_0}\right)^2 - 1\right]^{1/2}} \tan^{-1}\left[\left(\frac{I}{I_0}\right)\tan\frac{\gamma}{2} + 1\right]\cdot\left[\frac{I^2}{I_0^2} - 1\right]^{1/2}$$

for $I > I_0$

이것을 γ 에 대해 풀면

$$\tan\frac{\gamma}{2} = \sqrt{1 - \left(\frac{I}{I_0}\right)^2}\ \tan\left\{\frac{\pi R I_0}{\phi_0}\left(\frac{I^2}{I_0^2} - 1\right)^{1/2}\cdot t\right\} - \frac{I_0}{I}$$

여기서 진동 주기를 생각하면 주기는

$$T = \frac{2\pi}{2\left(\frac{\pi R I_0}{\phi_0}\right)\left(\frac{I^2}{I_0^2} - 1\right)^{1/2}}$$

I - V [illegible]

— 측정불가, 또는 무의미

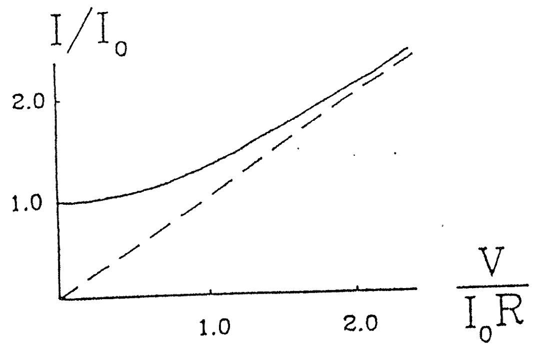
평균값이 의미 있다.

$$\langle V \rangle = \frac{\hbar}{2e} \left\langle \frac{d\gamma}{dt} \right\rangle$$

$$= \frac{\hbar}{2e} \frac{1}{T} \int_0^T \left(\frac{d\gamma}{dt}\right) dt$$

$$= \frac{\hbar}{2e} \cdot \frac{2\pi}{T}$$

$$= IR \left(1 - \frac{I_0^2}{I^2}\right)^{1/2} \qquad I > I_0$$



(그림 3-3) RSJ 모델에서 C=0인 경우의 I-V 특성곡선

만약 잡음을 고려하면

$$\frac{V}{R} + I_0 \sin\delta + C \frac{dv}{dt} = I + I_N(t)$$

적당히 [illegible]화 시키면

$$\beta_c \ddot{\delta} + \dot{\delta} = i - \sin\delta$$

pf.

$$\frac{V}{R} + I_0 \sin\gamma + C \frac{dv}{dt} = I + I_N(t)$$

$$\frac{d\gamma}{dt} = \frac{2eV}{\hbar}, \quad \Phi_0 = \frac{h}{2e}$$

$$V = \frac{\hbar}{2e} \frac{d\gamma}{dt}$$

$$\therefore \frac{\hbar}{2eR} \frac{d\gamma}{dt} + I_0 \sin\gamma + C \cdot \frac{\hbar}{2e} \frac{d^2\gamma}{dt^2} = I + I_N(t)$$

$$\frac{\hbar c}{2e} \ddot{\delta} + \frac{\hbar}{2eR} \dot{\delta} = I - I_0 \sin\gamma$$

$$= \frac{2e}{\hbar} \left\{ \left(\frac{2e}{h}\right)^{-1} \cdot \frac{1}{2\pi} (I - I_0 \sin\gamma) \right\}$$

$$= -\frac{2e}{\hbar} \frac{\partial}{\partial\delta} U$$

$$\text{where } U = -\frac{\phi_0}{2\pi} (I_0 \gamma + I_0 \cos\gamma)$$

$$\frac{\hbar c}{2e} \frac{d^2\gamma}{dt^2} + \frac{\hbar}{2eR} \frac{d\gamma}{dt} = I - I_0 \sin\gamma$$

$$t = \alpha t'$$

$$\frac{1}{I_0} \frac{\hbar c}{2e} \frac{d^2\gamma}{dt'^2} \frac{1}{\alpha^2} + \frac{\hbar}{I_0 2eR} \frac{1}{\alpha} \frac{d\gamma}{dt'} = \frac{I}{I_0} - \sin\gamma$$

$$\alpha = \frac{\hbar}{2 I_0 e R}$$

$$\text{than} \quad \frac{1}{I_0} \frac{\hbar c}{2e} \frac{4 I_0^2 e^2 R^2}{\hbar^2} = \frac{2 C I_0 e R^2}{\hbar} = \frac{2\pi C I_0 R^2}{\frac{h}{2e}}$$

$$= \frac{2\pi I_0 R^2 c}{\Phi_0} \equiv \beta_c$$

l.t $i = \frac{I}{\ }$ $t' = \frac{t}{\ }$

then $\beta_c \frac{d^2\gamma}{dt'^2} + \frac{d\gamma}{dt'} = i - \sin\gamma$

β_c : Macumber parameter.

$\beta_c >> 1$ 이면 I-V 특성곡선이 hysteric 하다.

$\beta_c << 1$ over damping

감쇠가 충분하여 hysteresis 가 나타나지 않는다.

SQUID : $\beta_c < 1$ 에서 作動

I, V_c, V, R

Josephson Junction :

i, v_c, v, r

정의 : $r_D = \left(\frac{\partial v}{\partial i}\right)_{i_c}$: dynamic resistance.

Note : Josephson 소자에 자기장을 가하면

$$I_o(\Phi_a) = I_o(0) \cdot \frac{\sin(\pi \Phi_a/\Phi_o)}{\pi \Phi_a/\Phi_o}$$

Fundamental limits on SQUID Technology

Superconductivity and the Josephson Effect

Cooper pair 들의 Macroscopic quantum state

$$\psi(\vec{r},t) = |\psi(\vec{r},t)| \cdot e^{i\phi(\vec{r},t)}$$

ψ is a single valued function

$\Phi = n\Phi_o$. where $\Phi_o = \frac{h}{2e} = 2.07 \times 10^{-15}$ wb

Josephson effect

$I_s = I_o \sin\delta$

where $\delta = \phi_1 - \phi_2$

if $I < I_o$ phase difference is time independent

Voltage across the junction $= 0$

$I > I_o$ $V \neq 0$

$$\dot{\delta} \equiv \omega = \frac{2eV}{\hbar} = \frac{2\pi V}{\Phi_o}$$

$\therefore$ Supercurrent oscillate at a freq

$\nu = \frac{\omega}{}$

I-V characteristics

: hysteretic

1963년 McCumber가 junction을 shunt 시킴으로 문제 해결.

RSJ (Resistively Shunted Junction)

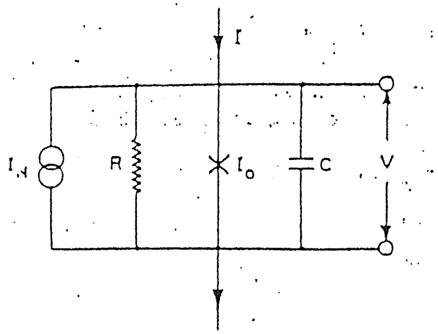


Fig. 1. Resistively shunted junction model.

$$\frac{V}{R} + I_0 \sin\delta + C\frac{dV}{dt} = I + I_N$$

Noise 항을 제거하면

$V = \frac{\hbar\dot{\delta}}{2e}$ 代入하면

$$\frac{\hbar C}{2e}\ddot{\delta} + \frac{\hbar}{2eR}\dot{\delta} = I - I_0 \sin\delta = -\frac{2e}{\hbar}\frac{\partial}{\partial\delta}U$$

where $U = -\frac{\Phi_0}{2\pi}(I\delta + I_0\cos\delta)$

$$\frac{\hbar C}{I_0 2e}\ddot{\delta} + \frac{\hbar}{I_0 2eR}\dot{\delta} = \frac{I}{I_0} - \sin\delta$$

$$\frac{\hbar C}{I_0 2e}\frac{d^2\delta}{d(\alpha t)^2} + \frac{\hbar}{I_0 2eR}\frac{d\delta}{d(\alpha t)} = \frac{I}{I_0} - \sin\delta$$

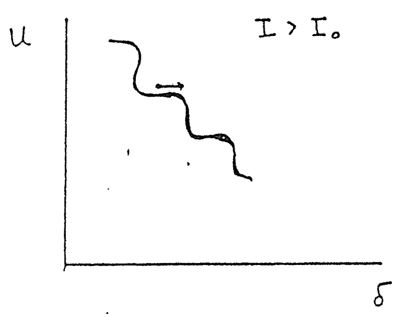
$\Rightarrow$ $\beta_c\ddot{\delta} + \dot{\delta} = i - \sin\delta = -\frac{\partial}{\partial\delta}U$ where $U = -i\delta + co$

where $\alpha = \frac{\hbar}{I_0 2eR} = \frac{h}{2\pi I_0 2eR}$ $\beta_C = \frac{2\pi I_0 R^2 C}{\Phi_0}$

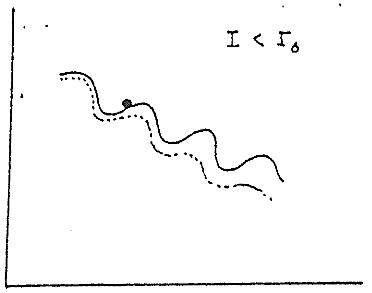
$$= \frac{\Phi_0}{2\pi I_0 R}$$

(문) $\beta_c\ddot{\delta} + \dot{\delta} = -\frac{\partial}{\partial\delta}U$, $U = -i\delta + \cos\delta$

$= i - \sin\delta$

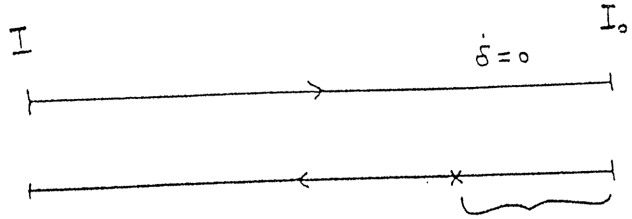


particle running down



..... instantaneous potential due to the fluctuatior

$\beta_c >> 1$ 인 경우 (전류를 I_0 까지 올렸다 내린다면



공이 움직이기 시작한다.
$\dot{\delta} \neq 0$ 이다.

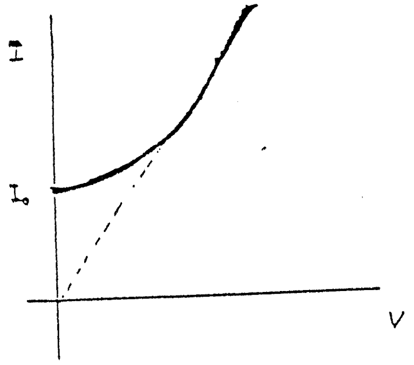
이점도 까지는 공이 계속움직인다
∵ Kinetic energy를 얻었으므로
∴ $\dot{\delta} \neq 0$
∴ Non zero Voltage
∴ hysteresis 의 원인

만약 $\beta_c << 1$ overdamped case
damping이 커서 Kinetic energy를 얻어도
instantly goes to zero
∴ No hysteresic

CROSS OVER $\beta_c = 1$

「exact value는 noise level에 따라 달라진다」

만약 C 가 zero 이면
$\dot{\delta} = i - \sin\delta$ 를 exact 하게 풀수있다.



Noise를 Washboard model에 넣자.

$$\frac{\hbar C}{2e}\ddot{\delta} + \frac{\hbar}{2eR}\dot{\delta} + I_0 \sin\delta = I + I_N(t)$$

Langevin equation

Thermal noise limit,

$$S_I(\nu) = 4K_BT/R$$

$I_N(t)$ 의 두가지 효과.

1. $I < I_0$

Ball을 가끔움직이게 한다.

- Induces a series of voltage pulses randomly spaced in t

∴ $< S_V(\nu_m) > \neq 0$

∴ I-V characteristic이 조금 변한다.

∴ 「Noise Rounded」

만약 Josephson Junction 에

$V(t) = V_{dc} + V_s \cos \omega_s t$ 를 걸어준다면

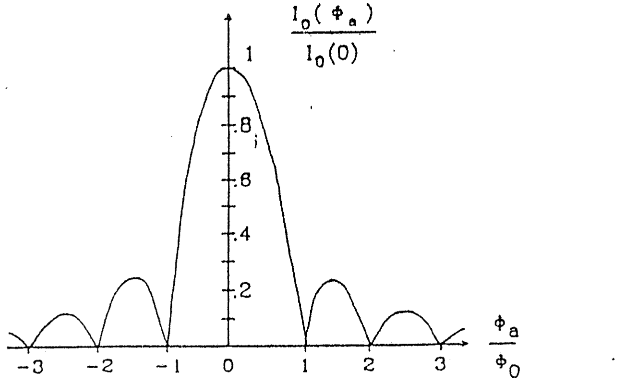
$$I(t) = I_0 \sin\left\{ \frac{2eV_{dc}}{\hbar} t + \frac{2eV_s}{\hbar \omega_s} \sin \omega_s t + \varphi_0 \right\}$$

Bessel 방정식을 이용하면

$$\cos(A \sin\delta) = \sum_{n=-\infty}^{\infty} J_n(A) \cos n\delta$$

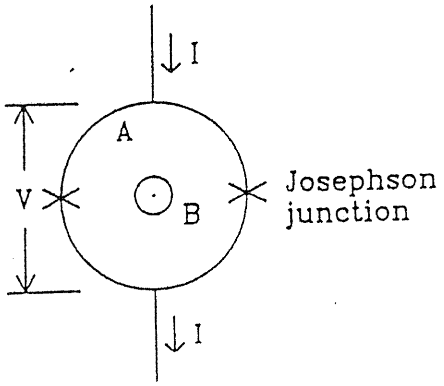
$$\sin(A \cos\delta) = \sum_{n=-\infty}^{\infty} J_n(A) \sin n\delta$$

$$= I_0 \sum_{n=-\infty}^{\infty} (-1)^n J_n (2eV_s/\hbar\omega_s) \sin\{(\omega_j - n\omega_s)t + \delta_0\}$$



(그림 3-4) 조셉슨 접합에서의 자기장에 따른 임계전류의 변화

SQUID의 기능



(그림 3-5) dc SQUID의 구성

SQUID : 자속을 전압으로 전환시켜 주는 세상에서 가장 민감한 감지기

10^{-9} gauss 정도의 뇌 또는 근육 에서의 자기장 측정 ; 지구의 표면저항, 전자기적 탐사 지자기의 변화측정, 지진 다 자장

전류계 : 10^{-14} 암페어 까지 측정 가능.

현재의 잡음 : fundamental limit to energy sensitivity

dc SQUID의 이론적 고찰

$$I_i = \frac{\hbar}{2e} C \frac{d^2\theta_i}{dt^2} + \frac{\hbar}{2eR} \frac{d\theta_i}{dt} + I_c \sin\theta_i + I_{0i}$$

$$I = I_1 + I_2$$

$L(I_1 - I_2)$ 을 통과하는 자

et $\nu = \frac{\theta_1+\theta_2}{2}$, $\phi = \frac{\theta_1-\theta_2}{2}$

$\therefore \theta_1 = \nu + \phi$

$\theta_2 = \nu - \phi$

$$I_1 = \frac{\hbar c}{2e}\frac{d^2}{dt^2}(\nu+\phi) + \frac{\hbar}{2eR}\frac{d}{dt}(\nu+\phi) + I_c[\sin\nu\cos\phi + \cos\nu\sin\phi] + I_{n_1}$$

$$I_2 = \frac{\hbar c}{2e}\frac{d^2}{dt^2}(\nu-\phi) + \frac{\hbar}{2eR}\frac{d}{dt}(\nu-\phi) + I_c[\sin\nu\cos\phi - \cos\nu\sin\phi] + I_{n_2}$$

$$I = I_1 + I_2$$

$$= \frac{\hbar c}{e}\frac{d^2\nu}{dt^2} + \frac{\hbar}{eR}\frac{d\nu}{dt} + 2I_c\sin\nu\cos\phi + I_{n_1} + I_{n_2}$$

$$\frac{2(\Phi-\Phi_a)}{L} = \frac{\hbar c}{e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{eR}\frac{d\phi}{dt} + 2I_c\cos\nu\sin\phi + I_{n_1} - I_{n_2}$$

Let $t = \alpha t'$, where $\alpha = \frac{\hbar}{2I_ceR}$, $i = \frac{I}{I_c}$

$$\phi_a = \pi\frac{\Phi_a}{\Phi_0}$$

then the above eq. will be

$$i - i_{n_1} - i_{n_2} = \frac{\hbar c}{2I_ce}\frac{4I_c^2e^2R^2}{\hbar^2}\frac{d^2\nu}{dt'^2} + \frac{d\nu}{dt'} + \sin\nu\cos\phi$$

$$\frac{2\Phi_0}{2I_cL\pi}\cdot(\phi-\phi_a) = \frac{\hbar c}{2I_ce}\frac{d^2\phi}{dt^2} + \frac{d\phi}{dt} + \cos\nu\sin\phi + i_{n_1} - i_{n_2}$$

$$\beta_c = \frac{\hbar c}{2I_ce}\cdot\frac{4I_c^2e^2R^2}{\hbar^2} \qquad \beta_L \equiv \frac{2\pi I_cc}{\Phi_0}$$

$$= \frac{2I_ceR^2c}{\hbar}$$

$$= \frac{2\pi I_cR^2c}{\Phi_0}$$

따라서 dc SQUID 의 운동 방정식은?

$$\beta_c\frac{d^2\nu}{dt^{*2}} + \frac{d\nu}{dt^*} + \sin\nu\cdot\cos\phi = i + i_{n,\nu}(t^*)$$

$$\beta_c\frac{d^2\phi}{dt^{*2}} + \frac{d\phi}{dt^*} + \cos\nu\cdot\sin\phi + \frac{2}{\beta_L}(\phi-\phi_a) = i_{n,\phi}(t^*)$$

여기서 $i = I/2I_c$, $\phi_a = \pi\Phi_a/\Phi_0$

여기서 ν 와 ϕ 의 연관 관계는 다음과 같다.

$$\langle i_{n,\nu}(t^*)\rangle = \langle i_{n,\phi}(t^*)\rangle = 0$$

$$\langle i_{n,\nu}(t^*+\tau^*)\,i_{n,\nu}(t^*)\rangle = \langle i_{n,\phi}(t^*+\tau^*)\,i_{n,\phi}(t^*)\rangle$$

$\langle i_{n,0}(t^*)\, i_{n,\phi}(t^*) \rangle = 0$

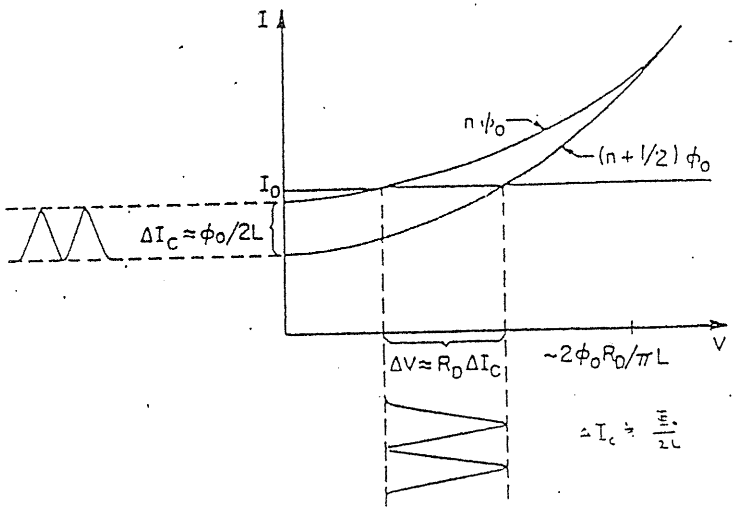
가정: 전류 잡음의 상관은 확률과정 (Stochastic Process) 만족

위의 2차 미분 방정식

: 매우 복잡하고 시간이 걸려 컴퓨터 Simulation 이용

또는 $\beta_c << 1$ 인 경우는 급수전개로 이용하여

순환 전류 (Circulating Current), 전달함수 (transfer function) $\partial V / \partial \Phi_a$ 등을 계산.



(그림 3-6) SQUID의 I-V 곡선

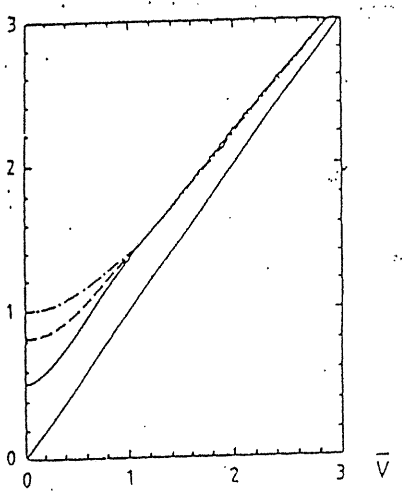
가정: SQUID 의 inductance 가 무시된다면

$\beta_L << 1$ 이고 $\beta_c \approx 0$ 이면 환에서의

총자속은 $\Phi \approx \Phi_a$

$$\bar{V} = \frac{1}{\tau} \int_0^{\tau} V \, dt$$

$$\approx \frac{RI}{2} \left[1 - \left(\frac{2I_c}{I} \cos \frac{\pi \Phi_a}{\Phi_0} \right)^2 \right]^{1/2}$$



(그림 3-7) dc SQUID에서의 전류-전압특성곡선
$\beta_c = 0.3,\ \beta_L = \pi$, 아래로부터 $\varphi_a = 0.5,\ 0.25,\ 0$

만약 $\beta_L = 0$ 의 가정을 완화하면

풀이가 매우 어렵지만 근사적으로

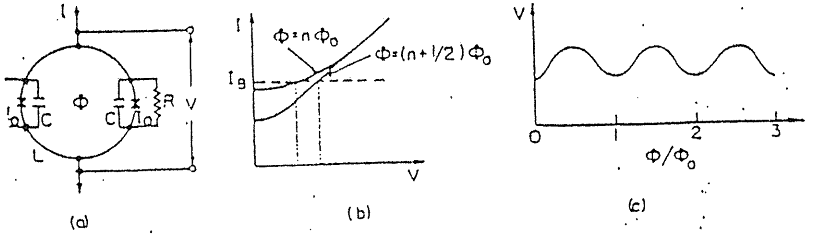
$\beta_L = \pi$ 에서, $\frac{\partial V}{\partial \Phi_a} \sim \frac{R}{2L}$, 동적저항은 $R_{dyn} = \frac{\partial \bar{V}}{\partial I} \sim R/\sqrt{2}$

D.C. SQUID

2개의 Josephson junction

Assume $\beta_c \ll 1$

I-V is non-hysteretic



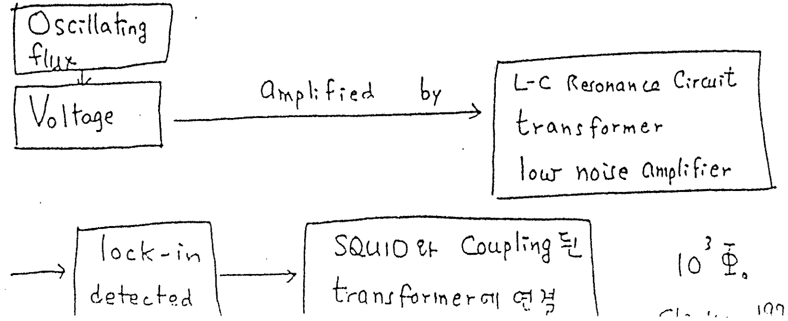
12. (a) Configuration of dc SQUID; (b) current-voltage (I-V) characteristic with $\Phi = n\Phi_0$ and $(n + \frac{1}{2})\Phi_0$, where n is an integer; (c) V vs. Φ at constant bias current.

flux to Voltage transformer

Maximum V $\quad (n+\frac{1}{2})\Phi_0$

$\frac{\partial V}{\partial \Phi}$ 의 maximum $\quad \frac{(2n+1)}{4}\Phi_0$

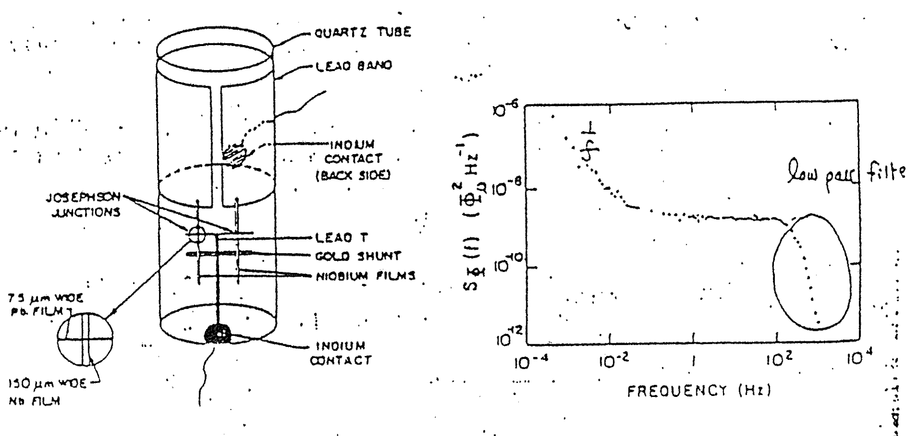
SQUID 에서는 보통 flux-locked loop mode로 작동.



$10^3 \Phi_0$

SQUID의 noise

$S_\phi = \frac{S_V(f)}{V_\Phi^2} \qquad$ where $V_\Phi = \frac{\partial V}{\partial \Phi}$



(a) (b)

Fig. 13. (a) Configuration of cylindrical dc SQUID; (b) spectral density of SQUID flux noise, $S_\Phi(f)$, for typical cylindrical SQUID (Clarke et al., 1976).

$S_\Phi \sim 3.5 \times 10^{-5}\,\Phi_0 \;\; H_z^{-\frac{1}{2}}$

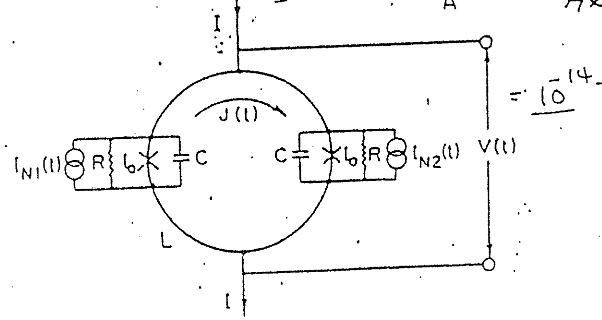
Area가 7 mm² 이다.

이것은 magnetic field sensitivity가 $10^{-14}\;TH_z^{-\frac{1}{2}}$ 에 해당한다.

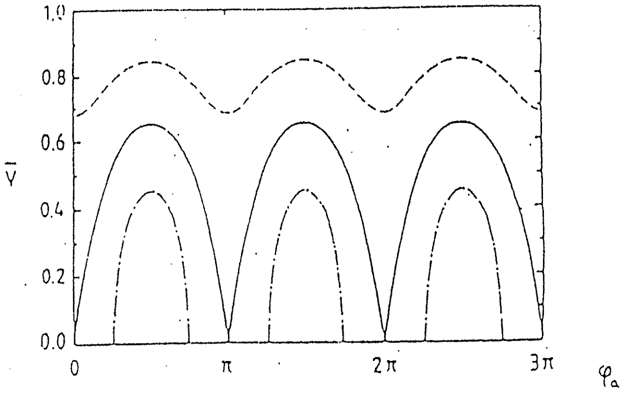
Thermal noise in the D.C. SQUID

$S_\Phi = B \cdot A \quad B = \frac{S_\Phi}{A} = \frac{3.5\times10^{-5} \times 2.07\times10^{-15} \cdot \text{tesla}\,\cancel{m^2}/\sqrt{H_z}}{7\times10^{-6}\cdot \cancel{m^2}}$

$= 10^{-14}\;TH_z^{-1/2}$

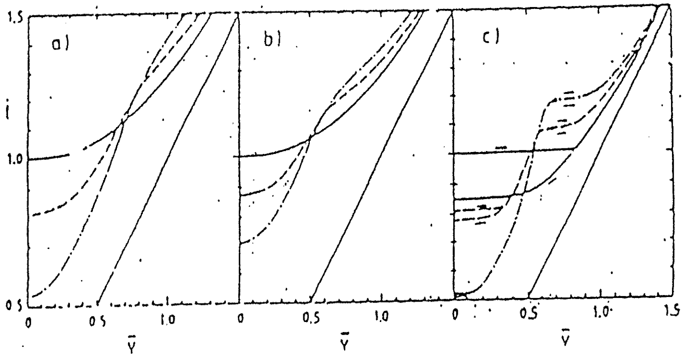


전과정이 정현모양 일수록 더 좋은 SQUID를 만들수 있다.



(그림 3-8) dc SQUID에서의 전압과 자속과의 관계

$\beta_c = 0.3, \beta_L = \pi$, 위로부터 i = 0.8, 1.0, 1.2



(그림 3-9) dc SQUID에서의 전압-전류의 특성곡선

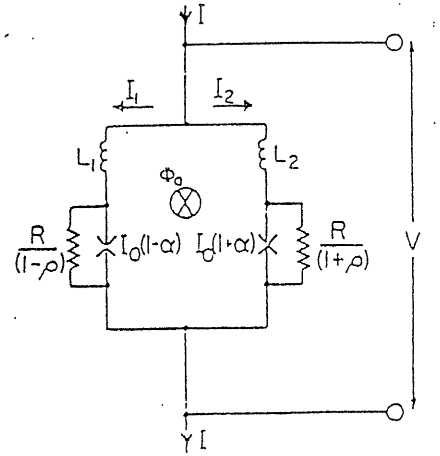
(a) $\beta_c = 0.7, \beta_L = \pi$, (b) $\beta_L = 0.7, \beta_L = 2\pi$, (c) $\beta_c = 1.6$,

$\beta_L = \pi$ 직선은 $\Phi_a = 0$, 점선은 $\Phi_a = \pi/4$, 그외는 $\Phi_a = \pi/2$이다.

β_L이 증가할수록 찌그러들고

β_c 증가하면 hysteresis가 나타난다.

비대칭적 SQUID의 등가회로

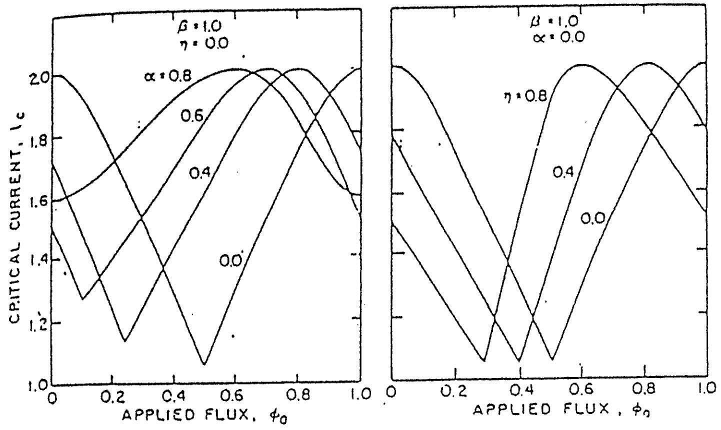


(그림 3-10) 비대칭적 SQUID의 등가회로

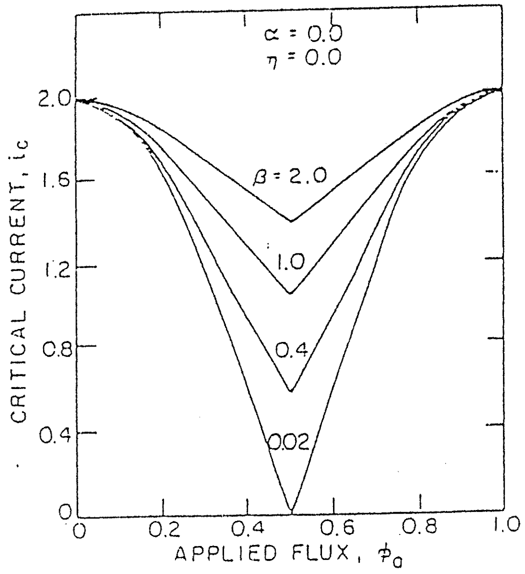
I_1, I_2 $I_1 = I_0(1-\alpha)$, $I_2 = I_0(1+\alpha)$

L_1, L_2 $L_1 = (1-\eta)\cdot\frac{L}{2}$, $L_2 = (1+\eta)\cdot\frac{L}{2}$

$\frac{R}{1-\rho}, \frac{R}{1+\rho}$, $\boxed{L_1 + L_2 = L}$ α: I_0의 비대칭정도.



(그림 3-11) 임계전류와 외부 자장 사이의 관계



(그림 3-12) β에 따른 임계전류와 외부자장 사이의 관계

언제가 가장 좋은 점인가

$\beta \sim 1$ 정도 , 잡음 적게

Autonomous SQUID : 입력코일이나 결합
임피던스 문제 고려안함

SQUID 내에서의 잡음 문제를 다루기 위하여서는 결합된 Langevin 방정식을 풀어야 한다

잡음은 주로 분기 저항에서 발생한다고 생각

수치해석적인 기법을 사용하여 잡음에 의해 깍여진 I-V 특성곡선을 구해야 한다

Ambegakor - Baratoff : 특수경우 Analytically.

Tesche : 무시하기 Computer Programming

전압잡음, 이의 Spectrum, 전류잡음 Spectrum ....

SQUID 의 운동 방정식

$$j = (\delta_1 - \delta_2 - 2\pi \Phi_a)/\pi\beta - \eta i/2$$

$$V = \frac{(1+\eta)}{2}\frac{d\delta_1}{d\theta} + \frac{(1-\eta)}{2}\frac{d\delta_2}{d\theta}$$

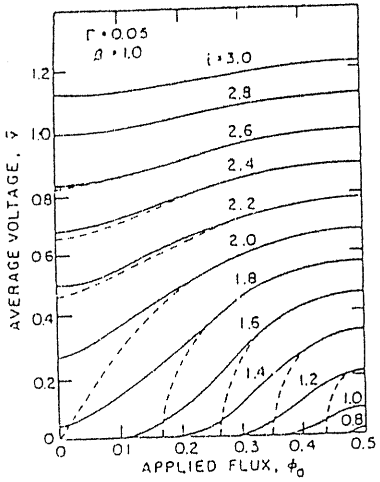
$$\frac{d\delta_1}{d\theta} = \frac{i/2 - j - (1-\alpha)\sin\delta_1}{(1-\rho)} + V_{1n}$$

$$\frac{d\delta_2}{d\theta} = \frac{i/2 + j - (1+\alpha)\sin\delta_2}{(1+\rho)} + V_{2n}$$

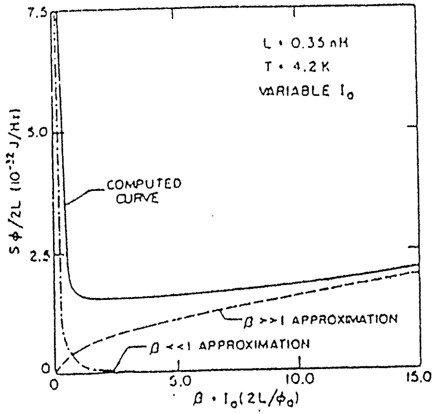
각각의 Spectrum 밀도

$$S V_n = 4\Gamma$$

$$\Gamma = 2\pi k_B T / I_0 \Phi_0$$



(그림 3-14) 외부에서 주어진 자장과 평균전압 사이의 관계



(그림 3-15) $S\phi/2L$과 β 사이의 관계

잡음을 줄이기 위해서는

$\frac{S_\Phi}{2L}$ 과 β 사이에서

β = 1 정도가 적당

$\beta \equiv I_0(2L)/\Phi_0$

$S_\phi/_{2L} \approx 2k_BT \cdot L/R \qquad \beta \gg 1$

$S_\phi/_{2L} \approx k_BTL/_{R\beta^2} \qquad \beta \ll 1$

SQUID 제작 조건

(1) $\frac{S_\phi}{2L}$을 줄이기 위하여서 $\sqrt{LC}$를 줄이자

너무 줄이면

국소적 발열효과,

Coupling이 너무 작아진다.

(2) hysteresis를 줄이기 위해 $\beta_c < 1$

즉 $2\pi I_0 R^2 C \leq \Phi_0$

초미세 가공법의 발달에 따라

$C \approx 1-200$ PF 정도 제작 가능

즉 분기저항 R :

$$\frac{2\pi I_0 R^2 C}{\Phi_0} < 1$$

(3) SQUID 변조를 크게 하기 위해서

β ~ 1 즉 $\beta = 2LI_0/\Phi_0 \sim 1$ 인

SQUID 만들자.

실제적 dc SQUID의 종류와 기능.

1964. R.C. Jaklevic

2개의 Josephson Junction 에서 초전도

양자 간섭 효과 발견

Nb 뾰족한 머리 점접촉으로 접합 제조

; 기계적 접촉이기에 매우 비대칭적이였다.

항상 동일한 신호가 나오지 않는다.

Zimmerman : 죠셉슨 접합 1개만 이용하는 rf

SQUID 연구 제안

SHE : 상업적 rf SQUID 제작 판매

1974. John Clarke - 박막형 dc SQUID 제조

민감도 : 상업적 rf SQUID와 비슷

Tesche : dc SQUID의 잡음은 $10^3 - 10^4$ 정도

더 낮춰, 양자극한 까지 갈수 있다 주장

1970말 - 1980 년대 까지

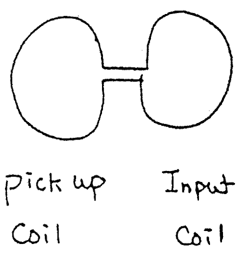
양자적 극한까지의 잡음을 줄이기 위한 여러종류의

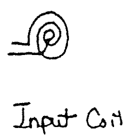
디자인이 제안 잡음에너지 $\varepsilon \simeq h$

문제점 : 입력코일과의 효과적인 결합을 이룰수 없다.

동작점 범위도 매우 좁은 영역에 국한

실제적 응용가능성은 희박해지다.





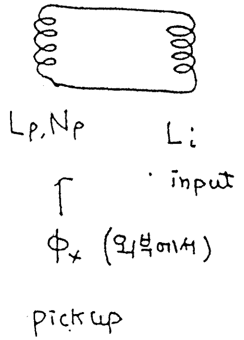
$L_i = L_p$ 일때 signal 의 maximum transfer 이룬다.

Pick up Coil에서 noise current 가 i 이면

이때의 Noise energy는 $\frac{1}{2} L i_n^2$

여기서 $L = \frac{\Phi}{I}$ $\therefore$ $E_{noise} = \frac{1}{2} L \left(\frac{\Phi_n^a}{L} \right)^2 = \frac{\Phi_n^{a\,2}}{2L}$

Φ_n^a 때문에 생겨난 flux



$$N_p \phi_x + (L_p + L_i) i = const$$

$$\underset{squid}{\delta \phi} = M \delta_i = - \frac{M N_p}{L_p + L_i} \delta \phi_x$$

pick-up coil

M : Squid와 input coil 사이의

$$\frac{MN_p}{L_p+L_i} = \frac{MN_p}{\lambda_p N_p^2 + L_i}$$

$$0 = \frac{d}{dN_p} \frac{M(\lambda_p N_p^2+L_i) - MN_p(2\lambda_p N_p)}{(\lambda_p N_p^2+L_i)^2}$$

$$\therefore \quad L_i = \lambda_p N_p^2$$

즉 $L_i = L_p$ 일때 Signal transfer가 maximum이 된다.

이때 Transfer efficiency는

$$= \frac{MN_p}{2L_i} = \frac{1}{2\lambda_p^{1/2}} \cdot \frac{M}{L_i^{1/2}} = \frac{1}{2\lambda_p^{1/2}} K L^{1/2}$$

L : SQUID의 inductance

K : Coupling constant

Nb 조셉슨 접합

1. 기계적으로 매우 강하고 높은 tensile strength 갖는다.
2. T_c 가 높고 Nb_2O_5 는 250°C 이하에서 매우 안정된 절연체 이다

문제점

1. 산소와 반응 잘함 NbO 금속으로 T_c가 매우 낮다
2. Nb_2O_5 와 전극 Nb 사이에 깨끗한 경계 형성 안됨

따라서 leakage current가 매우 크다.

Selective niobium onodization Process

종래의 Nb/Nb_2O_5/Nb 접합특성보다 현저히 향상

가장 안정한 터널 접합이면서 캐피시턴스가 낮은 Nb/Al-AlO_x-(Al)/Nb 조셉슨 접합 제조

Selective niobium etching process

Self-aligned contact process 등의 접합 기술의 발전 — 누설전류가 현저히 낮으면서 갭전위가 날카롭고 임계전류가 높은 조셉슨 접합 등장.

Modulation Coil.

Thermal fluctuation Effect:

$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar}.$$

$$C\frac{dV}{dt} = I - I_c \sin\varphi \; - \frac{V}{R_N} + \hat{I}(t)$$

$\uparrow$
Noise term

"White Noise"

$\langle \tilde{I}(t) \rangle = 0$

$\langle \hat{I}(t)\,\hat{I}(t+T) \rangle = \frac{2k_BT}{R_N}\,\delta(T)$ $\rightarrow \tilde{I}(t)$ are completely uncorrelated.

$$p = m\frac{d\varphi}{dt} = \left(\frac{\hbar}{2e}\right)^2 C\dot{\varphi}$$

$$\dot{p} = -\frac{du}{d\varphi} - \gamma_0 p + \hat{i}(t) \qquad \gamma_0 \equiv \frac{1}{R_N C}$$

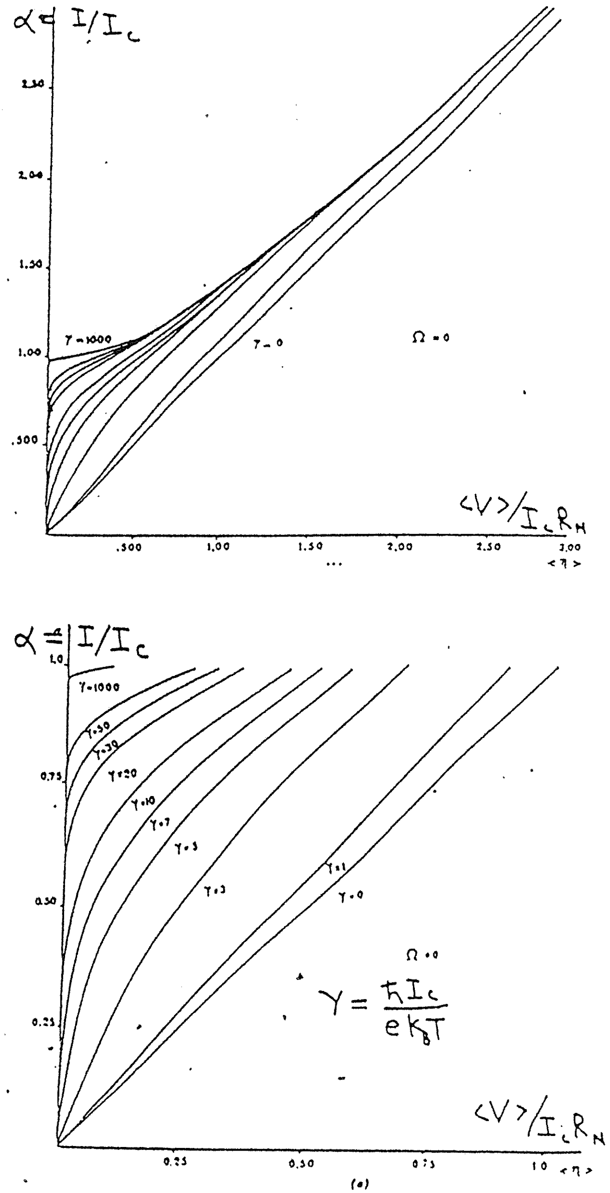
$$\hat{i} \equiv \frac{\hbar}{2e}\,\tilde{I}(t)$$

$$U(\varphi) = -\frac{\hbar I_c}{2e}(\alpha\varphi + \cos\varphi)$$

$$= -J_0(\alpha\varphi + \cos\varphi)$$

$$= -\frac{1}{2}\gamma k_BT(\alpha\varphi + \cos\varphi)$$

$$\gamma \equiv \frac{2J_0}{k_BT} = \frac{\hbar I_c}{ek_BT}$$



(a)

Figure 6.22 Voltage-current characteristics in the presence of thermal fluctuations obtained by numerical integration of (6.4.12) (zero capacitance case). The various curves correspond to different values of the parameter $\gamma = \hbar I_1(T)/(ek_BT)$. Voltage and current are in reduced units: $\alpha = I/I_1$ and $\langle\eta\rangle = \langle V\rangle/I_1R$. (a) and (b) correspond to the same values of γ but in different scales

Shapiro Steps :

when V (≠0) is applied across a junction , the phase difference φ precesses at the rate

$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar} \sim 484\ \mathrm{MHz}/\mu V$$

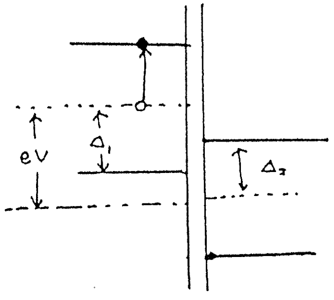
Cooper pairs tunnel with emission of a photon of the freq. above.

Conversely, a microwave field of freq. $\frac{2eV}{\hbar}$ is applied to the jnc, biased at V, Cooper pairs tunnel with the absorption of photons.

→ both dc and RF biased, phase locking at

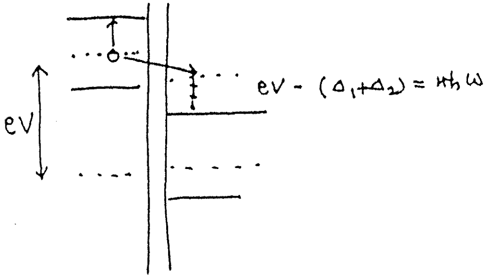
$$\langle V_n \rangle = n\frac{h\nu}{2e} \quad , \quad n = 0, 1, 2, \cdots$$

어떻게 phonon을 흡수하나?



$\Delta_1 + \Delta_2 - eV = \hbar\omega$

photon - absorption Process



$eV - (\Delta_1 + \Delta_2) = \hbar\omega$

photon - emission process

dc + RF biased

$$\hbar\frac{d\varphi}{dt} = 2e\,(\,V + v\cos(\omega t + \theta))$$

Assuming that the jnc. is voltage fed

$$\varphi(t) = \frac{2eV}{\hbar}t + \frac{2ev}{\hbar\omega}\sin(\omega t + \theta) + \varphi_0$$

then

$$I = I_c \sin\varphi$$

$$= I_c \sin\left\{\frac{2eV}{\hbar}t + \frac{2ev}{\hbar\omega}\sin(\omega t + \theta) + \varphi_0\right\}$$

Mathematical Identity

$$e^{ic\sin x} = \sum_{n=-\infty}^{\infty} J_n(c)\, e^{inx}$$

$$J_{-n}(x) = (-1)^n J_n(x)$$

OR

$$\cos(c\sin x) = \sum_{n=-\infty}^{\infty} J_n(c)\cos nx$$

$$= J_0(c) + 2\sum_{n=1}^{\infty} J_{2n}(c)\cos 2nx$$

$$\sin(c\sin x) = \sum_{n=-\infty}^{\infty} J_n(c)\sin nx$$

$$= 2\sum_{n=1}^{\infty} J_{2n-1}(c)\sin(2n-1)x$$

Therefore

$$I = I_c \sin \left\{ \frac{2ev}{\hbar} t + \frac{2ev}{\hbar\omega} \sin(\omega t + \theta) + \varphi_0 \right\}$$

$$= I_c \left[\sin\left(\frac{2eV}{\hbar} t + \varphi_0 \right) \cos\left\{ c \sin(\omega t + \theta) \right\} \right.$$

$$+ \cos\left(\frac{2eV}{\hbar} t + \varphi_0 \right) \sin\left\{ c \sin(\omega t + \theta) \right\}$$

$$= I_c \left\{ \sin\left(\frac{2eV}{\hbar} t + \varphi_0 \right) \left[J_0(c) + 2 \sum_{n=1}^{\infty} J_{2n}(c) \cos 2n(\omega t + \theta) \right] \right.$$

$$\left. + \cos\left(\frac{2eV}{\hbar} t + \varphi_0 \right) 2 \sum_{n=1}^{\infty} J_{2n-1}(c) \sin(2n-1)(\omega t + \theta) \right\}$$

$$= I_c \left\{ J_0(c) \sin(\quad) \right.$$

$$+ 2J_1 \cos(\quad) \sin(\omega t + \theta) \begin{array}{l} \nearrow -J_1(c) \sin(\quad) \\ \searrow J_1(c) \sin(\quad) \end{array}$$

$$+ \cdots$$

$$= I_c \sum_{n=-\infty}^{\infty} (-1)^n J_n(c) \sin\left(\frac{2ev}{\hbar} t + \varphi_0 - n\omega t - n\theta \right)$$

$$= I_c \sum_{n=-\infty}^{\infty} (-1)^n J_n\left(\frac{2ev}{\hbar\omega} \right) \sin\left\{ \left(\frac{2ev}{\hbar} - n\omega \right) t - n\theta + \varphi_0 \right\}$$

i) If the supercurrent is not synchronous to the RF signal → Jnc. serves as a mixer, generating intermediate freq.

$$\omega_i = \left| \frac{2ev}{\hbar} - n\omega \right|$$

with amplitude

$$I_0 J_n\left(\frac{2ev}{\hbar\omega} \right)$$

ii) $\langle I(t) \rangle$ vanishes, unless the phase of the Josephson osc. is locked to the phase of the applied signal → spike-like voltage steps in $\langle I \rangle - \langle v \rangle$ characteristics

at $$\langle v \rangle = \frac{n\hbar\omega}{2e} \qquad n = 0, 1, 2, 3, \cdots$$

with the magnitude

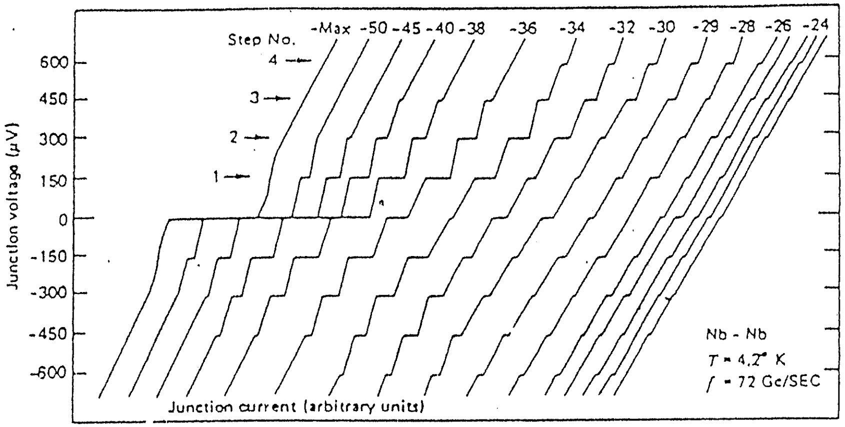
$$I_n = (-1)^n I_c J_n\left(\frac{2ev}{\hbar\omega} \right) \sin(\varphi_0 - n\theta)$$

Step width

$$I_n^{(step)} = I_c J_n\left(\frac{2ev}{\hbar\omega} \right)$$

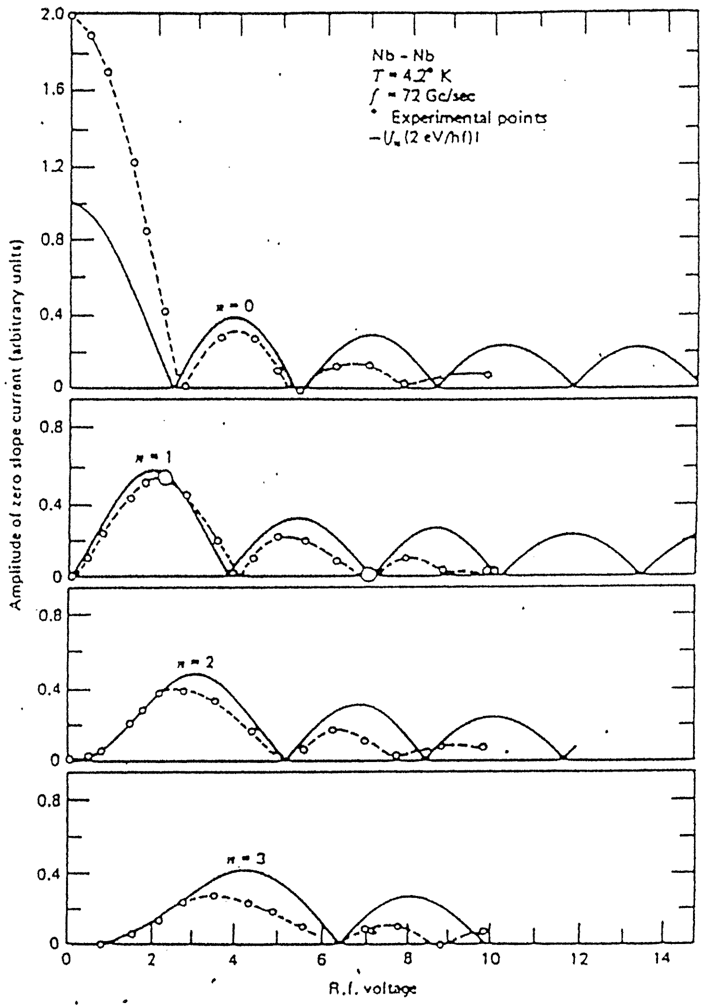
Zero voltage supercurrent

$$I_0^{(step)} = I_c J_0\left(\frac{2ev}{\cdot} \right) \xrightarrow{\frac{2ev}{\hbar\omega} \ll 1} I_c\left(1 - \frac{2e^2v^2}{\hbar^2\omega^2} \right)$$



(a)

Figure 11.1 (*a*) Voltage-current curves for an Nb-Nb point contact Josephson junction exposed to a 72 Gc/sec signal at various power levels. (*b*) Data from (*a*) plotted to show how the current in several constant voltage steps varies as the applied r.f. voltage is varied. The data points from the *n*th step are compared with the amplitude of the *n*th order Bessel function. The data are fitted to the theoretical curves at the two points denoted by double circles. The r.f. voltage across the junction is expressed in units of $h\nu/2e$ or 149 μV/div. (After Grimes and Shapiro 1968.)



$V \neq 0$: ac effect

$$J_x(y,t) = J_c \sin \varphi(y,t)$$

$$\frac{\partial \varphi}{\partial y} = \frac{2\pi}{\phi_0}(2\lambda+d)h$$

$$\frac{\partial \varphi}{\partial t} = \omega(t) = \frac{2eV}{\hbar}$$

Neglecting the screening due to Josephson current

$h \to H$

$V = V_0$ everywhere

$$\varphi(y,t) = \varphi_0 + \omega_0 t + k_0 y$$

$$\omega_0 = \frac{2eV_0}{\hbar}, \quad k_0 = \frac{2\pi(2\lambda+d)H}{\phi_0}$$

The periodic current distribution moves in the y direction with a phase velocity,

$$v_0 = \frac{\omega_0}{k_0} = \frac{cV_0}{(2\lambda+d)H}$$: Vortex motion

$$\nabla \times \vec{e} = -\frac{1}{c}\frac{\partial \vec{h}}{\partial t}$$

h : extends λ into the s.c.

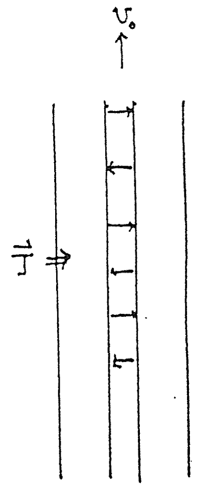
e : negligible except in the barrier

by integration

$$\oint \vec{e}\cdot d\vec{e} = -\frac{1}{c}\frac{\partial}{\partial t}\int \vec{h}\cdot d\vec{s}$$

$$e_x d - \left(e_x + \Delta y \cdot \frac{\partial e_x}{\partial y}\right)d = -\frac{1}{c}\frac{\partial}{\partial t} h_z (2\lambda+d)\Delta y$$

$$\frac{\partial e_x}{\partial y} = \frac{1}{c}\,\frac{2\lambda+d}{d}\,\frac{\partial h_z}{\partial t}$$



and

$$\nabla \times \vec{h} = \frac{4\pi}{c}\vec{J} + \frac{\epsilon}{c}\frac{\partial \vec{e}}{\partial t}$$

$$\frac{\partial h_z}{\partial t} = \frac{4\pi}{c}J_x + \frac{\epsilon}{c}\frac{\partial e_x}{\partial t}$$

$$\frac{\partial^2 h_z}{\partial t \partial y} = \frac{4\pi}{c}\frac{\partial J_x}{\partial t} + \frac{\epsilon}{c}\frac{\partial^2 e_x}{\partial t^2}$$

$$\frac{1}{c}\,\frac{2\lambda+d}{d}\,\frac{\partial^2 h_z}{\partial y \partial t} = \frac{\partial^2 e_x}{\partial y^2}$$

$$\therefore \frac{4\pi}{c}\frac{\partial J_x}{\partial t} + \frac{\epsilon}{c}\frac{\partial^2 e_x}{\partial t^2} = \frac{cd}{2\lambda+d}\frac{\partial^2 e_x}{\partial y^2}$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{(2\lambda+d)\epsilon}{c^2 d}\frac{\partial^2 V}{\partial t^2} + \frac{4\pi}{c^2}(2\lambda+d)\frac{\partial J_y}{\partial t}$$

$$V = ed$$

OR

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{\bar{c}^2} \frac{\partial^2}{\partial t^2} \right) V = \frac{4\pi}{c^2} (2\lambda + d) \frac{\partial J_x}{\partial t}$$

where $\bar{c}^{\,2} = \frac{c^2}{\epsilon} \frac{d}{2\lambda + d} \ll c^2$

$$\bar{c} \xrightarrow[d \simeq 10Å]{\lambda \simeq 500Å} c/20$$

$\epsilon = 4$

for $\nu = 10^{10}$ Hz

$\lambda = 3$ cm in free space

but $\lambda_{jnc} \sim 1$ mm

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{\bar{c}^2} \frac{\partial^2}{\partial t^2} \right) \frac{\hbar}{2e} \frac{\partial \varphi}{\partial t} = \frac{4\pi}{c} (2\lambda + d) \frac{\partial}{\partial t} (J_c \sin\varphi)$$

OR

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{\bar{c}^2} \frac{\partial^2}{\partial t^2} \right) \varphi = \frac{1}{\lambda_J^2} \sin\varphi$$

where $\lambda_J^2 = \frac{c\, \phi_0}{8\pi^2 J_c (2\lambda + d)}$

Take into account the dissipation term,

$J_x = J_c \sin\varphi + \sigma_N(V)\, V$

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{\bar{c}^2} \frac{\partial^2}{\partial t^2} - \frac{\beta}{\bar{c}^2} \frac{\partial}{\partial t} \right) \varphi = \frac{\sin\varphi}{\lambda_J^2}$$

$\beta = \sigma_N / C$

$C = \frac{\epsilon A}{4\pi d}$

i) Weak coupling limit

$\lambda_J \longrightarrow \infty$

$\varphi \sim e^{i(\omega t - ky)}$

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{\bar{c}^2} \frac{\partial^2}{\partial t^2} \right) \varphi = 0$$

electromagnetic phase waves with phase velocity $\bar{c}$.

if $v_0 \hat{=} \bar{c} \longrightarrow v_0^2 \equiv \frac{c^2 V_0^2}{(2\lambda+d)^2 H^2} = \frac{c^2}{\epsilon} \left(1 + \frac{2\lambda}{d}\right)^{-1} \equiv \bar{c}$

$$V_0^2 = \frac{d\,(2\lambda + d)\, H^2}{\epsilon_0}$$

- Electromagnetic waves couple strongly to the Josephson current → a peak in the average current.
- Maximum dissipation peak.

ii) $\omega \ll \lambda_J$: φ uniform over the junction

$\frac{d^2\varphi}{dt^2} + \frac{\bar{c}^2}{\lambda_J^2} \sin\varphi = 0$ eq. of motion of a pendulum

$\frac{d^2\varphi}{dt^2} + \frac{\bar{c}^2}{\lambda_J^2} \sin\varphi = 0$ eq. of motion of a pendulum

$$\omega_J^2 : \frac{\bar{c}^2}{\lambda_J^2} = \frac{c^2}{\epsilon} \frac{d}{2\lambda + d} \cdot \frac{8\pi^2 J_c (2\lambda + d)}{c\, \phi_0 \frac{hc}{2e}}$$

$$= \frac{8\pi e d J_c}{\epsilon \hbar} = \left(\frac{4\pi d}{\epsilon A} \right) \left(\frac{2e A d J_c}{\hbar} \right)$$

$J_c = \frac{2e\hbar |\Psi|^2}{\ }$ $J = J_c \sin\varphi \sim J_c \varphi$ $= J_c d\varphi = 2e|\Psi|^2 \frac{\hbar}{\ } \varphi$

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{\bar{c}^2}\frac{\partial^2}{\partial t^2}\right) V = \frac{4\pi}{c^2}(2\lambda + d)\frac{\partial J_x}{\partial t}$$

We deal with a, s.t. $v/v_0 \ll 1 \Rightarrow J_x = J_c \sin(\omega_0 t - kx)$

Sol.

$$V = V_0 \cos(\omega_0 t - k_0 y + \theta)$$

$$V_0 = \frac{\{4\pi(2\lambda+d)/\omega_0\} J_c}{\left\{\left[1-\left(\frac{k_0\bar{c}}{\omega}\right)^2\right]^2 + \left(\frac{1}{Q}\right)^2\right\}^{1/2}}$$

$$\theta = \tan^{-1}\frac{1/Q}{1-\left(\frac{k_0\bar{c}}{\omega_0}\right)^2}$$

$$\frac{\partial\varphi}{\partial t} = \frac{2eV}{\hbar}$$

ECK, Scalapino, Taylor
PRL 13, 15 (1964)

$$\varphi = \omega_0 t - k_0 y + \frac{v_0}{V_0}\sin(\omega_0 t - k_0 y + \theta)$$

$$J_x = J_c \sin\varphi$$

$$J_{DC} = J_c \frac{v_0}{V_0}\sin\theta$$

$$= J_c \cdot \frac{4\pi(2\lambda+d)J_c}{\omega_0 V_0} \qquad \frac{1/Q}{\left[1-\left(k_0\bar{c}/\omega_0\right)^2\right]^2 + \left(\frac{1}{Q}\right)^2}$$

Resonance maximum at

$$\bar{c} = \frac{\omega_0}{k_0}$$

phase velocity associated with current distribution

= phase velocity of the electromagnetic fields

※ Uniform φ over the junction (barrier)

1) $h \propto \frac{d\varphi}{dy} = 0$ No magnetic field in the barrier.

2) Electric field ⊥ plane of the barrier

3) longitudinal plasma waves, originating from energy exchange.

$$\frac{\hbar J_c(1-\cos\varphi)}{2e} \longleftrightarrow \frac{(2en)^2}{2C}$$

K.E P.E

4) $\omega_J = \left(\frac{2eI_c}{\hbar c}\right)^{1/2} = \frac{\bar{c}}{\lambda_J} \ll \omega_p$

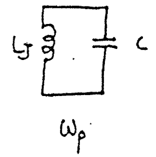
ordinary plasma freq. for metals

due to small density of charge carriers in the barrier (peculiar to weak s.c.)

5) $V = L_J \frac{dI}{dt} = L_J I_c \cos\varphi \frac{d\varphi}{dt}$

$$= \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

$$L_J = \frac{\hbar}{2eI_c\cos\varphi} \underset{\varphi\simeq 0}{\simeq} \frac{\hbar}{2eI_c}$$



$$\omega_J = \frac{1}{\sqrt{LC}} = \left(\frac{2eI_c}{\hbar c}\right)^{1/2}$$

$$\frac{d^2 \delta\varphi}{dt^2} + \omega_J^2 \sin(\varphi_0 + \delta\varphi) = 0$$

$$\frac{d^2}{dt^2}\delta\varphi + \underset{\omega^2}{(\omega_J^2 \cos\varphi_0)}\,\delta\varphi = 0 \qquad \text{linearized}$$

$$\omega^2 = \omega_J^2 \cos\varphi_0$$

$$= \omega_J^2 \cos\left[\sin^{-1}\left(\frac{J_a}{J_c}\right)\right]$$

$$= \omega_J^2 \left[1 - \left(\frac{J_a}{J_c}\right)^2\right]^{1/2}$$

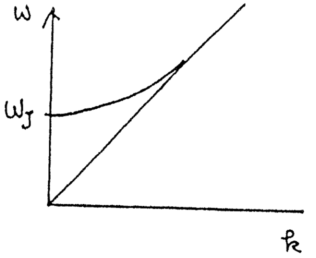
iii) linearized about $\varphi = 0$

$$\frac{\partial^2\varphi}{\partial y^2} - \frac{1}{\bar{c}}\frac{\partial^2\varphi}{\partial t^2} = \frac{\varphi}{\lambda_J^2} \quad : \quad \varphi \sim e^{i(\omega t - k y)}$$

$$-k^2 + \frac{\omega^2}{c^2} = \frac{1}{\lambda_J^2}$$

OR

$$\omega^2 = \omega_J^2 + k^2 \bar{c}^2$$



ω_J : lowest freq. which allows the preparation of e.m waves inside the jnc.

if $\omega = 0 \rightarrow k = \pm i/\lambda_J$

$$\varphi \sim e^{\pm y/\lambda_J} \quad (y \gtrless 0)$$

· Limit of sensitivity of DC SQUID

1) SQUID 자체의 Josephson Noise에 연유

2) In practice, the limit is set by the Noise in the room - temperature circuit.

$$\langle \delta V^2 \rangle = 4K_B T R' B$$

$$\langle \delta I^2 \rangle = \frac{\langle V^2 \rangle}{R'^2 + \omega^2 L^2} = \frac{4K_B T R' B}{R'^2 + \omega^2 L^2}$$

$$\omega \ll R'/L \qquad = 4K_B T B / R'$$

$$\langle \delta\phi^2 \rangle = \langle \delta I^2 \rangle L^2 = 4K_B T L^2 B / R'$$

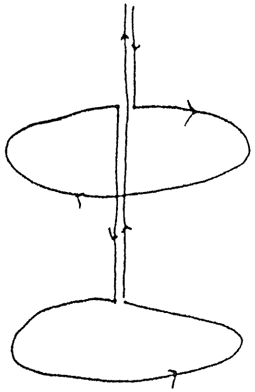
$$\longrightarrow \quad 0.5 \times 10^{-5} \phi_0$$

In the case of high T_c SQUID, the operating temperature is 10 - 100 times higher.

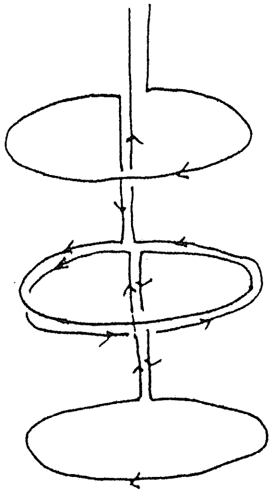
$$\sqrt{\langle \delta\phi^2 \rangle}\Big|_{\text{high } T_c} \sim 3 \sim 10 \text{ of } \delta\phi \text{ of the Low } T_c \text{ SQUID.}$$

Current at the weak link

Magnetic field gradiometers



$\sim \frac{\partial H_z}{\partial z}$

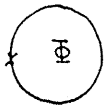


$\sim \frac{\partial^2 H_z}{\partial z^2}$

If Ring is not superconducting : $\phi = \phi_x$

RF (Single Contact SQUID)

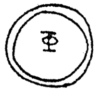
- Mechanically stable
- Easy to fabricate



total phase change around the ring

$$\Delta\varphi = \frac{1}{\hbar}\oint \vec{p}\cdot d\vec{l}$$

$$= -\frac{2e}{\hbar c}\oint \vec{A}\cdot d\vec{l} = 2n\pi$$



OR $\Delta\Phi = n\phi_0$

$\Phi_{max} = \frac{1}{2}\phi_0$

Now Put a weak link in the s.c. ring

$I_c^{w.l} \ll I_c^{loop}$

$$\Delta\varphi = \theta - \frac{2e}{\hbar c}\Phi = 2n\pi$$

$$\downarrow$$

$$-\frac{m}{\rho_s e\hbar}\int_{-a}^{a} j_s \cdot dl \qquad \theta - 2\pi\cdot\frac{\Phi}{\Phi_0} = 2n\pi$$

Current at the weak link

$I_s = - I_c \sin\theta$

$= - I_c \sin\left(2\pi \frac{\phi}{\phi_0}\right)$

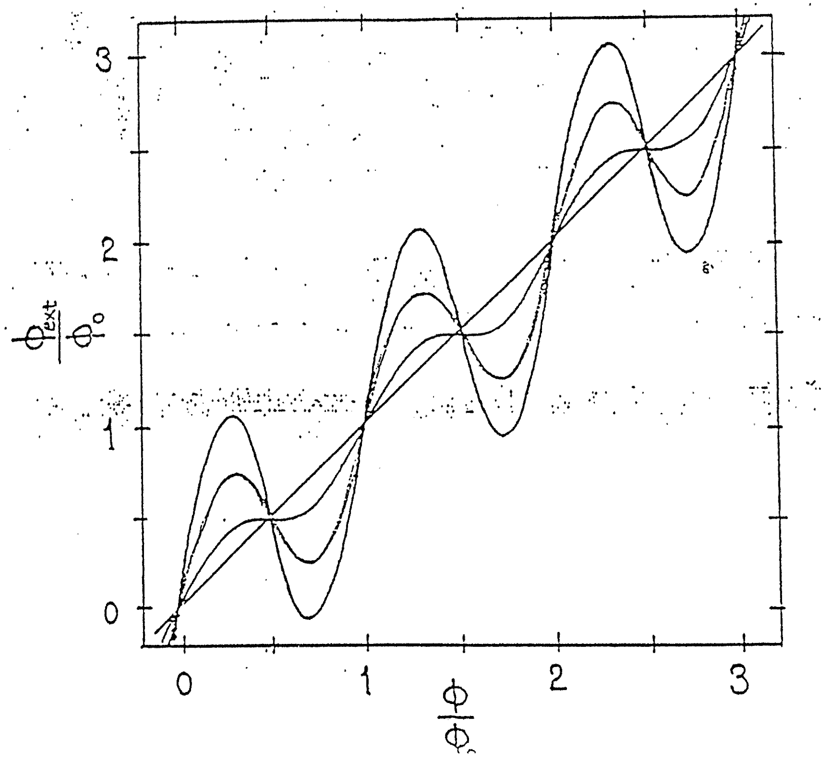
1~2 µA for typical point contact jnc.

ϕ = total flux

$= \phi_{ext} + L I_s$

$= \phi_{ext} - L I_c \sin\left(2\pi \frac{\phi}{\phi_0}\right)$

$$\frac{\phi}{\phi_0} + \frac{L I_c}{\phi_0} \sin\left(2\pi \frac{\phi}{\phi_0}\right) = \frac{\phi_{ext}}{\phi_0}$$



If Ring is not superconducting : $\phi = \phi_x$

Suppose the ring is S.C. & starts with $\phi_x = 0$

① $\phi = 0$

② increase ϕ_x (with $\phi = 0$) until $i \to i_c$

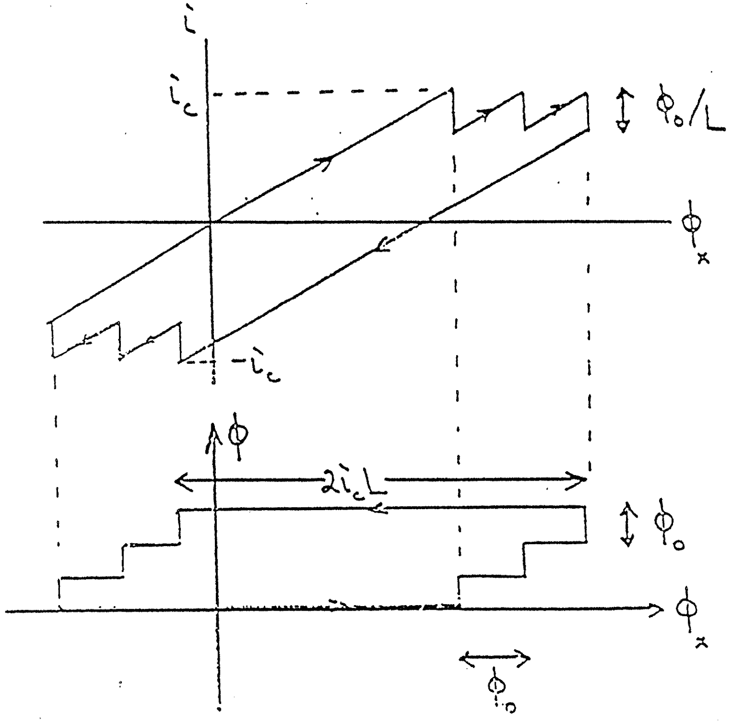
③ ring becomes normal

ϕ_0 admitted inside the ring, $\phi = \phi_0$

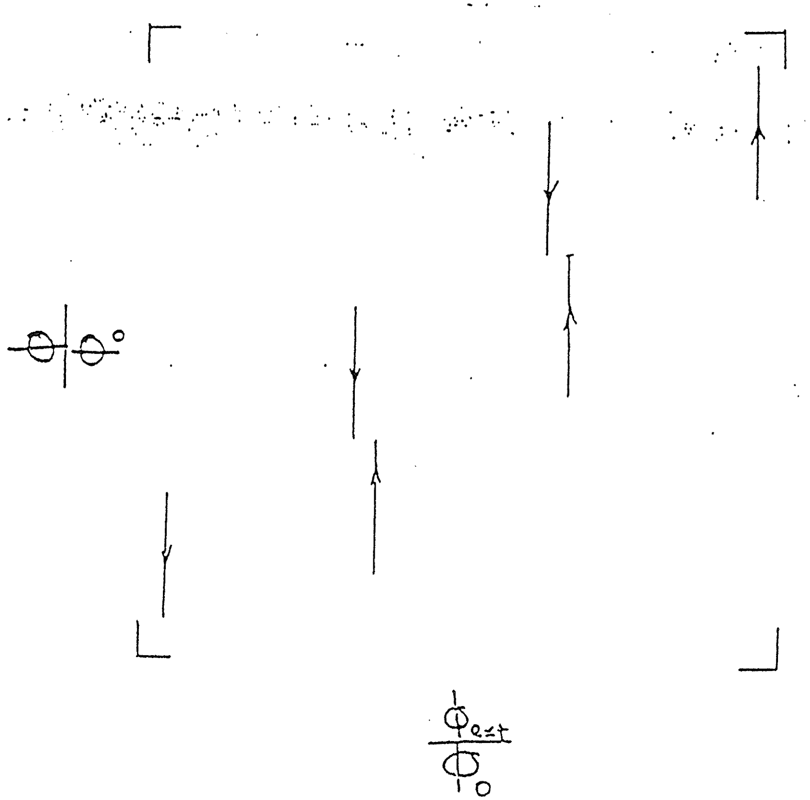
④ $i \to i - \phi_0/L$

⑤ ring becomes S.C. again with $\phi = \phi_0$

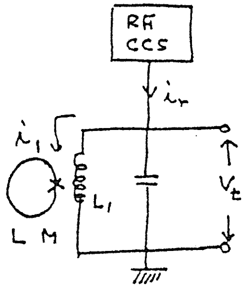
⑥ as ϕ_x increased ① - ⑤ repeated



$\phi = \phi_x - iL = n$



Q1) How do we bias this ring with a weak link to make a useful device?



$i_1 = i_{DC} + i_t$

$i_t = Q\, i_r$

: In steady state only ignoring dissipation

$Q = 1.$

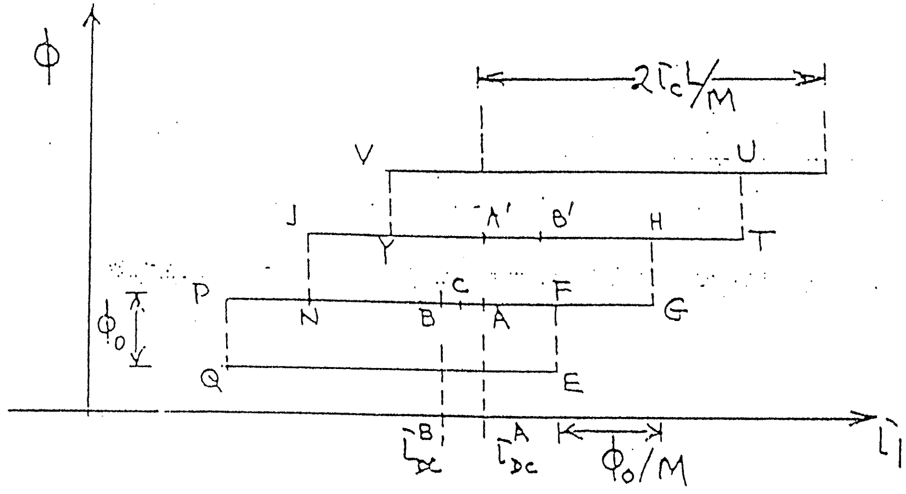
$\phi_x = M\, i_1$: Tank circuit injecte flux ϕ_x into the ring

$V_t = \omega L_1 i_t$

i_r : a convenient controllable parameter

V_t : conveniently measurable parameter.

Using the relation $\phi_x = M i_1$, we can convert ϕ vs ϕ_x to ϕ vs i_1 (6-6)



Q2). What happen to the system for a certain unique Values of Dc bias current

1) $i_{DC} = i_{DC}^{A}$

$i_r < \overline{AG}$ $\quad \phi = \text{const}$

$i_r > \overline{AG}$ $\quad$ energy □ out of the tank circuit

$i_r > \overline{AT}$  odd # hysteresis.

ii) $i_{DC} = i_{DC}^{B}$

$i_r > \overline{BG}$  ,

even # of hysteresis enclosed.

Q3) What happen to the Voltage V_t as i_r varies?

Suppose $i_{DC} = i_{DC}^{A}$

$i_r < \overline{AG}$ $\quad$ no loop enclosed

$V_t = \omega L_1 i_t \propto i_r$ linear

$i_r > \overline{AG}$

① a loop is enclosed

② energy absorbed from the tank circuit.

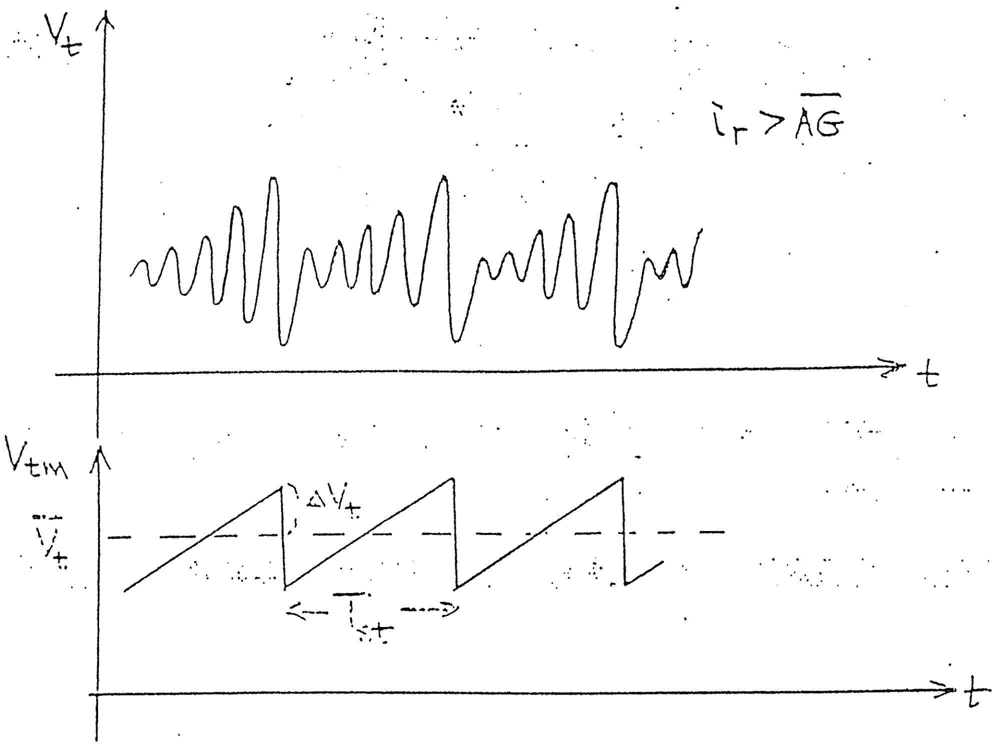
$\Delta E = \frac{1}{L} \oint \phi \, d\phi_x$

$= 2 i_c \phi_0$

③ instantaneous decrease of V_t

i.e, i_t is out of equilibrium with i_r, $i_t \doteqdot i_r$

flowed by recovery with τ determined by R, C & L

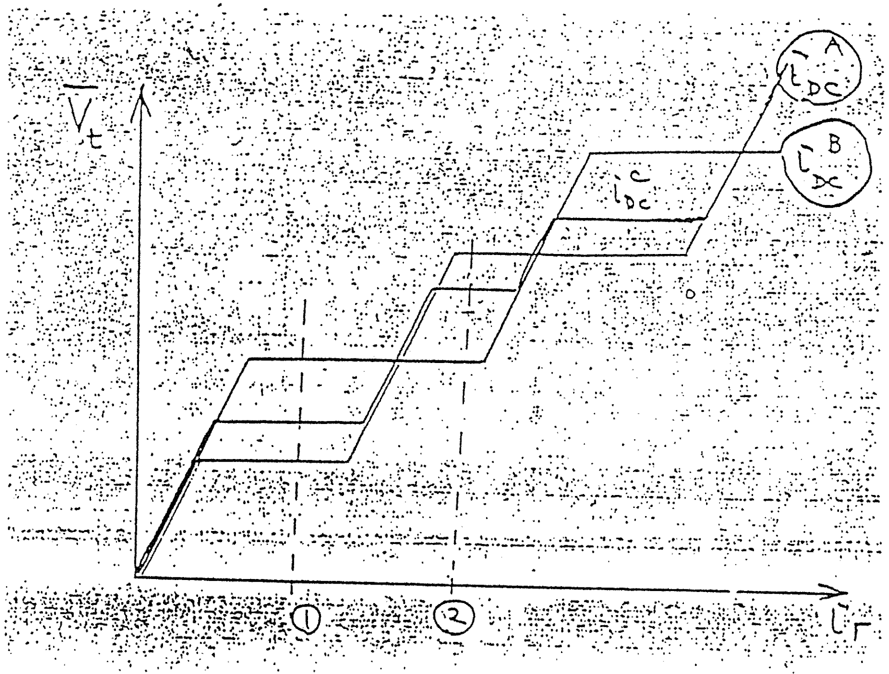


As one increases i_r, ΔV_t, $\overline{V_t}$ remain unchanged, but $T_{\sigma T}$ gets shorter.

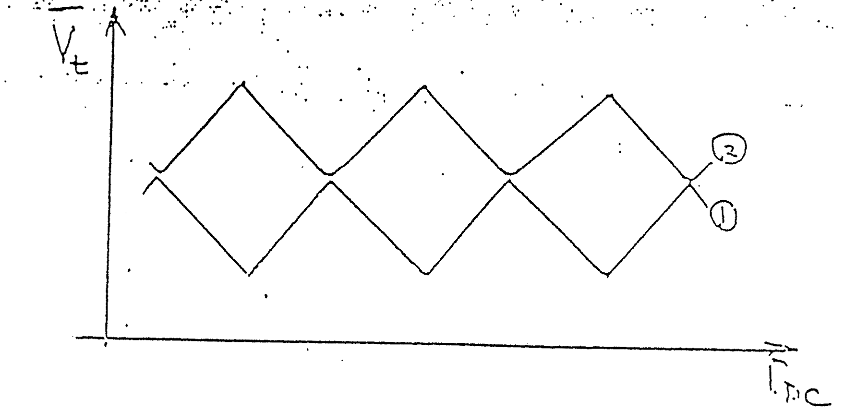
Eventually one reaches the point, where

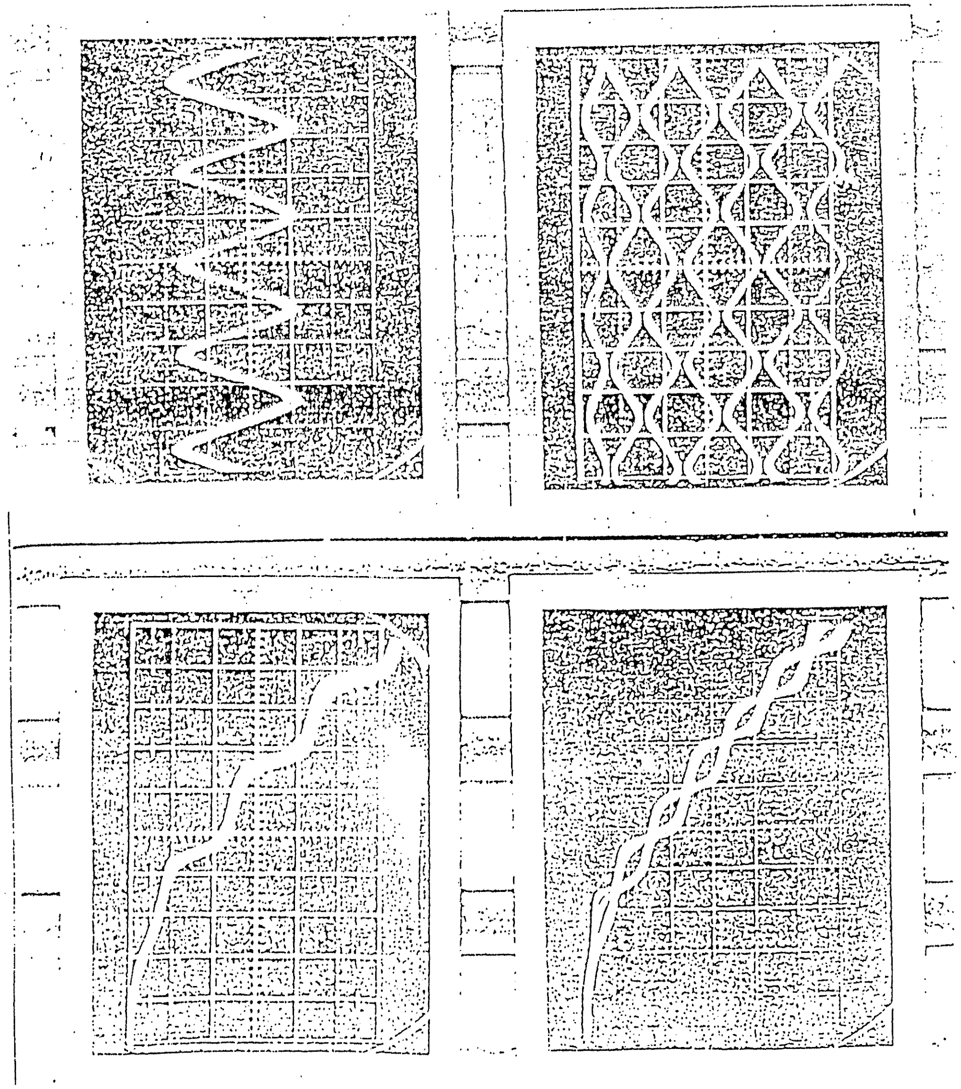
$T_{\sigma T} = T_{R\bar{H}}$

then $\overline{V_t}$ starts to increase again

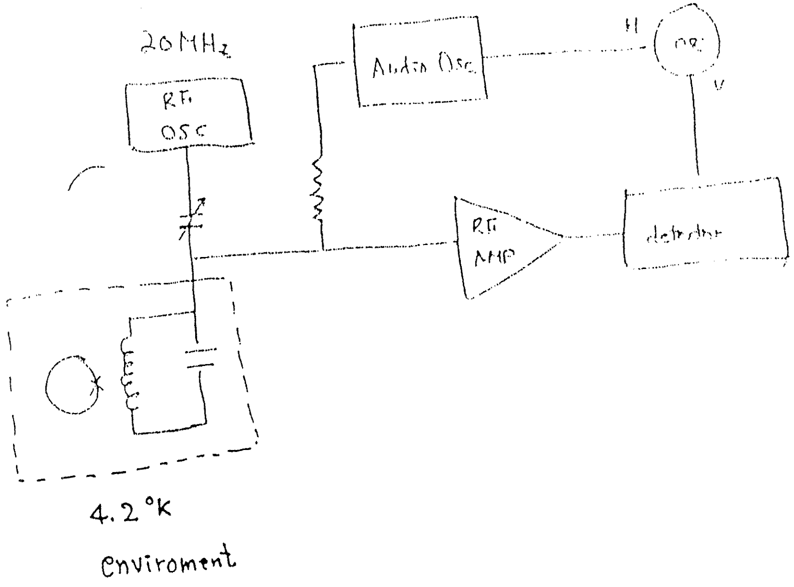


If we fix i_r and change i_{DC}





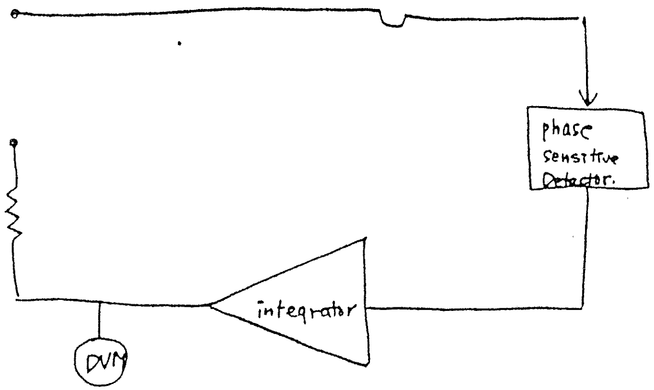
Q4) How can we detect flux change?



① triangular pattern on the OSC.

② $\Delta\phi$ in the SQUID moves triangular pattern on the screen

By counting the # of shift ⇔ measure the flux change



PSD mode with feedback

① Audio Osc. provides both DC bias i_{DC} and reference to Lock-in

② $\Delta\phi$ in SQUID shift V_t vs i_p

③ inducing phase change between reference & output

④ PSD output changes

⑤ Measure this.

SHE CORPORATION

MODEL BMS BASIC SQUID SYSTEM

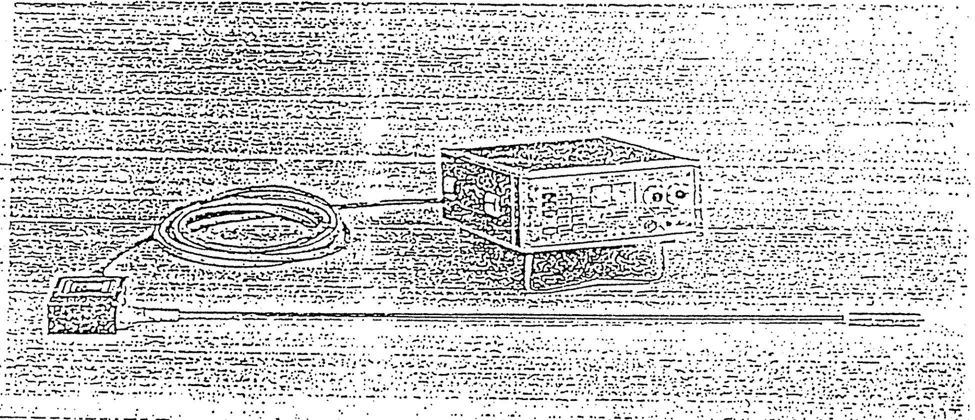


Fig. 1. Model BMS Basic SQUID System

FEATURES

- Large Dynamic Range
- Simple, Reliable Operation
- Unsurpassed dc Stability
- Fully Remote Tuning and Operation

The Model BMS is the most versatile, easiest to use SQUID-based measurement system available. It incorporates a number of unique, important features that dramatically improve performance, both in the laboratory and under the most difficult field conditions. Other SHE SQUID systems use the BMS as their core element, along with special input circuits and additional electronics, to measure a wide variety of parameters including resistance, inductance, voltage, magnetic field, and magnetic susceptibility. The specifications given here describe fully the Model BMS Basic Measuring System and also provide the basis for the specifications of other SHE SQUID systems.

The major components of the Model BMS are shown in Fig. 1.

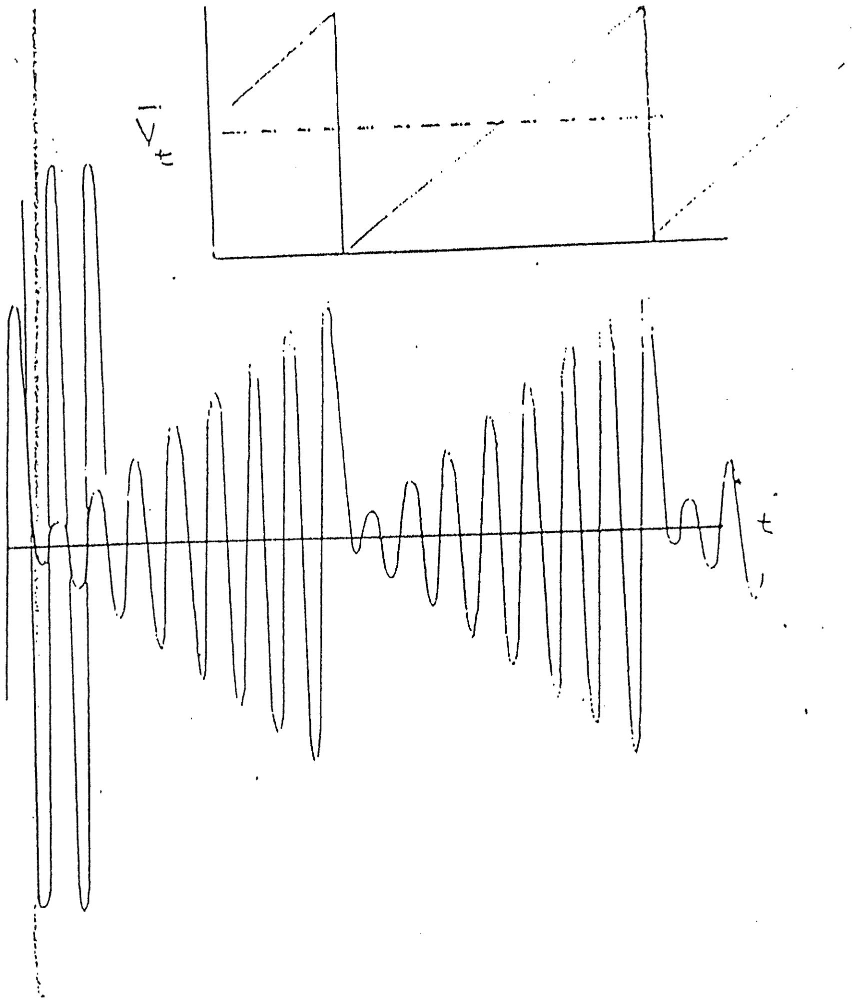
- Model SP or SPO probe with TSQ SQUID sensor
- Model 300 *rf* head
- Model 30 control unit

An optional version of the BMS, the BMSX, is available which utilizes a lower noise *rf* head, the Model 300X, and a specially selected SQUID sensor, the Model TSQX, to provide a significantly lower noise level. The BMSX can be directly substituted for the BMS in all applications.

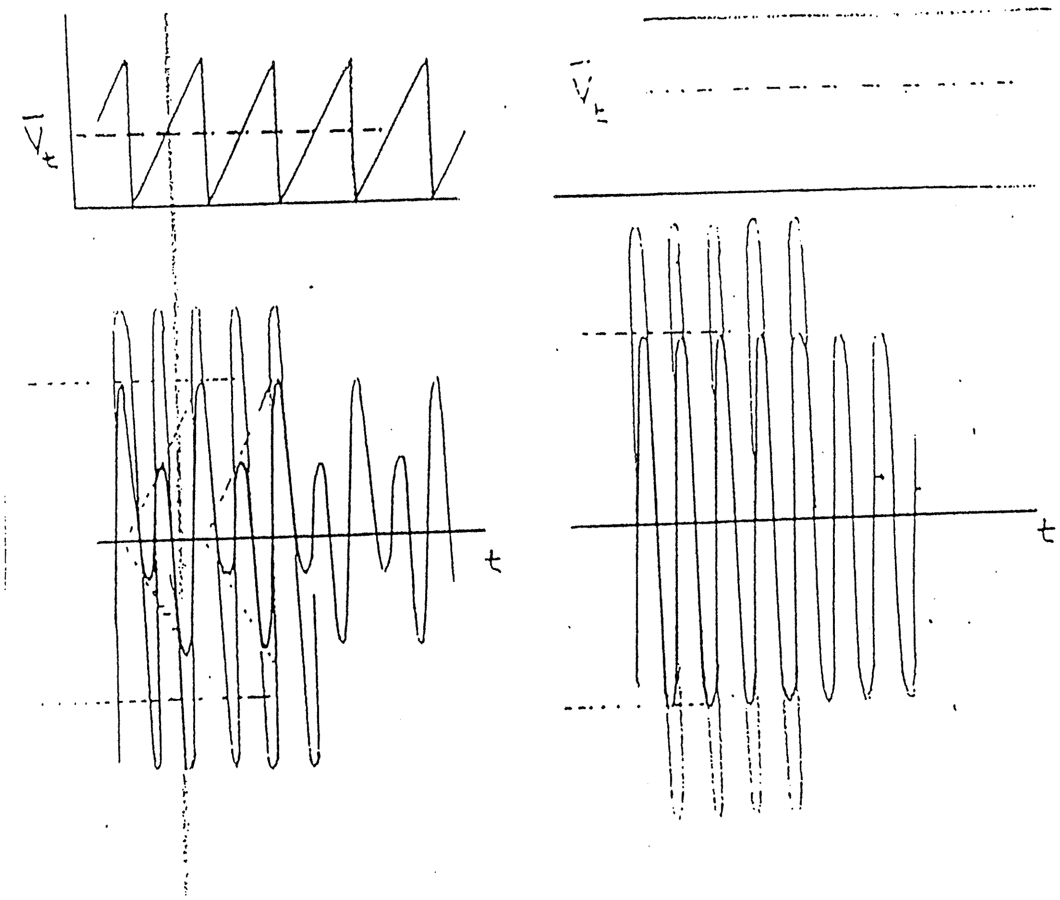
BASIC OPERATING PRINCIPLES

The Model BMS is an ultra-low-noise, current sensitive amplifier whose output voltage changes in proportion to the change in current at its input. The input signal is joined to the SQUID sensor, located in liquid helium at the bottom of the SQUID probe, by a pair of superconducting screw terminals. Current through these terminals couples magnetic flux into the SQUID sensor via an integral 2 μH input coil. The sensor also contains an *rf* coil which is used to inject a 19 MHz bias signal from the Model 300 *rf* head. The amplitude of the *rf* bias in the sensor is modulated by the input signal, detected by the *rf* head, and transmitted via an interconnecting cable to the Model 30 control unit. There it is used to generate a negative feedback signal which cancels the flux change in the sensor caused by the input signal. The amplitude of this feedback signal is therefore proportional to the input current and is used to generate the system output voltage.

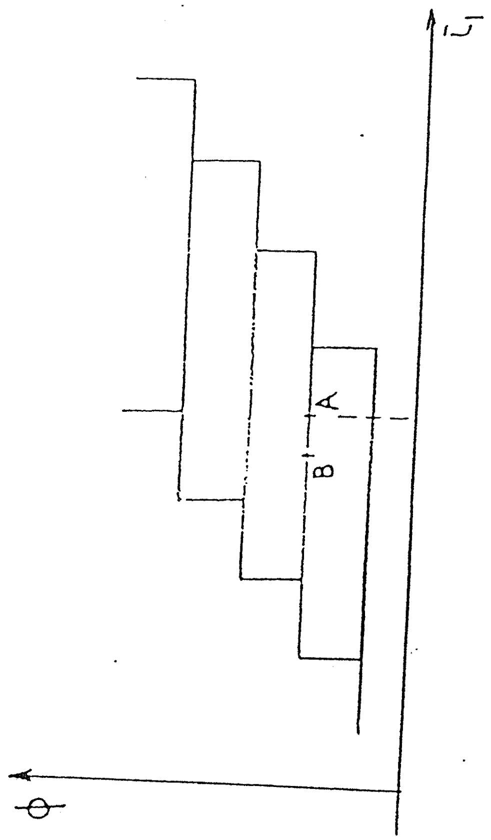
6-71

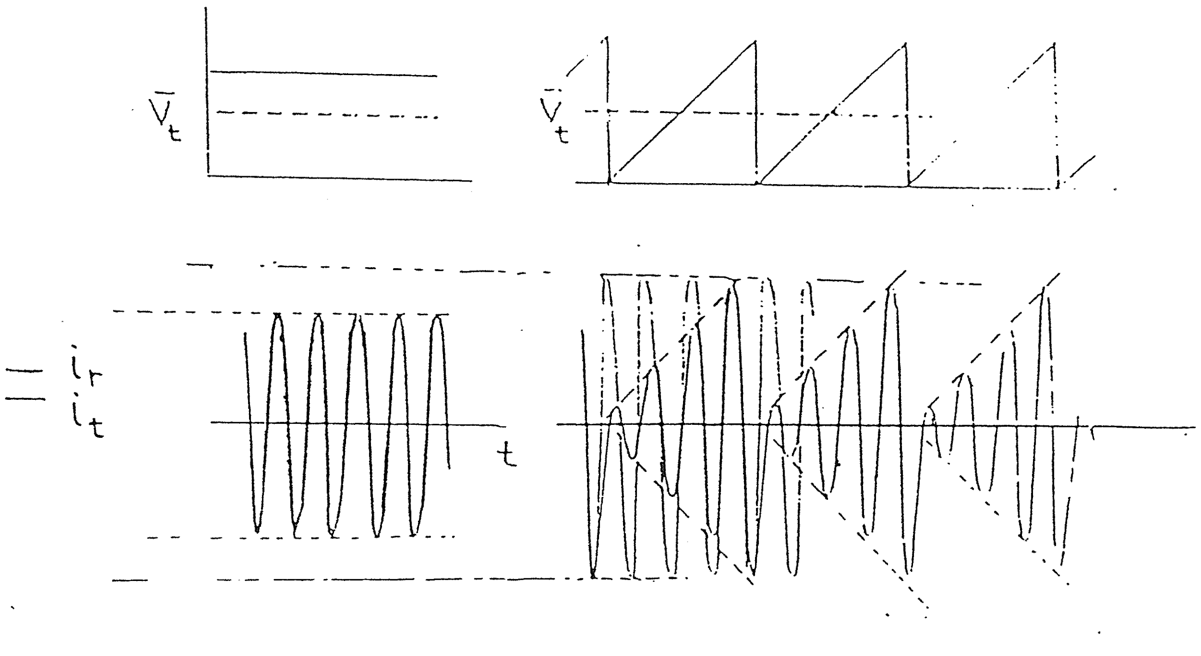


6-72



6-7





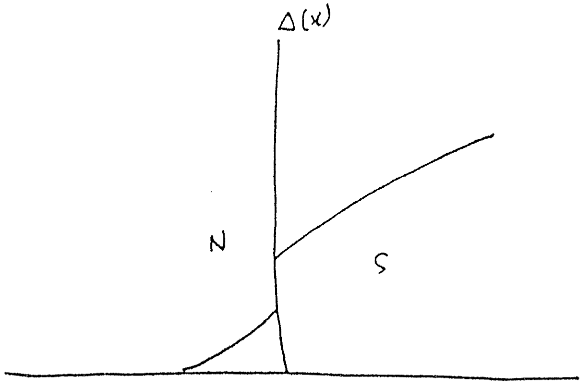
$i_r < A\bar{G}$ $i_r > A\bar{G}$

SNS junctions :

at the S-N interface - Proximity Effect.

- Reducing the S.C. Character of S.C.
- extending S.C. prop. into the normal metals.
- varying DOS, e-e int, etc → Gorkov theory

  (not BCS)



Order parameter in the N-region,

$$\Delta_N(x) = \Delta_N(0_-)\, e^{-|x|/\xi_N(T)}$$

where
$$\xi_N(T) = \begin{cases} \left(\frac{\hbar D_N}{2\pi K_B T}\right)^{1/2}, & l_N \ll \xi_N(T) \\ \frac{\hbar v_{FN}}{2\pi K_B T} & l_N \gg \xi_N(T) \end{cases}$$

$D_N = \frac{1}{3} v_{FN}\, l_N$: diffusion coefficient

v_{FN} : Fermi velocity

l_N : mean free path

} In the normal metals.

When "N" is a normal metal with $T_{cN} < T_{cs}$

$$\xi_N(T) = \left(\frac{\hbar D_N}{2\pi K_B T}\right)^{1/2} \left(1 + \frac{2}{\ln(T/T_{cN})}\right)$$

boundary Conditions : dirty limit

$$\frac{\Delta_N(0_-)}{N_N} = \frac{\Delta_s(0^+)}{N_s}$$ De Gennes, 1964

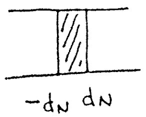
and
$$D_N \left(\frac{d\Delta_H}{dx}\right)_{0^-} = D_s\left(\frac{d\Delta_s}{dx}\right)_{0^+}$$

SNS junctions:

$I = I_c \sin\varphi$. $I_c(T) \sim (T_{cs}-T)^2 e^{-2d_N/\xi_N}$

$\sim (T_{cs}-T)^2$

$T \simeq T_{cs}$



Cf: SIS junction

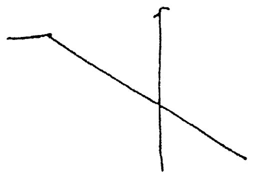
$$I_c(T) = \frac{\pi\Delta(T)}{2eR_N}\tanh\left(\qquad - \frac{\Delta(T)}{2k_BT}\right)$$

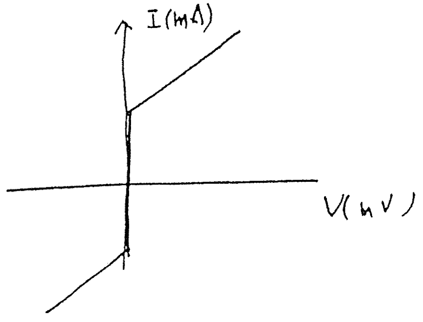
$\sim (T_{cs}-T)$

approached by J. Clarke

→ Simplification of De Genne's approach

$$\left.\begin{matrix} N_s = N_N \\ \xi_s = \xi_N \end{matrix}\right\} \rightarrow \begin{matrix} \Delta_N(0_-) = \Delta_S(0_+) \\ \left(\frac{d\Delta_N}{dx}\right)_{0-} = \left(\frac{d\Delta_S}{dx}\right)_{0+} \end{matrix}$$





$$I_c(T) \sim |\Delta_0(T)|^2 \left(\frac{\xi_N(T)}{\xi^2_{GL}(T)}\right) e^{-2d_N/\xi_N(T)}$$

J. Clarke

$\Delta_0(T) \sim (T_{cs}-T)^{1/2}$

$\xi_{GL}(T) \sim (T_{cs}-T)^{-\frac{1}{2}}$

$\xi_N(T) \sim T^{-\frac{1}{2}} \sim$ const. near Tcs

$\sim (T_{cs}-T)^2 \qquad T \doteq T_{cs}$

$\sim e^{-\text{Const}\cdot\sqrt{T}} \qquad T \to 0$

R vs. T of an SNS junction.

fraction of the length of the junction which retains S.C. Coherence in spite of thermal noise

$$= \frac{W}{2d_N} \qquad \frac{\hbar I_c}{2e}\frac{\xi_N}{w} > k_BT$$

assuming

$$\frac{R}{R_N} = 1 - \frac{W}{2d_N}$$

$$= 1 - \left(\frac{a\xi_N}{d}\right) \ln\left\{\frac{\hbar I_c(0)\left(1-\frac{T}{T_{cs}}\right)^2 \xi_N}{2ewk_BT}\right\}$$

OR

$$\left(1-\frac{R}{R_N}\right)\sqrt{T} \sim \ln\left\{\left(1-\frac{T}{T_c}\right)^2 T^{-3/2}\right\} + \text{Const.}$$

6.6. Array of Josephson Junction.

여러개의 J.J.

2차원 —— 3차원

granular Superconductor.

Ceramic HTSC

$\boxed{\text{그림 } 6\text{-}13}$

한 Plaquette 에서

$$\sum_{\text{plaquette}} \gamma_i = 2\pi \Phi/\Phi_0 \pmod{2\pi}$$

$$= 2\pi f \pmod{2\pi}$$

$$= 2\pi (f-n)$$

일반적으로

$$\sum_{\text{Contour}} \gamma_i = 2\pi \sum_{\substack{\text{enclosed}\\ \text{Cells}}} (f_j - n_j)$$

Restriction : Neglect macroscopic field screening effects

$$B = H$$

$$f_i \doteq f = Ha^2/\Phi_0$$

where H is the field applied normal to the array, yielding a uniformly frustrated array.

$$E = E_J \sum_{\text{array}} (1 - \cos\gamma_i)$$

Note :

$$\lambda_\perp = \lambda_{eff}^2 / d$$

$$J_s d = \frac{c}{4\pi \lambda_{eff}^2} \cdot d \cdot \frac{\Phi_0}{2\pi} \nabla\varphi$$

둘에서 $\lambda_\perp = \dfrac{c\,\Phi_0}{8\pi^2 I_c}$

which diverges as $I_c \to 0$

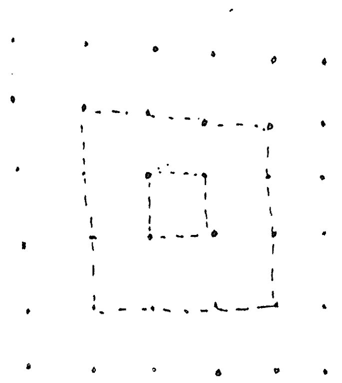
$\lambda_\perp \gg R$ (시료크기)

Screening effect can be safely ignored

$\lambda_\perp \lesssim a$

Screening effect is important.

6.6.1. Arrays in Zero Magnetic field.



처음 $|\gamma_i| = \frac{2\pi}{4} = \frac{\pi}{2}$

계속하면 $\gamma_i \sim \frac{a}{r}$

만약 γ_i 가 작으면 이 J.J에서의 Energy는

$$E_J \cdot \frac{\gamma_i^2}{2} \approx E_J \cdot a^2/2r^2$$

$$\therefore E = \int E_J \cdot \frac{a^2}{\cancel{2}r^2} \left(\cancel{2}\pi \frac{r}{a}\right)\left(\frac{dr}{a}\right)$$

$$\sim \pi E_J \frac{a^2}{a^2} \ln \frac{r}{a}$$

$$\therefore \boxed{E = \pi E_J \ln R/a}$$

R is the outer limit of the integration.

R : radius of the Array

언제까지 成立하나?

$R < \lambda_\perp$ 이어야 한다.

The Kosterlitz - Thouless transition.

두개의 Core를 포함 하는 경우. $(\uparrow, \downarrow)$

$$E_{12} = 2\pi E_J \ln (R_{12}/a)$$

이 Energy는 2D Coulomb gas model과 비슷

$$\mathcal{E} = \frac{4\pi Q}{2\pi r} = \frac{2Q}{r}$$

$$F = 2Q^2/R_{12}$$

$$\therefore E = 2Q^2 \ln\left(\frac{R_{12}}{a}\right)$$

따라서 Energies of Vortices maps onto that of a 2D Coulomb gas of charges of magnitude

$$Q = (\pi E_J)^{1/2}$$

언제 bound pair 가 풀어지나

$$\Delta F = E - TS$$

$$= 2\pi E_J \ln N - T \cdot (2k \ln N)$$

$$\therefore kT = \pi E_J$$

- Kosterlitz 와 Thouless 가 처음으로 R.G. 결과 발표

- Beasley, Mooij, Orlando

  $\lambda_\perp > R$ 이면 Superconductor 에서도 일어난다

- 다른 방법으로 T_{KT} 아는법

$$\langle R_{12}^2 \rangle \propto \int_1^\infty R^2 e^{-E(R)/kT} 2\pi R \, dR$$

$$\propto \int_1^\infty R^3 e^{-2\pi E_J \ln R/kT} dR$$

$$= \int_1^\infty R^3 R^{-2\pi E_J/kT} dR$$

which is infinite if $T \geq T_{KT}$,

where $T_{KT} = \frac{\pi}{2} E_J(T_{KT})$

$$kT_{KT} = \frac{\pi}{2} E_J(T_{KT})$$

$$= \frac{\pi}{2} \cdot \frac{\hbar^2 n_s(T_{KT}) d}{m^*}$$

$$= \frac{\Phi_0^2}{32\pi^2} \frac{d}{\lambda_{eff}^2(T_{KT})}$$

$$= \frac{\Phi_0^2}{32\pi^2 \lambda_\perp(T_{KT})}$$

$$= \frac{\Phi_0^2}{8\pi c^2 L_{K\square}(T_{KT})}$$

$L_{K\square}$: kinetic inductance per square of the film.

Kosterlitz - Thouless transition temperature 를

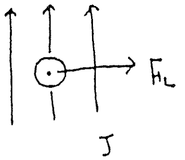
I-V 에서 알아낼수 있는 방법

Current Induced depairing

- nonlinear current - Voltage characteristics
- Competition between Lorentz force & logarithmic attractive force of a vortex pair

Lorentz force on a vortex by a transport current (per unit length)

$F_L = \frac{1}{c} J \phi_0$



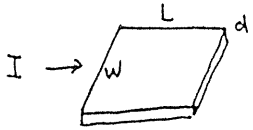
Neglecting the renormalization effect between vortices

$U(r) = U_0(r) - F_L r$

$= 2E_c + g^2 [\ln \frac{r}{\xi} - 2m v_s r/\hbar]$

where $g^2 = \pi n_s \hbar^2 / 2m$

effective vortex charge



$F_L = \frac{1}{c} \frac{I \phi_0}{wd} d = \frac{1}{c} J_s \phi_0 d$

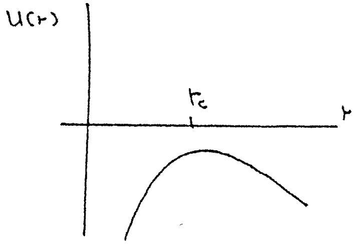
$= \frac{1}{c} n_s^{3D} e v_s \phi_0 d$

$= \frac{1}{c} n_s e v_s \phi_0$

$F_L r = g^2 2m v_s r/\hbar$

$\frac{\partial U}{\partial r} = 0 \longrightarrow r_c$

$r_c = \frac{\hbar}{2m v_c} = \frac{\hbar n_s e}{2m J_s}$



the interaction energy at the saddle

$U(r_c) = 2E_c + g^2 [\ln \frac{r_c}{\xi} - 1]$

$\simeq 2E_c - g^2 \ln(\frac{J_s}{J_0})$

$J_0 \simeq \frac{\hbar n_s e}{2m\xi}$

$\frac{r_c}{\xi} = \frac{J_0}{J_s}$

the classical thermally - activated escape rate over this saddle point,

$\sim e^{-U(r_c)/K_B T} \sim (\frac{J_s}{J_0})^{g^2/K_B T}$

$\sim (\frac{r_c}{\xi})^{g^2/K_B T}$

$\ell_J = \ln(r/\xi)$

$= \ln(J_0/J)$

The density of current - induced single vortices

→ balance between vortex pair breaking & the recombination.

$\dot{n}_{f,J} = \Gamma - \quad \alpha n_{f,J}^2$

in equilibrium $\quad n_{f,J} \sim \Gamma^{1/2}$

$$R = 2\pi \xi^2 n_{f,J} R_N$$

$$\sim \left(\frac{J_s}{J_0}\right)^{\xi^2/2k_BT}$$

$$= \left(J_s/J_0\right)^{\pi K}$$

$$\text{OR} \quad = e^{-\ell_J \pi K}$$

좀더 정확히 Halperin & Nelson

$$R = (2\pi K - 4)\left(\frac{J_s}{J_0}\right)^{\pi K} R_N$$

$$J_0 = \frac{k_B T_c e}{\hbar \xi_c} \simeq \frac{\hbar n_s e}{2m\xi}$$

$$\because \xi^2 = 2\pi k_B T K$$

then $V \simeq IR \sim I^{1+\pi K(T)} = I^{a(T)}$

$$a(T) = 1 + \frac{\xi^2}{2k_BT} = 1 + \pi K(T)$$

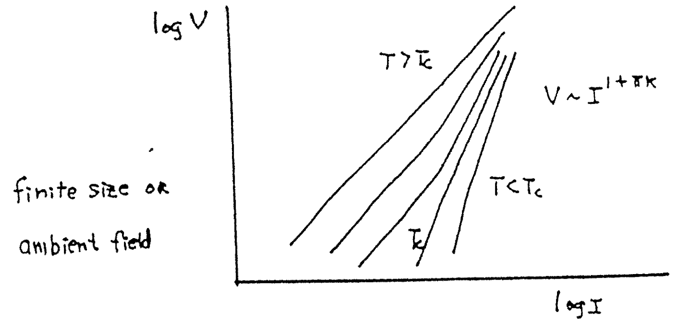
$$= 1 + \frac{\pi n_s \hbar^2}{4m k_B T}$$

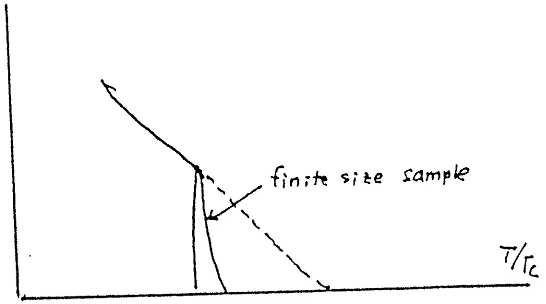
- $a(T) = \begin{cases} 1 & T > T_c \quad \text{only for infinite system} \\ 3 & T = T_c \quad \text{universal jump condition} \end{cases}$

- Extended to the case of renormalized int. by replacing n_e by n_s^R : the renormalized superelectron density.

- the relation can be used to determine $K(T)$ experimentally in the low current level.

- $$\frac{d \ln R(T)}{d \ln I} = \pi K(T)$$





- if $\ell = \infty$ $\quad (\ell \equiv \ln r/\xi)$

πK, a universal jump $0 \to 2$ at T_c

- if ℓ = finite, the jump smears out.

Assuming that the smearing is only due to finite size of the sample.

$$\frac{\Delta K}{\Delta T} = -\frac{4}{\pi^2}\ell_0, \quad T \simeq T_c$$

$$\ell_0 = \frac{1}{2}\ln\left(\frac{K}{K_c}\right)$$ by Hikami & Tsuneto

For example in $Tl_2Ba_2CaCu_2O_8$

$$\ell_0 = 5.3$$

Cf. r_c can be obtained from the force relation also.

$$U = 2E_c + 2\pi k_B T\, K(r)\, \ln\left(\frac{r}{\xi}\right)$$

$$F = -\frac{\partial U}{\partial r}$$

$$= -2\pi k_B T\, K(r) \cdot \frac{1}{r}$$

force balance condition

$$2\pi k_B T \cdot K \cdot \frac{1}{r_c} = \frac{I}{wd}\,\frac{\phi_0}{c}\, d$$

$$r_c : \frac{2\pi c w k_B T\, K(r_c)}{I\phi_0} \qquad \left(\doteq \frac{2\pi c w k_B T}{I\phi_0} \cdot \frac{n_s \hbar^2}{4m k_B T} \right)$$

$$= \frac{2ecw}{I\hbar c}\;\frac{n_s\hbar^2}{4m}$$

$$= \frac{2ew}{Jwd} \cdot \frac{n_s\hbar}{4m}$$

$$= \frac{e n_s \hbar}{n_s^{2D} e v_s}\;\frac{1}{d} \cdot \frac{1}{2m}$$

$$= \frac{\hbar}{2m v_s}$$

i) $r_c > w$

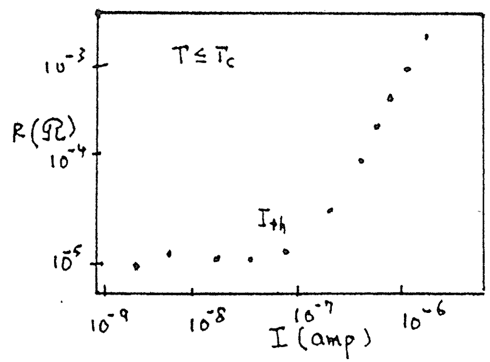
the onset of depairing in a "finite film":

$r_c \simeq w$

$$I_{th} = \frac{2\pi c k_B T\, K(w)}{\phi_0} = \frac{2e k_B T\, K(w)}{\hbar}$$

Fiory, Hebard, and Glaberson (1983)

In / InOx film



I_{th} (theoretical) = 55 nA

I_{th} (exp) $\simeq$ 20 nA

※ mechanism of I_{th}

- finite size effect
- ambient magnetic field effect

ii) if $r_c \ll w$, the resistance increase soly due to the current - induced depairing.

$$R \sim I^{\pi K + 1}$$

Magnetoresistivity : R_H

$R = 2\pi \xi^2 n_{f,H} R_N$ flux flow resistance

$n_{f,H} \phi_0 = BA$

$\simeq HA$ for $H \gg H_{c_1}$

$r_H \cong \left(\frac{A}{n_f}\right)^{1/2} = \left(\frac{\phi_0}{H}\right)^{1/2}$

or $l_H = \ln\left(\frac{r_H}{\xi}\right)$

$= \frac{1}{2} \ln\left(\frac{\phi_0}{\xi^2} \frac{1}{H}\right)$

$\simeq \frac{1}{2} \ln\left(\frac{H_{c_2}}{H}\right)$

In analogy to the current-induced depairing case:

$R(J,T) \sim e^{-l_J \pi K(T)}$

$R(H,T) \sim e^{-l_H \pi K(T)}$

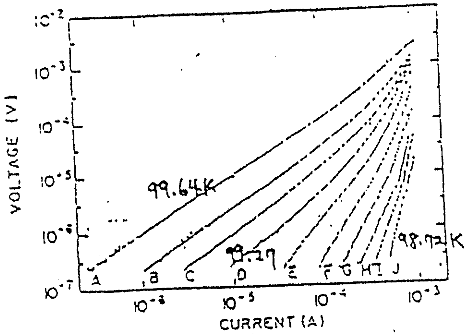
$= \left(\frac{H}{H_{c_2}}\right)^{\pi K(T)/2}$ → consistent with the fact that $R(H) \sim H$ at $T = T_c$

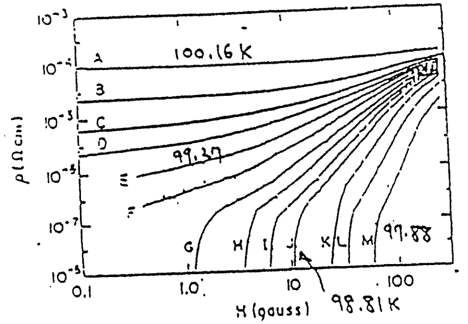
$\ln R \sim \frac{\pi K}{2} \ln\left(\frac{H}{H_{c_2}}\right)$

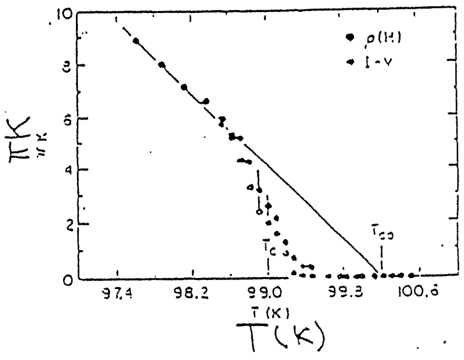
$2 \frac{d \ln R(T)}{d \ln H} = \pi K(T)$

김동호 박사 졸업 논문

$Tl_2Ba_2CaCu_2O_8$ film.







KT transition in S.C. Films (2D)

interaction energy between vortices

$$V(r) \simeq \frac{\phi_0^2}{4\pi^2\lambda_\perp} \ln\left(\frac{r}{\xi}\right) \quad , \quad \xi \ll r \ll \lambda_\perp$$

$$\lambda_\perp = \frac{2\lambda^2}{d}$$

$$= 2\pi J \ln\left(\frac{r}{\xi}\right) = 2\pi k_B T \, K \ln(r/\xi)$$

$$= g^2 \ln r/\xi$$

$$2\pi J = \frac{\phi_0^2}{4\pi^2\lambda_\perp} = \frac{\pi n_s \hbar^2}{2m} = g^2$$

$$n_s = n_s^{3D} d$$

$$K \equiv \frac{J}{k_B T} = \frac{n_s \hbar^2}{4 m k_B T}$$

$$\lambda_\perp \equiv \frac{2\lambda^2}{d} = \frac{2\lambda^2(0)}{d}\left(\frac{\xi_0}{\ell}\right) \Big/ \frac{\Delta(T)}{\Delta(0)} \tanh\frac{\beta\Delta(T)}{2}$$

$$k_B T_c = \frac{\pi J(T_c)}{2} = \frac{\phi_0^2}{16\pi\lambda_\perp^2(T_c)}$$

$$\frac{T_c}{T_{c0}} \cong (1 + 0.17 R_\square \epsilon_c / R_c)^{-1}$$

$$\epsilon_c = \frac{\pi}{2}\frac{J(T_c)}{k_B T_c} \simeq 1.2 \pm 0.1 \quad \text{for HgXe film}$$

$$R_c = \frac{\hbar}{e^2} = 4.11 \ k\Omega/\square$$

Table I. Basic Properties of a Single Quantum Vortex in Bulk and Thin Film Superconductors ($n = 1, \lambda \gg \xi$)

	Bulk		Film	
K_s	$\frac{c}{4\pi}\frac{\phi_0 d}{2\pi\lambda^2}\frac{1}{r}$	$r \ll \lambda$	$\frac{c}{4\pi}\frac{\phi_0 d}{2\pi\lambda^2}\frac{1}{r}$	$r \ll \frac{2\lambda^2}{d}$
	$\frac{c}{4\pi}\frac{\phi_0 d}{2\lambda^2(2\pi\lambda)^{1/2}}\frac{e^{-r/\lambda}}{r^{1/2}}$	$r \gg \lambda$	$\frac{c}{4\pi}\frac{\phi_0}{\pi}\frac{1}{r^2}$	$r \gg \frac{2\lambda^2}{d}$
U	$\left(\frac{\phi_0}{4\pi\lambda}\right)^2 d\left(\ln\frac{\lambda}{\xi} + \ln 2 - \gamma\right) + \pi d\xi^2\frac{H_c^2}{8\pi}$		$\left(\frac{\phi_0}{4\pi\lambda}\right)^2 d\left(\ln\frac{\lambda}{\xi} + \ln\frac{4\lambda}{d} - \gamma\right) + \pi d\xi^2\frac{H_c^2}{8\pi}$	
U_{ij}	$\left(\frac{\phi_0}{2\pi\lambda}\right)^2 d\left(\ln\frac{\lambda}{r_{ij}} + \ln 2 - \gamma\right)$	$r_{ij} \ll \lambda$	$\left(\frac{\phi_0}{2\pi\lambda}\right)^2 d\left(\ln\frac{\lambda}{r_{ij}} + \ln\frac{4\lambda}{d} - \gamma\right)$	$r_{ij} \ll \frac{2\lambda^2}{d}$
	$\frac{\phi_0^2}{8\pi(2\pi)^{1/2}}\frac{d}{\lambda^2}\left(\frac{\lambda}{r_{ij}}\right)^{1/2} e^{-r_{ij}/\lambda}$	$r_{ij} \gg \lambda$	$\left(\frac{\phi_0}{2\pi}\right)^2\frac{1}{r_{ij}}$	$r_{ij} \gg \frac{2\lambda^2}{d}$
M	$\frac{\phi_0}{4\pi}d$		$\approx\frac{\phi_0}{4\pi}R$	

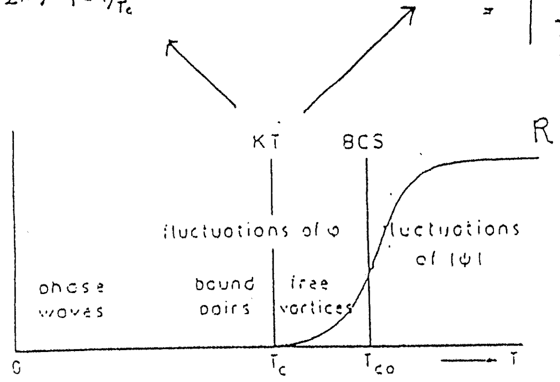
$\xi_- =$ average separation between bound vortices

$$\ell_- \equiv \ln(\xi_-/\xi) = \frac{1}{2\pi}\sqrt{\frac{b}{1-T/T_c}}$$

$$\xi_+ = a\xi\, e^{\left(b'\frac{T_{co}-T}{T-T_c}\right)^{1/2}}$$

= average separation between free vortices

$$\ell_+ \equiv \ln(\xi_+/\xi) = \sqrt{\frac{b}{\frac{T}{T_c}-1}}$$



T_{co} 결정 방법

$$\sigma - \sigma_N = \frac{e^2}{16\hbar d}\,\frac{1}{\frac{T}{T_{co}}-1}$$ 에서 T_{co} 결정

$T_c < T < T_{co}$

dissipation due to flux flow resistance of free vortices.

$$R = 2\pi\xi^2 n_f R_N = AR_N \exp\left(-2\sqrt{\frac{b}{\frac{T}{T_c}-1}}\right)$$

$$n_f \simeq \frac{1}{\xi_+^2}$$

$T \gtrsim T_c$

T_c, b는 실험으로 결정

In order to study the vortex excitation (In a fully renormalized way), we use the Nelson - Kosterlitz recursion relations :

$$\frac{d}{d\ell}K^{-1} = 4\pi^3 y^2$$

$$\frac{dy}{d\ell} = (2-\pi K)\,y$$

where $\ell = \ln(r/\xi)$

$$y = e^{2\ell - u/2k_BT} = \left(\frac{r}{\xi}\right)^2 e^{-u/2k_BT}$$: pair - excitation prob

$K = \beta J$

K and y at any given r can be obtained from an initial condition.

initial cond

$K(0) \propto \frac{T_{co}}{T} - 1$

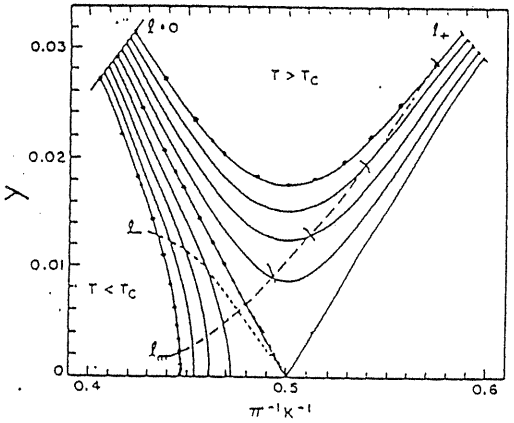
$y(0) = y_0\, e^{-cK(0)}$

↑

related to vortex core energy

$y_0 \simeq 1$

$c \simeq 1$



linearized form:

Near $T \simeq T_c$, $K \simeq \frac{2}{\pi}$

Defining $x \equiv \frac{2}{\pi K} - 1$

$$\longrightarrow \pi K = \frac{2}{1+x} \simeq 2(1-x)$$

then

$$\frac{dx}{d\ell} = 8\pi^2 y^2$$

$$\frac{dy}{d\ell} = 2xy$$

$$x\frac{dx}{d\ell} = 8\pi^2 x y^2$$

$$y\frac{dy}{d\ell} = 2xy^2$$

$$x\frac{dx}{d\ell} - 4\pi^2 y \frac{dy}{d\ell} = 0$$

$$\therefore \; x^2 - 4\pi^2 y^2 = c(T)$$

= an invariant

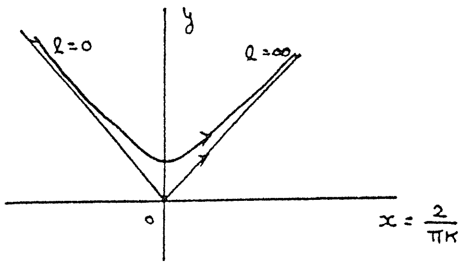
$$= x_0^2 - 4\pi^2 y_0^2$$

where

$x_0 = x(\ell=0)$, $y_0 = y(\ell=0)$

then

$$\frac{dx}{d\ell} = 8\pi^2 y^2 = 2(x^2 - c)$$



i) $C = 0$ at $T = T_c$

$x = \pm 2\pi y$

or $\pi K = 2 + 4\pi y$

$x = 0$, $y = 0$ → fixed pt. of the sys. as $r \to \infty$

$$\left.\begin{array}{ll} T > T_c, & \pi K(\infty) = 0 \\ T = T_c, & \pi K(\infty) = 2 \end{array}\right) \text{universal jump}$$

$y_R(\ell = \infty) = 0$ no bound vortices at infinite separation → all finite bound vortices

$$\frac{dx}{d\ell} = 8\pi^2 y^2 = 2x^2$$

$$\frac{dx}{x^2} = 2d\ell$$

$$-\frac{1}{x} - \frac{1}{x_0} = 2\ell$$

$$x = -\frac{1}{2\ell - \frac{1}{x_0}} \xrightarrow[\text{large } \ell]{\text{limit}} -\frac{1}{2\ell}$$

$y \to \frac{1}{4\pi\ell}$ $\pi K = 2 + \pi$

ii) $C<0$ $(T>T_c)$

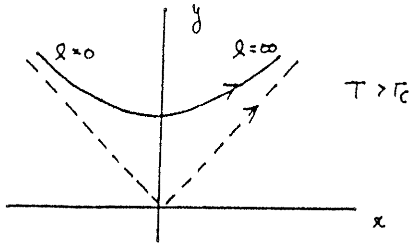
The curve turning away from the origin

$\ell \to \infty$: $x_R = \infty \longrightarrow K_R = 0$ interaction totally screened.

$y_R = \infty$: plenty of free vortices at large separation.

$$x = -D\left[\tan\{2D\ell + \tan^{-1}(-D/x_0)\}\right]^{-1}$$
$$2\pi y = D\left[\sin\{2D\ell + \tan^{-1}(-D/x_0)\}\right]^{-1}$$

$$D \equiv |c|^{1/2}$$



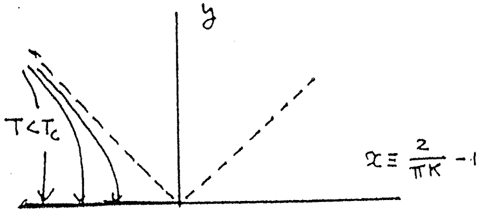
iii) $C>0$ $(T<T_c)$

fixed point at $y_R = 0$

$x_R = -\sqrt{c}$

Corresponding to somewhat stronger int. between bound vortices.

$$x = -D\left[\tanh\{2D\ell + \tanh^{-1}(-D/x_0)\}\right]^{-1}$$
$$2\pi y = D\left[\sinh\{2D\ell + \tanh^{-1}(-D/x_0)\}\right]^{-1}$$



length Scales

$\ell_m = \ln\left(\frac{W}{\xi}\right)$: sample size

$\ell_- = (2\pi K(\infty) - 4)^{-1}$: average distance between bound vortices at $T<T_c$

$$\simeq \frac{1}{2\pi}\left(\frac{b}{1-T/T_c}\right)^{\frac{1}{2}}, \quad T \leq T_c$$

$y(\ell_+) \equiv y(0)$

ℓ_+ : average distance between free vortices

$$\ell_+ = \left(\frac{b}{\frac{T}{T_c} - 1}\right)^{1/2}$$

$T \geq T_c$

$$R_T \sim e^{-2\ell_+}$$

$\rightarrow \ell_J = \ln(J_0/J)$: onset of current-induced depairing

$$R_J \sim e^{-\pi K \ell_J}$$

$\rightarrow \ell_H = \frac{1}{2}\ln\left(\frac{H_{c2}}{H}\right)$: distance between field generated vortices

$$R_H \sim e^{-\pi K \ell_H}$$

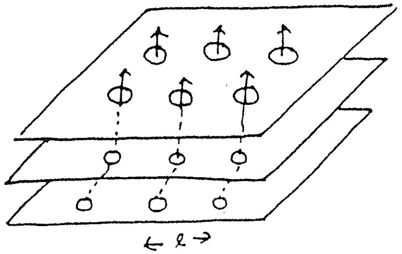
$\rightarrow \ell_\omega = \frac{1}{2}\ln(14D/\omega\xi^2)$

Controllable characteristic length scale

$$R_\omega \sim \omega y^2(\ell_{\omega\omega})$$

※ Layered - Structure System

(artificial multi-layer system
HTSC system.



$\left(\begin{array}{l} \text{independent vortex-} \\ \text{pair configuration} \\ \text{on a layer} \end{array} \right) : \left(\begin{array}{l} \text{multi-layer vortex} \\ \text{ring configuration} \end{array} \right)$

$r < r_0$ $\qquad\qquad$ $r > r_0$

$$r_0 = \xi \left(K/K_c \right)^{1/2}$$

$$r \gtrsim r_0$$

$$\text{OR} \quad \ell \gtrsim \ell_0 = \frac{1}{2} \ln \left(K/K_c \right)$$

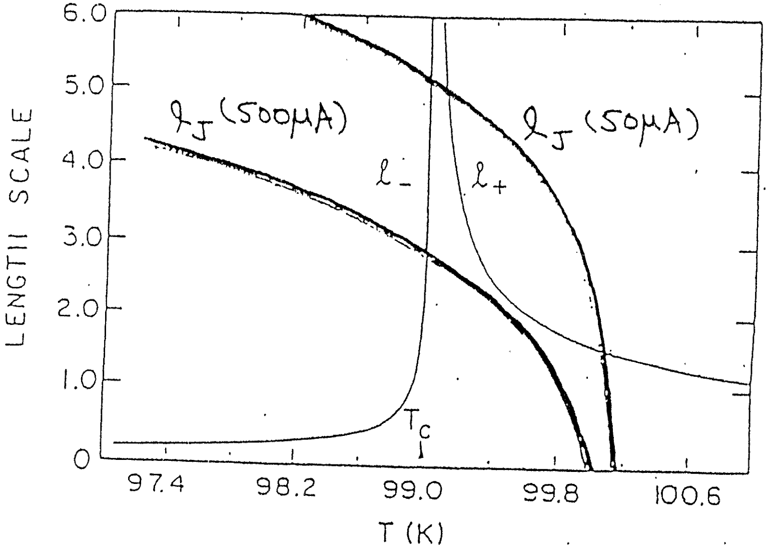
If $\ell_m > \ell_0$ (or $w > r_0$), the jump smears out due to the finiteness of ℓ_0

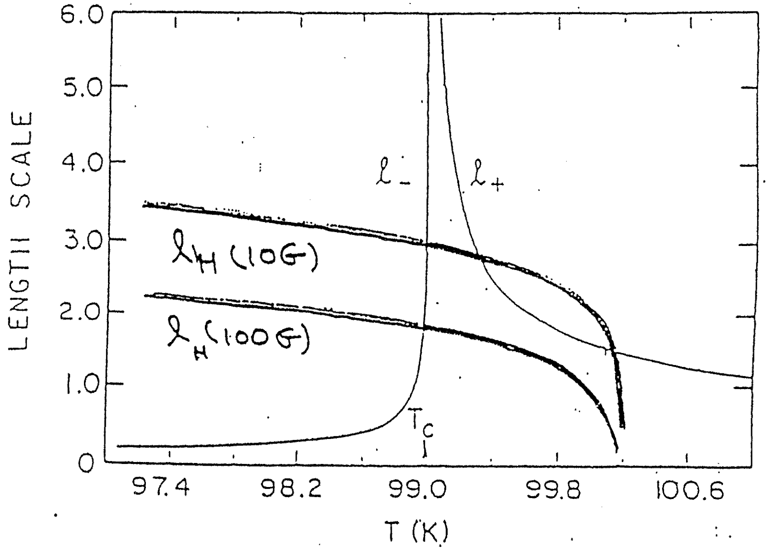
$$\frac{\Delta K}{\Delta T} = - \frac{4}{\pi^2} \ell_0 \quad , \quad T \geq T_c$$

ex) in $Tl_2Ba_2CaCu_2O_8$ → $\ell_0 = 5.3$

Hikami & Tsuneto

Prog. Theoretical Physics ($\underline{B}$, 387 (1980))


LENGTII SCALE
6.0
5.0
4.0
3.0
2.0
1.0
0
97.4
98.2
99.0
99.8
100.6
T (K)
l_J (500μA)
l_J (50μA)
l_-
l_+
T_c

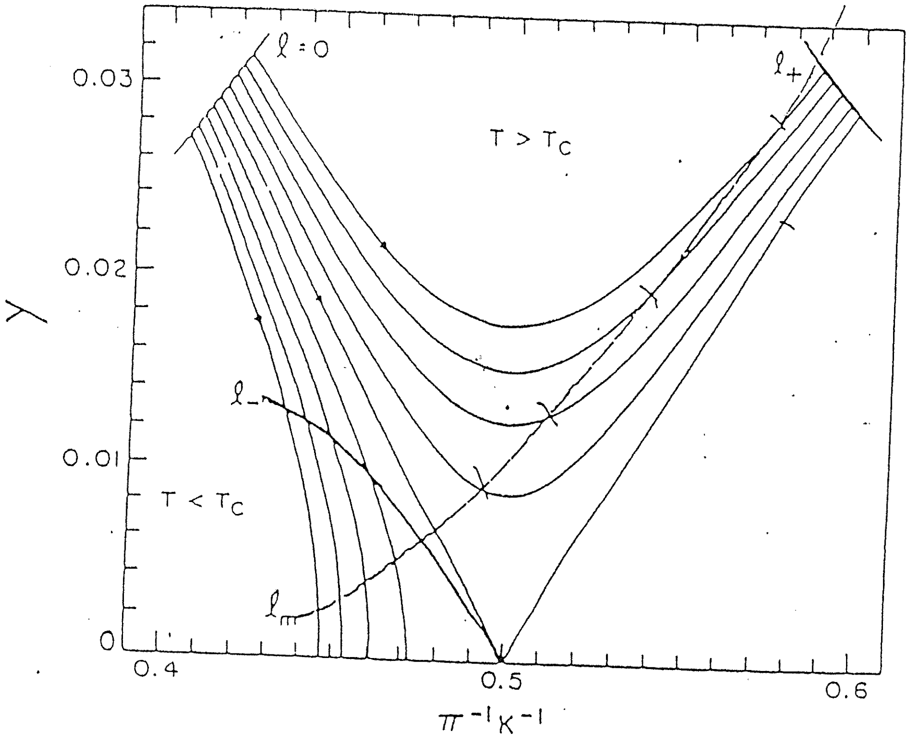

LENGTII SCALE
6.0
5.0
4.0
3.0
2.0
1.0
0
97.4
98.2
99.0
99.8
100.6
T (K)
l_-
l_+
l_H (10G)
l_H (100G)
T_c

fully renormalized case

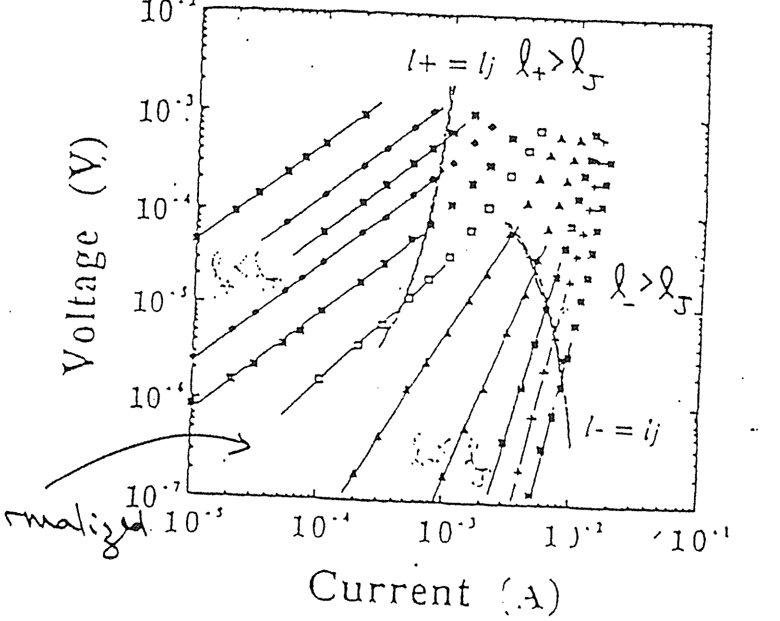
$\ell_m > \ell_+$
ℓ_-

$\ell_J > \ell_-$

$\ell_H > \ell_m, \ell_-$



YBCO epitaxial film (c ⊥ sub.)



fully renormalized regime

Teshima et al. Physica C 199, 149(1992)

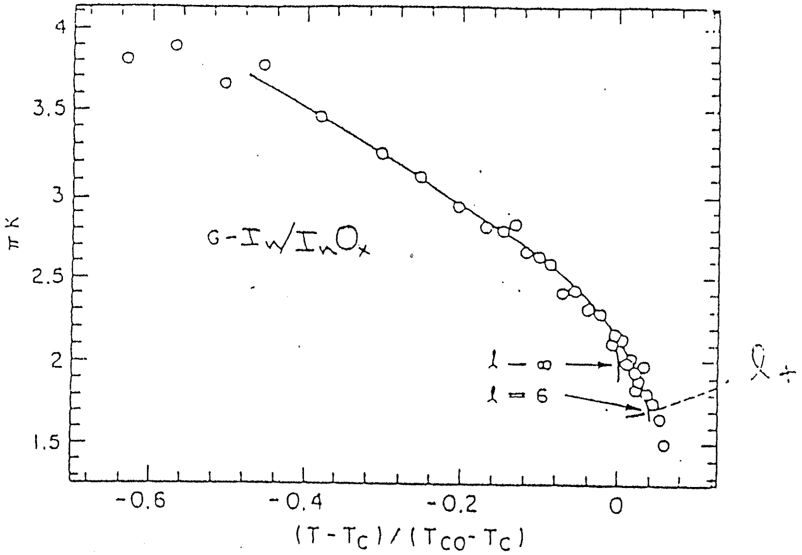


FIG. 13. Reduced stiffness constant vs reduced temperature for sample A-2. Fitted curves are for $l_c = 6$, $l \rightarrow \infty$, and $l = l_+$ (dashed curve).

Fiory, Hebard, & Glaberson, PRB $\underline{28}$, 5075 (1983)

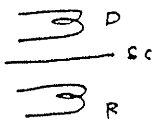
AC Response:

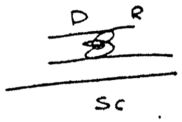
$$i\hbar \frac{\partial}{\partial t}\psi = 2\mu\dot{\psi}$$ ψ : S.C. order parameter

$$= 2(\mu_c + eV)\psi$$

$$-\hbar\frac{\partial\phi}{\partial t} = 2\mu \quad \Leftarrow \quad \psi = |\psi|e^{i\phi}$$

$$\frac{4\pi\lambda^2}{c}\vec{J}_s + \vec{A} = \frac{\phi_0}{2\pi}\nabla\phi \quad \Leftarrow \quad \text{G.L. eq.}$$





$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} - \nabla V$$

$$= \frac{4\pi\lambda^2}{c^2}\frac{\partial\vec{J}_s}{\partial t} + \nabla\left(\frac{\mu_c}{e}\right)$$

$$= L_K \frac{\partial\vec{K}_s}{\partial t} + \nabla\left(\frac{\mu_c}{e}\right)$$ $\vec{K}_s = \vec{J}_s \cdot d$

$$L_K = \frac{2\pi\lambda_\perp}{c^2} = \frac{m}{n_s e^2}$$

$$= \underbrace{L_K \frac{\partial\vec{K}_s}{\partial t}}_{\text{superfluid background}} + \underbrace{\frac{\phi_0}{\eta_v c}\hat{z}\times\vec{K}_v}_{\text{flowing vortices}} = Z\vec{K}$$

$K_N \ll K_s$, as

long as $\omega \ll \frac{R_N}{L_K}$

Since $\vec{K_v} \propto \vec{K} \simeq \vec{K_s}$

$$Z = i\omega L_K + \underbrace{Z_v}_{\text{Vortex Impedance}} = i\omega L_K \underbrace{\epsilon(\omega)}_{\text{Complex}}$$

OR $= R + i\omega L$

R_f : $T > T_c$ $\qquad L_K = \dfrac{m}{n_s e^2} \qquad T \ll T_c$

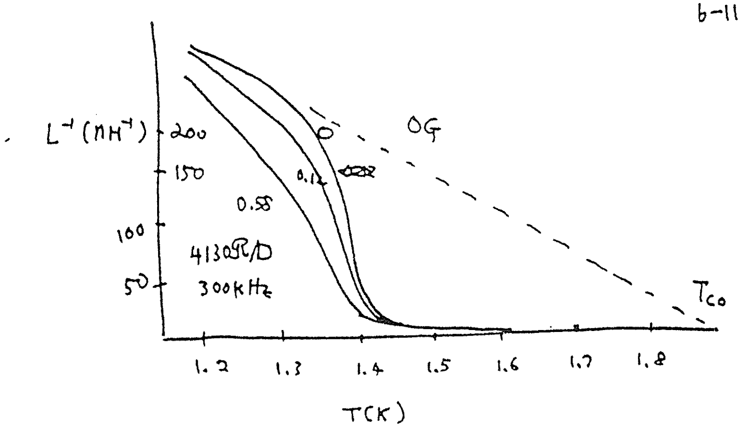
$\left.\begin{matrix} R_b(\omega) \\ R_p(\omega) \end{matrix}\right\} T \leq T_c \qquad \left.\begin{matrix} L_b \\ L_p \end{matrix}\right\} T \sim T_c$

$\epsilon(\omega) = \epsilon_f + \epsilon_b \qquad 0 < T < T_c$

$T < T_c \qquad \epsilon(\omega) = \epsilon_b$

$T = 0 \qquad \epsilon(\omega) = 1$ no vortex exists

$\vec{K} = -\dfrac{\vec{A}}{c L_K}$ London eq.



i) $B=0$ data

$T < T_c$ outside the critical region

$(1.2 \leq T \leq 1.3K)$

$L^{-1} \simeq L_K^{-1} \propto T_{co} - T$

ii) $\lambda(T_c)\, T_c = \dfrac{\phi_0^2}{16\pi^2 k_B} = 1.96$ cm K

Universal jump cond.

iii) $\lambda_\perp = \dfrac{2\lambda^2}{d} = \dfrac{2\lambda_L^2(0)}{d} \left(\dfrac{\xi_0}{\ell}\right) \left\{ \dfrac{\Delta(T)}{\Delta(0)} \tanh\left(\dfrac{\beta\Delta(T)}{2}\right) \right\}^{-1}$

iv) $B \neq 0$.

$L^{-1} \rightarrow 0$ at temp. where $\ell_\omega \sim \ell_H$ &

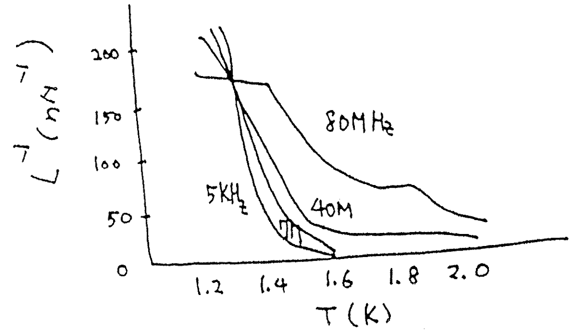
$\ell_+ > \ell_H$

$\ell_\omega < \ell_H < \ell_+(T)$: response dominated by bound pairs

$\ell_H < \ell_\omega < \ell_+(T)$: by field generated vortices

threshold : $(14D/\omega)^{1/2} \simeq (\phi_0/B)^{1/2}$

V) ω - dependence



$$r_\omega = (14D/\omega)^{1/2}$$

$T < T_c$: $\xi_+ > r_\omega$ always

Vortex - pair polarization picture valid

$T > T_c$: $r_\omega = \xi_+(\omega)$: Crossover at T_ω

$T_c < T < T_\omega$: $r_\omega < \xi_+$ bound pair effect dominates

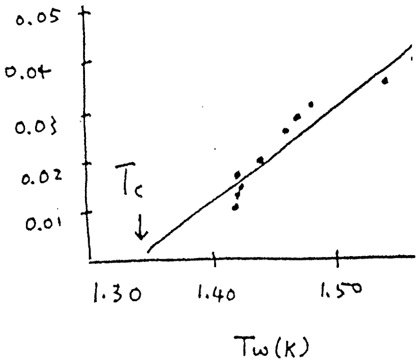
$T > T_\omega$: $r_\omega > \xi_+$ free Vortex effect

the bend in $L^{-1}(\omega)$ takes place at T_ω

$$r_\omega = \xi_+(T_\omega)$$
$$= \xi\, e^{b\left(\frac{T_\omega}{T_c}-1\right)^{-\frac{1}{2}}}$$

$$l_\omega = \ln\left(\frac{r_\omega}{\xi}\right) = b\left(\frac{T_\omega}{T_c}-1\right)^{-\frac{1}{2}}$$

$$\therefore\ l_\omega^{-2} = b^{-2}\left(\frac{T_\omega}{T_c}-1\right)$$



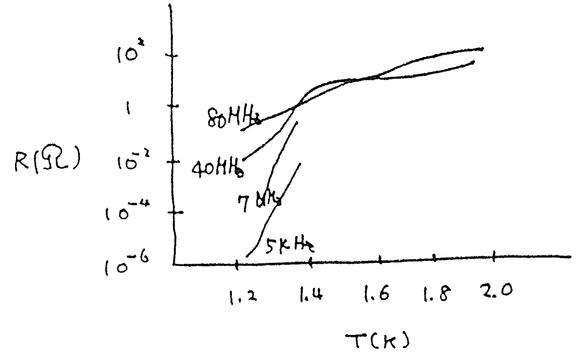
$$r_\omega \leftarrow D$$

① $$\frac{D}{K_BT} = \frac{1}{\eta} = R_\square^f \cdot \frac{c^2}{B\phi_0}$$
$$= 2\pi\xi^2 n_f R_N^\square \frac{c^2}{B\phi_0}$$
$$= 2\pi\xi^2 c^2 R_N^\square / \phi_0^2$$

$n_f = n_f^{ext}$ 일때만 成立

② $D \leftarrow$ from $\dfrac{dH_{c2}}{dT}$

Vi) rf resistance



$r_\omega < \xi_+(T)$

$$R \propto \exp\left[-\frac{\phi_0^2 \ln(r_\omega/\xi) + \frac{1}{4}}{4\pi^2 \epsilon(r_\omega)\lambda_\perp(T)}\right]$$

at $T = T_c$

$R = 2\pi^3 \omega L_K \ell_\omega^{-2}$: theory

$\propto \omega^{+0.9}$: exp

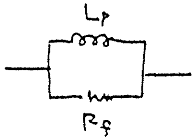
Vii) Pinning & lattice melting

$$\mu = \frac{v}{F_L} = \frac{c^2 z_v}{B\phi_0}$$

Vortex motion in a uncorrelated harmonic pinning potential

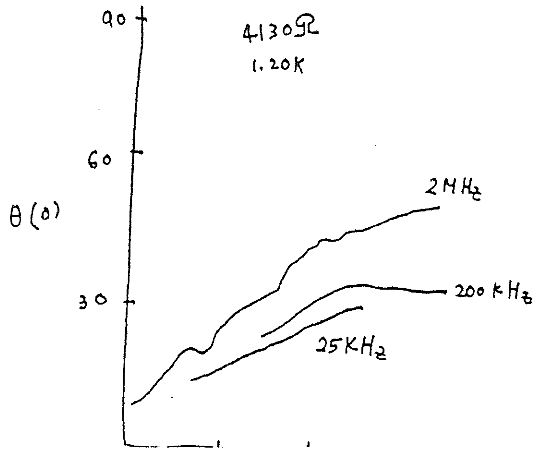
① $Z_v \propto B$

② $Z_v \equiv$



$R_f \propto \eta_f$

$\Theta = \tan^{-1}[\omega(L - L_K)/R]$



viii)

lower sheet resistance film

→ stronger vortex correlation

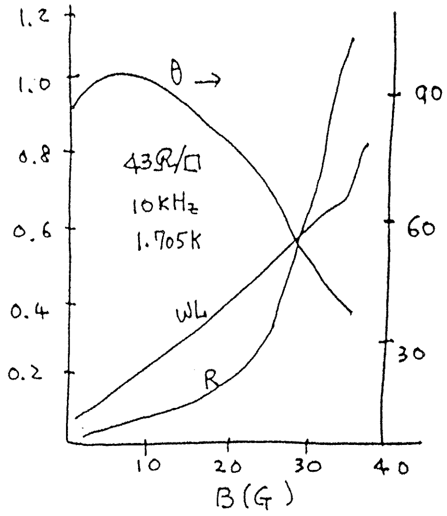
→ higher melting temperature

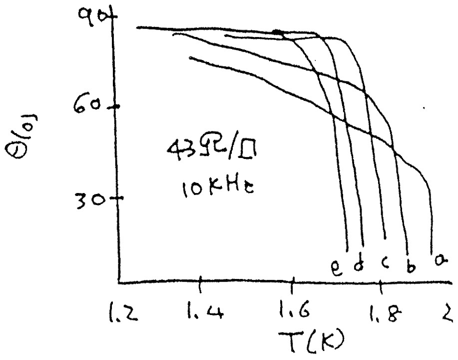
$$T_M = \Phi_0^2 A_1 / 64\sqrt{3}\,\pi^3 k_B\, \lambda_\perp(T_M)$$

$$: \ \xi < a_0 < \lambda_\perp$$

$0.40 < A_1 < 0.75$

In a high $R_\square$ film : $\lambda_\perp \propto R_\square$





○ : milestone paper (mostly static resp.)

∨ : ac response

△ : high T_c S.C.

References : KT transition in S.C. films

○ 1. Halperin and Nelson, J. Low Temp. Phys. 36, 599 (1979)

○∨ 2. Fiory, Hebard, & Glaberson, PRB 28, 5075 (1983)

○ 3. Kadin, Epstein & Goldman, PRB 27, 6691 (1983)

○ 4. Garland and Lee, PRB 36, 3638 (1987)

△ 5. Techima et al. physica C 199, 149,(1992)

∨ 6. Hebard and Fiory, PRL 44, 291 (1980)

∨ 7. Hebard and Fiory, Proceedings of the "International Conference on Ordering in 2D. (1980)

∨ 8. Fiory and Hebard, PRB 25, 2073 (1982)

∨ 9. Hebard and Fiory, physica 109C 1108, 1637 (1982)

△∨ 10. Martin et al. PRL 62, 677 (1989)

△∨ 11. Fiory et al. PRL 61, 1419 (1988)

△ 12. Kim et al. PRB 40, 8834 (1989)

△ 13. Onogi, Ichiguchi, and Aida, Solid state comm. 69, 991 (1989)

△ 14. Ying and Kwok, PRB 42, 2242 (1990)

Chap 8.

Fluctuation Effects in Classical Superconductors.

G-L 이론 : ψ minimum을 찾아간다.

온도에 의해 다른 값을 가질 수 있다.

1. Thermally activated flux creep

finite resistance below T_c

저항이 어떻게 생기나?

Metastable persistent current in a ring

— flux의 jump가 어떻게 생기나?

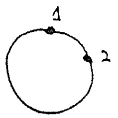
2. $T > T_c$

$\langle \psi \rangle = 0$ 지만 $\langle \psi^2 \rangle \neq 0$.

즉 thermodynamic fluctuations give rise to superconducting effect.

8.1. 초전도 wire에서 저항이 어떻게 나타나나?

persistent current in a ring.



$$\oint \nabla \varphi \cdot ds = 2n\pi$$

$\varphi_{12} = \varphi_1 - \varphi_2$ 계속 Constant 이어야 한다.

크기는 Supercurrent의 크기에 의존한다

φ_{12}는 Constant mean field를 중심으로 fluctuate 한다.

total current = Const.

noise normal current도 Compensate 하면서

$\omega \neq 0$ 이면 항상 noise voltage가 있고 $\sim \omega^2$

$\overline{V}$ 는 더이상 zero가 아니다.

만약 저항이 생겨나면

$\langle \varphi_{12} \rangle \sim$ increase with time

Steady 하지 않다.

해결책 — Phase slip

한두 군데에서 Spatially localized place에서 phase slip 생기난다.

Steady state가 되기 위해서는 phase slip이 일어나 $\frac{2eV}{\hbar}$ 의 율로 일어나야 한다.

1 D wire 이것을 이해하기 쉽게 1 D wire 생각하자

$d \ll \xi$, $d \ll \lambda$

1. fluctuation effect 무시하자.

$\boxed{V=0}$ $|\psi| = \text{const.}$ for 초전도 상태

$\psi(x) = |\psi| e^{i\varphi(x)} = |\psi| e^{iqx}$

그림 8.1

만약 $V \neq 0$ 이면

$$\frac{d\varphi_{12}}{dt} = \frac{2eV}{\hbar} \quad , \quad \text{Steady Cranked}$$

초전도 에서는 $\vec{E} = \frac{\partial}{\partial t}(\Lambda \vec{J}_s)$

J 의 acceleration을 기술한다.

Langer 와 Ambegaokar 의 phase-slip process 의 주요요점. 계산은 따라가지 않고

$J(x) \propto |\psi(x)|^2 v_s(x)$ be constant

$\psi(x) = |\psi(x)| e^{i\varphi(x)}$ then

$$|\psi(x)|^2 \frac{d\varphi}{dx} = \text{const} \propto I$$

만약 어떤 부분의 $|\psi(x)|^2$ 이 매우 작으면

$\frac{d\varphi}{dx}$ 가 매우 커진다.

Saddle point free energy increment 계산하다.

$\Delta F = \frac{8\sqrt{2}}{3} \frac{H_c^2}{8\pi} A\xi$ 만큼씩 변한다.

$I=0$ 전류가 없으면 $\Delta\varphi_{12} = 2\pi$ or -2π 가 된다.

$I \neq 0$ $\delta F = \Delta F_+ - \Delta F_- = \frac{h}{2e} I$

$$\frac{d\varphi_{12}}{dt} = \Omega \left[\exp\left(-\frac{\Delta F_0 - \delta F/2}{kT}\right) - \exp\left(\frac{\Delta F_0 + \delta F/2}{kT}\right) \right]$$

$$= 2\Omega\, e^{-\Delta F_0/kT} \sinh \frac{\delta F}{2kT}$$

$$V = \frac{\hbar\Omega}{e} e^{-\Delta F_0/kT} \sinh \frac{hI}{4ekT}$$

Small current 인 경우

$$R = \frac{V}{I} = \frac{\pi\hbar^2\Omega}{2e^2kT} e^{-\Delta F_0/kT}$$

Valid for $I \leq I_0 = \frac{4ekT}{\hbar}$

$= 0.013\ \mu A/k$

Higher current

$$V = \frac{\hbar\Omega}{2e} e^{-\Delta F_0/kT} e^{I/I_0}$$

attempt freq. Ω

Langer and Ambegaokar 의 이론

그냥 손으로 넣었다.

McCumber and Halperin.

Time dependent GL theory

$$\mathcal{R} = \frac{L}{\xi}\left(\frac{\Delta F_0}{KT}\right)\frac{1}{\tau_s}$$

where $\frac{1}{\tau_s} = \frac{8K(T_c-T)}{\pi\hbar}$

온도 $T \to T_c$ 가 되면서 작아진다.

따라서 McCumber-Halperin prefactor.

실험 Lukens, Warburton, Webb

whiskers. – 0.5 μm diameter.

10^6 정도의 범위에서 맞더라.

293. 그림 8.2

$I = 0.2 \times 10^{-6}$ A

$V = 10^{-13}$V

100 phase slippages per second.

1 millidegree 는 낮추면 10^{-11} per second

1 in 10^9 years

normal resistance ———— no resistance

three millidegree

8.3. Superconductivity above T_c in Zero-dimensional system.

S.C. above T_c in zero dimensional Sys.

Ψ : Const. over the particle volume V

$$F_s - F_n = \left(\alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4\right)V$$

$$\alpha \equiv \alpha_0 (t-1)$$

i) $T < T_c$

$$F_0 = F_s - F_n$$

$$\xrightarrow{\text{min}} \left[\alpha\left(-\frac{\alpha}{\beta}\right) + \frac{1}{2}\beta\cdot\frac{\alpha^2}{\beta^2}\right]V$$

$$= -\frac{\alpha^2}{2\beta}V = -\frac{\alpha_0^2}{2\beta}(1-t^2)V = -\frac{H_c^2}{8\pi}V$$

where

$$|\psi_0|^2 = -\frac{\alpha}{\beta} = \frac{\alpha_0(1-t)}{\beta}$$

fluctuations about ψ_0 :

$$\frac{\partial F}{\partial \psi} = (2\alpha\psi + 2\beta\psi^3)V$$

$$\rightarrow \left.\frac{\partial F_0}{\partial \psi}\right|_{\psi_0} = \left[2\alpha\psi + 2\beta\left(-\frac{\alpha}{\beta}\right)\psi\right]V = 0$$

$$\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_0} = (2\alpha + 6\beta\psi^2)V\Big|_{\psi_0}$$

$$= \left\{2\alpha + 6\beta\left(-\frac{\alpha}{\beta}\right)\right\}V$$

$$= -4\alpha V$$

$$= 4\alpha_0(1-t)V$$

$$\langle F - F_0 \rangle = \frac{1}{2}\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_0}(\delta\psi)^2 \approx KT$$

$$= 2\alpha_0(1-t)V(\delta\psi)^2$$

$$(\delta\psi)^2_< \approx \frac{K_BT}{2\alpha_0(1-t)V} = \frac{K_BT}{-2\alpha V}$$

$$\frac{(\delta\psi)^2}{\psi_0^2} \approx -\frac{K_BT}{2\alpha V}\left(-\frac{\beta}{\alpha}\right)$$

$$= \frac{K_BT}{2V}\left(\frac{\beta}{\alpha^2}\right) = \frac{K_BT}{2V}\frac{4\pi}{H_c^2} = \frac{2\pi K_BT}{H_c^2 V}$$

$$= \frac{2\pi K_BT}{H_c^2(0)(1-t)^2 V} \xrightarrow{S_n} \frac{10^{-20}}{(1-t)^2 V}$$

Very small unless

① T very close to T_c

② the size : very small $d \leq 1000\text{Å}$

Divergence of $(\delta\psi)^2$ at T_c is cutoff by the quadratic term

$$\frac{1}{4!}\frac{\partial^4 F}{\partial \psi^4}(\delta\psi)^4 V \approx K_BT$$

$$\frac{1}{24}(12\beta)(\delta\psi)^4 V$$

$$(\delta\psi)^2_{T_c} = \left(\frac{2K_BT}{\beta V}\right)^{1/2}$$

ii) $T \geq T_c$

$\alpha > 0$ $F_0 \equiv F_{min} = 0$ at $\psi_0 = 0$

$$\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_0} = (2\alpha + 6\beta\psi^2)V\Big|_{\psi_0=0} = 2\alpha V$$

$$= 2\alpha_0(t-1)V$$

$$\langle F - F_0 \rangle = \frac{1}{2}\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_0}(\delta\psi)^2 \approx K_BT$$

$$\therefore (\delta\psi)^2_> \approx \frac{K_BT}{\alpha V} = \frac{K_BT}{\alpha(t-1)V} = 2(\delta\psi)^2_<$$

Once again the divergence is cutoff very near T_c :

$$F = (\alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4)V = k_BT$$

$$|\psi|^2 = \frac{-\alpha + \sqrt{\alpha^2 - 4(\frac{\beta}{2})(-\frac{k_BT}{V})}}{\beta}$$

$$= (\delta\psi)^2$$

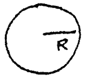
$$(\delta\psi)^2 = \frac{\alpha}{\beta}\left\{\left(1 + \frac{2\beta k_BT}{\alpha^2 V}\right)^{1/2} - 1\right\}$$

$$\xrightarrow{T \to T_c} \frac{\alpha}{\beta}\left(\frac{2\beta k_BT}{\alpha^2 V}\right)^{1/2} = \left(\frac{2k_BT}{\beta V}\right)^{1/2}$$

$$\xrightarrow{T > T_c} \frac{\alpha}{\beta}\left(\frac{\beta k_BT}{\alpha^2 V}\right) = \frac{k_BT}{\alpha V}$$

Observation of zero-dim S.C. fluctuation:

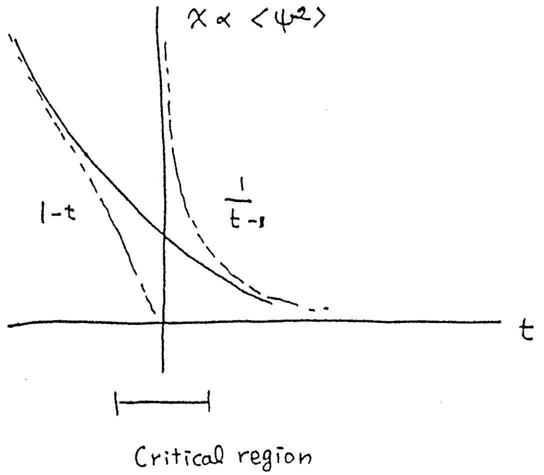
$$\xi_0 < R \ll \lambda$$



$$\chi = -\frac{1}{40\pi}\frac{R^2}{\lambda^2}$$

$$= -\frac{1}{40\pi}\cdot\frac{4\pi e^{*2}}{mc^2}\underbrace{\langle\psi^2\rangle}_{(\delta\psi)^2 \text{ above } T_c} R^2$$

$$\propto \begin{cases} \frac{1}{t-1}, & t > 1 \rightarrow \text{fluctuation effect} \\ 1-t, & t < 1 \rightarrow \text{S.C. mean field effect takes over.} \end{cases}$$



Mean field description cannot be applied for the critical region.

Spatial Variation of fluctuations.

(More in-depth analysis)

Consider a bulk specimen far above T_c

→ quadratic term neglected

$$f - f_n = \alpha |\psi|^2 + \frac{\hbar^2}{2m^*} \left| \left(-i\nabla - \frac{2\pi \vec{A}}{\Phi_0} \right) \psi \right|^2$$

linearized GL eq. for a small ψ (or $\delta\psi$)

$$\left(-i\nabla - \frac{2\pi\vec{A}}{\Phi_0} \right)^2 \psi = - \frac{2m^*\alpha}{\hbar^2} \psi$$

$$= - \frac{1}{\xi^2} \psi \qquad \boxed{-} : \text{note the sign}$$

i) $\vec{A} = 0$ case:

$$\psi(\vec{r}) = \sum_{\vec{k}} \psi_{\vec{k}} \, e^{i\vec{k}\cdot\vec{r}}$$

$$f = \sum_{k}\sum_{k'} \alpha \, \psi_k \psi_{-k'} \, e^{i(\vec{k}-\vec{k}')\cdot\vec{r}} + \frac{\hbar^2}{2m} \sum_{k}\sum_{k'} \vec{k}\cdot\vec{k}' \psi_k \psi_{-k'} \, e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}$$

when integrated over the unit volume

$$\int_{\substack{\text{unit}\\\text{vol.}}} f \, d^3r = f \text{ again}$$

$$= \sum_{k} \alpha |\psi_k|^2 + \frac{\hbar^2}{2m^*} \sum k^2 |\psi_k|^2$$

$$= \sum \left(\alpha + \frac{\hbar^2}{2m^*} k^2 \right) |\psi_k|^2$$

Probability of finding a given distribution of the Fourier Component.

$$W(\{\psi_k\}) = e^{-f/k_BT}$$

$$= \prod_{k} \underbrace{e^{-\left(\alpha + \frac{\hbar^2k^2}{2m^*}\right)|\psi_k|^2/k_BT}}_{\text{Gaussian distribution}}$$

$T > T_c$

$$\langle |\psi_k|^2 \rangle \simeq \frac{k_BT}{\alpha + \frac{\hbar^2k^2}{2m^*}}$$

$$= \frac{2m^*}{\hbar^2} \frac{k_BT}{k^2 + \frac{1}{\xi^2}} \qquad \left(\frac{1}{\xi^2} = \frac{2m^*\alpha}{\hbar^2} \right)$$

equivalent to assigning a thermal energy k_BT to each orthogonal mode. (i.e, equipartition thm).

the correlation fc.

$$g(\vec{r}, \vec{r}') \equiv \langle \psi^*(\vec{r}) \, \psi(\vec{r}') \rangle$$

$$= \left\langle \sum_{k,k'} \psi_k^* \psi_{k'} \, e^{i(\vec{k}'\cdot\vec{r}' - \vec{k}\cdot\vec{r})} \right\rangle$$

introduce : $\left.\begin{aligned} \langle \vec{r} \rangle &\equiv \frac{\vec{r}+\vec{r}'}{2} \\ \vec{R} &\equiv \vec{r}' - \vec{r} \end{aligned}\right\} \rightarrow \begin{aligned} \vec{r}' &= \langle \vec{r} \rangle + \tfrac{1}{2}\vec{R} \\ \vec{r} &= \langle \vec{r} \rangle - \tfrac{1}{2}\vec{R} \end{aligned}$

$$\therefore\ g(\vec{r},\vec{r}\,') = \left\langle \sum_{k,k'} \Psi_k^* \Psi_{k'} \exp\left(\frac{i(\vec{k}+\vec{k}')}{2}\cdot\vec{R}\right) \exp\left(-i(\vec{k}-\vec{k}')\cdot\langle\vec{r}\rangle\right)\right\rangle$$

Taking an average over $\langle\vec{r}\rangle$

$$g(\vec{r},\vec{r}\,') = g(\vec{r}-\vec{r}\,') = g(\vec{R})$$

$$= \sum_k \langle|\Psi_k|^2\rangle\, e^{i\vec{k}\cdot\vec{R}}$$

$$= \sum_k \frac{2m^*}{\hbar^2}\frac{K_BT}{k^2+\frac{1}{\xi^2}}\, e^{i\vec{k}\cdot\vec{R}}$$

$V=1$

$$= \frac{1}{(2\pi)^3}\frac{2m^*K_BT}{\hbar^2}\int \underbrace{d^3k}_{2\pi\, \sin\theta d\theta\, k^2dk} \frac{e^{i\vec{k}\cdot\vec{R}}}{k^2+\frac{1}{\xi^2}}$$

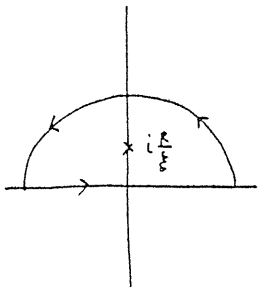
θ integration

$$\int_{-1}^{1} d\mu\, e^{ikR\mu} = \frac{2}{kR}\sin kR$$

K integration

$$\int_0^\infty dk\cdot k\cdot\frac{\sin kR}{k^2+\frac{1}{\xi^2}}\frac{2}{R} = \frac{2}{R}\int_0^\infty dx\cdot\frac{x\sin x}{x^2+R^2/\xi^2}$$

$kR \equiv x$



$$\int_0^\infty dx\cdot\frac{x\sin x}{x^2+R^2/\xi^2} = \int_0^\infty dx\cdot\frac{1}{2i}\frac{x(e^{ix}-e^{-ix})}{x^2+R^2/\xi^2}$$

$$= \int_0^\infty \frac{dx}{2i}\frac{x e^x}{x^2+R^2/\xi^2} + \int_0^{-\infty}\frac{-dx'}{2i}\frac{(-x')(-e^{ix'})}{x'^2+R^2/\xi^2}$$

$$= \frac{1}{2i}\int_{-\infty}^\infty dx\,\frac{x e^{ix}}{x^2+R^2/\xi^2}$$

$$= \frac{1}{2i}\oint dz.\frac{z e^{iz}}{z^2+R^2/\xi^2}$$

$$= \pi\ \mathrm{Res}\left(i.\frac{R}{\xi}\right)$$

$$= \pi.\frac{i\frac{R}{\xi}\cdot e^{-R/\xi}}{2i\,R/\xi} = \frac{\pi}{2}e^{-R/\xi}$$

$$\therefore\ g(R) = \frac{1}{(2\pi)^2}\cdot\frac{2m^*K_BT}{\hbar^2}\frac{2}{R}\left(\frac{\pi}{2}e^{-R/\xi}\right)$$

$$= \frac{m^*K_BT}{2\pi\hbar^2}\frac{1}{R}e^{-R/\xi}$$

The local values of Ψ, in the fluctuation regime are correlated only over a distance $\xi(T)$

Effect of a Magnetic Field.

(On the spatial distribution of fluctuations)

G.L. eq. $(-i\nabla - \frac{2\pi\vec{A}}{\phi_0})^2\psi = -\frac{1}{\xi^2}\psi \quad \cdots\cdots (1)$

introduce ψ_υ : Complete set of orthonormal Order parameter

$\mathcal{H}$: pseudohamiltonian operator.

$$H\psi_\upsilon = \frac{\hbar^2}{2m^*}\left[(-i\nabla - \frac{2\pi\vec{A}}{\phi_0})^2 + \frac{1}{\xi^2}\right]\psi_\upsilon = \mathcal{E}_\upsilon\psi_\upsilon \quad \cdots\cdots (2)$$

$$\text{eg. }(2) \xrightarrow{\mathcal{E}_\upsilon = 0} \text{eg. }(1)$$

Recalling the results of sec. 4-8,

$$\psi = e^{ik_y y} \cdot e^{ik_z z} f(x)$$

$$\left(-i\nabla - \frac{2\pi\vec{A}}{\phi_0}\right)^2\psi_\upsilon = \left(\frac{2m^*}{\hbar^2}\mathcal{E}_\upsilon - \frac{1}{\xi^2}\right)\psi_\upsilon$$

$$-f''(x) + \left(\frac{2\pi H}{\phi_0}\right)^2\left(x - \frac{\phi_0}{2\pi H}k_y\right)^2 f(x) = \left(\frac{2m^*}{\hbar^2}\mathcal{E}_\upsilon - \frac{1}{\xi^2} - k_z^2\right)f(x)$$

$$\therefore \quad (n+\tfrac{1}{2})\hbar\omega_c = \mathcal{E}_\upsilon - \frac{\hbar^2}{2m^*}\left(\frac{1}{\xi^2} + k_z^2\right)$$

$$\therefore \quad \mathcal{E}_\upsilon = (n+\tfrac{1}{2})\hbar\omega_c + \frac{\hbar^2}{2m^*}\left(\frac{1}{\xi^2} + k_z^2\right)$$

$\omega_c = \frac{2eH}{m^*c}$ Cyclotron freq.

expanding $\psi(\vec{r}) = \sum_\upsilon C_\upsilon \psi_\upsilon(\vec{r})$ orthonormal

$$f = \alpha|\psi|^2 + \frac{\hbar^2}{2m^*}\left|(-i\nabla - \frac{2\pi\vec{A}}{\phi_0})\psi\right|^2$$

$$F = \int f\, d^3r$$

$$= \alpha\sum_{\upsilon,\upsilon'} C_\upsilon C_{\upsilon'}^* \int \psi_\upsilon(\vec{r})\psi_{\upsilon'}^*(\vec{r})\, d^3r$$

$$+ \frac{\hbar^2}{2m^*}\int \left|(-i\nabla - \frac{2\pi\vec{A}}{\phi_0})\psi\right|^2 d^3r$$

$$= \alpha\sum_\upsilon |C_\upsilon|^2 + \frac{\hbar^2}{2m^*}\sum_{\upsilon,\upsilon'} C_\upsilon C_{\upsilon'}^* \int \psi_{\upsilon'}^*(-i\nabla - \frac{2\pi}{\phi_0}\vec{A})^2\psi_\upsilon\, d^3\vec{r}$$

$$= \alpha\sum_\upsilon |C_\upsilon|^2 + \frac{\hbar^2}{2m^*}\sum_{\upsilon,\upsilon'} C_\upsilon C_{\upsilon'}^* \int\left(\frac{2m^*}{\hbar^2}\mathcal{E}_\upsilon - \frac{1}{\xi^2}\right)\psi_\upsilon\psi_{\upsilon'}^*\, d^3\vec{r}$$

$$= \sum_\upsilon \left(\alpha + \mathcal{E}_\upsilon - \frac{1}{\xi^2}\frac{\hbar^2}{2m^*}\right)|C_\upsilon|^2$$

$$= \sum_\upsilon |C_\upsilon|^2\mathcal{E}'_\upsilon$$

Assigning K_BT to each normal mode

$$|C_\upsilon|^2\mathcal{E}'_\upsilon = K_BT$$

$$|C_\upsilon|^2 = K_BT/\mathcal{E}'_\upsilon$$

$$W(\{\psi_\upsilon\}) = \prod_\upsilon e^{-|C_\upsilon|^2\mathcal{E}'_\upsilon/K_BT}$$

Review

Take the Symmetric Gauge : $\vec{A} = \frac{1}{2}\vec{H}\times\vec{r}$

$$H\,\psi_\upsilon = \frac{\hbar^2}{2m^*}\left\{ \left(-i\nabla - \frac{2\pi\vec{A}}{\phi_0}\right)^2 + \frac{1}{\xi^2}\right\}\psi_\upsilon = \varepsilon_\upsilon\,\psi_\upsilon$$

OR

$$H = \frac{1}{2m^*}\left(\vec{P} - \frac{2e}{c}\cdot\frac{1}{2}\vec{H}\times\vec{r}\right)^2 + \frac{\hbar^2}{2m^*\xi^2}$$

$$= \frac{P^2}{2m^*} - \frac{e}{m^*c}\vec{H}\cdot\underbrace{\vec{r}\times\vec{p}}_{\vec{L}} + \frac{e^2}{2m^*c^2}\left\{r^2H^2 - (\vec{r}\cdot\vec{H})^2\right\} + \frac{\hbar^2}{2m^*}\cdot\frac{1}{\xi^2}$$

$$= \frac{1}{2m}(P_x^2+P_y^2) + \frac{e^2H^2}{2m^*c^2}(r^2 - z^2) - \frac{eH}{m^*c}L_z + \frac{1}{2m^*}P_z^2 + \frac{\hbar^2}{2m^*}\cdot\frac{1}{\xi^2}$$

$$= \underbrace{\frac{1}{2m^*}(P_x^2+P_y^2) + \frac{1}{2}m^*\omega_L^2(x^2+y^2)}_{\substack{2D \\ H_0:\ \text{harmonic Osc.}}} + \omega_L L_z + \frac{1}{2m^*}P_z^2 + \frac{\hbar^2}{2m^*}\cdot\frac{1}{\xi^2}$$

$$\omega_L \equiv -\frac{eH}{m^*c} = \frac{\omega_c}{2}$$

(H_0, L_z, P_z) Commutes with each other

$\longrightarrow$ common eigenstate available

$$H_0\,\psi_{n',m_\ell,P_z} = \hbar\omega_L(n'+1)\,\psi_{n',m_\ell,P_z}$$

$$L_z\,\psi_{n',m_\ell,P_z} = m_\ell,\ m_\ell,P_z$$

$$P_z\,\psi_{n',m_\ell,P_z} = P_z\,\psi_{n',m_\ell,P_z}$$

$$H\,\psi_{n',m_\ell,P_z} = \left\{\hbar\omega_L(n'+1) + \hbar\omega_L m_\ell + \frac{\hbar^2}{2m^*\xi^2} + \frac{P_z^2}{2m}\right\}\psi_{n',m_\ell,P_z}$$

$$= \left\{\hbar\omega_c\left(\frac{n'+m_\ell}{2} + \frac{1}{2}\right) + \frac{\hbar^2}{2m^*\xi^2} + \frac{P_z^2}{2m}\right\}\psi_{n',m_\ell,P_z}$$

We can show that $n' + m_\ell = 2n$ for $\psi_{n' m_\ell P_z}$ to be nonvanishing.

$$\therefore\ H\,\psi_{n' m_\ell P_z} = \left\{\hbar\omega_c\left(n+\frac{1}{2}\right) + \frac{\hbar^2}{2m^*\xi^2} + \frac{P_z^2}{2m}\right\}\psi_{n' m_\ell P_z}$$

To find the eigenfunction:

$$\frac{\partial}{\partial x} \equiv \partial_x, \qquad z = x+iy$$
$$\bar{z} = x - iy$$

$$H_0 = -\frac{\hbar^2}{2m^*}(\partial_x^2 + \partial_y^2) + \frac{1}{2}m^*\omega_L^2(x^2+y^2)$$

$$= -\frac{2\hbar^2}{m^*}\frac{\partial^2}{\partial z\partial\bar{z}} + \frac{1}{2}m^*\omega_L^2 z\bar{z}$$

Let $\psi(z,\bar{z}) = e^{\frac{m\omega_L}{2\hbar}z\bar{z}} f(z,\bar{z})$

then $H_0\,\psi(z,\bar{z}) = E_0\,\psi(z,\bar{z})$

$$\underbrace{\left\{-\frac{2\hbar^2}{m^*}\partial_z\partial_{\bar{z}} - \hbar\omega_L(z\partial_z + \bar{z}\partial_{\bar{z}})\right\}}_{E_0^{(n')} = \hbar\omega_L(n'+1)} f(z,\bar{z}) = (E_0 + \hbar\omega_L) f(z,\bar{z})$$

ground state $E_0^{(0)} = \hbar\omega_L$

$$f_0 = e^{-\frac{m^*\omega_L}{\hbar}z\bar{z}}$$

To find the excited states:

$$[h_0, \partial_z] = -\hbar\omega_L [z\partial_z, \partial_z] = -\hbar\omega_L \underbrace{[z, \partial_z]}_{-1} \partial_z$$

$$= \hbar\omega_L \partial_z$$

$$[h_0, \partial_{\bar z}] = \hbar\omega_L \partial_{\bar z}$$

$\therefore$ $\partial_z f_0$, $\partial_{\bar z} f_0$ $\longrightarrow$ Eigenfunctions of h_0 with eigenvalue

$$(E_0^{(0)} + \hbar\omega_L) + \hbar\omega_L = E_0^{(0)} + 2\hbar\omega_L$$

$\longrightarrow$ ∂_z, $\partial_{\bar z}$: ladder operators

$$\therefore \ f_{n,\bar n} = \partial_z^{n} \, \partial_{\bar z}^{\bar n} \ e^{-\frac{m^*\omega_L}{\hbar} z\bar z}$$

with

$$E_0 = \underbrace{\hbar\omega_L}_{} + \hbar\omega_L (n + \bar n)$$

Ground ~~state~~ energy

$$\psi_{n,\bar n} = e^{\frac{m^*\omega_L}{2\hbar} z\bar z} \left(\frac{\partial}{\partial \bar z}\right)^n \left(\frac{\partial}{\partial z}\right)^{\bar n} e^{-\frac{m\omega_L}{\hbar} z\bar z}$$

$$E = E_0 + \hbar\omega_L m_\ell$$

$$= \hbar\omega_L + \hbar\omega_L (n+\bar n) + \hbar\omega_L \overbrace{(n - \bar n)}^{m_\ell}$$

$$L_z \psi_{n,\bar n} = m_\ell \psi_{n,\bar n} = (n - \bar n)\psi_{n,\bar n}$$

$$E_n = (2n+1)\hbar\omega_L = (n + \tfrac{1}{2})\hbar\omega_c$$

$$\psi \sim e^{-\frac{m^*\omega_L}{2\hbar} z\bar z} = e^{-\frac{m^*\omega_L}{2\hbar} \underbrace{(x^2+y^2)}_{\rho^2}}$$

$$\varepsilon_{\parallel} = \varepsilon_{n,k_z} = (n+\tfrac{1}{2})\hbar\omega_c + \frac{\hbar^2}{2M^*}\left(\frac{1}{\xi^2} + k_z^2\right)$$

Put $\psi_\nu = f_{mn}(\rho)\, e^{im\varphi}\, e^{ik_z z}$

$$f(\rho) \sim f_1\, e^{-\frac{m^*\omega_L}{2\hbar}\rho^2} = e^{-\frac{\pi H}{2\phi_0}\rho^2}$$

irrespective of n and m.

Wave functions localized in a region satisfying

$$\pi H \rho^2 \sim \phi_0$$

$$g(\vec r, \vec r\,') \equiv \langle \psi^*(\vec r)\, \psi(\vec r\,') \rangle$$

$$= \sum_{\nu,\nu'} C_\nu^* C_{\nu'} \langle \psi_\nu^*(\vec r)\, \psi_{\nu'}(\vec r\,') \rangle$$

$$= \sum_{\substack{n,n' \\ m,m' \\ k_z,k_z'}} C^*_{n,k_z} C_{n',k_z'} \langle f_{mn}(0)\, f_{m,n'}(\rho)\, e^{i(k_z z - k_z' z')}\, e^{i(m\varphi - m'\varphi')}$$

Transforming to relative and C.M. Coordinates

$$\langle e^{i(k_z z - k_z' z')} \rangle = \delta_{k_z,k_z'}\, e^{ik_z(z-z')} = e^{ik_z z}\, \delta_{k_z,k_z'}$$

$$\langle e^{i(m\varphi - m'\varphi')} \rangle = \delta_{m,m'}\, e^{im(\varphi-\varphi')}$$

$$\therefore \ g(\rho, z) = \sum_{n,n',k_z} C^*_{n,k_z} C_{n',k_z} f^{(0)}_{0,n} f_{0,n'}(\rho)\, e^{ik_z z}$$

$$(\because f_1(0) \doteq 0 \ \text{ unless } \ m=0)$$

$$|C_{n,k_z}|^2 = k_B T / \varepsilon_{n,k_z} \sim (k_z^2 + k_{0,n}^2)^{-1}$$

$$k_{0,n}^2 \equiv \frac{1}{\xi^2} + \frac{2m^*\hbar\omega_c}{\hbar^2}(n + \tfrac{1}{2}) = \frac{1}{\xi^2} + \frac{(2n+1)2\pi H}{\phi_0}$$

$$g(\rho, z) \sim \int_0^{\infty} \frac{e^{ik_z z} k_z^2 \, dk_z}{k_z^2 + k_{0,n}^2} f_{0,n}^{(0)} f_{0,n'}(\rho)$$

$$\sim e^{-K_{0,n}|z|} e^{-\pi H\rho^2/2\phi_0}$$

dominant for $n = 0$

$$\longrightarrow e^{-K_{0,0}|z|} e^{-\pi H\rho^2/2\phi_0}$$

$$K_{0,0} = \left(\xi^{-2} + \frac{2\pi H}{\Phi_0} \right)^{1/2}$$

The radius of the "Correlated fluctuations' shrink below ξ as H z $\Phi_0/2\pi\xi^2$

$\chi \propto C \langle \psi^2 \rangle R^2$ R : radius of a particle

The susceptibility gets smaller in a finite field than in the limit of zero field.

. High field rapidly extinguish the fluctuations.

Fluctuation Diamagnetism above T_c

a superconductor above T_c

→ a collection of independent fluctuating droplets of superconductivity

$$\chi \sim -\frac{1}{40\pi} \frac{4\pi e^{*2}}{m^* c^2} \langle \delta\psi^2 \rangle \langle r^2 \rangle$$

↑ size of the droplet

$$= -\frac{1}{10} \frac{e^2}{m^* c^2} \frac{K_B T \langle r^2 \rangle}{\alpha V}$$

$$= -\frac{1}{5} (2\pi)^2 \cdot \left(\frac{2e}{hc} \right)^2 \cdot \frac{K_B T \langle r^2 \rangle \xi^2}{V} \qquad \xi^2 = \frac{\hbar^2}{2m^* \alpha}$$

$$= -0.8\pi^2 \frac{K_B T \xi^2 \langle r^2 \rangle}{\phi_0^2 V}$$

$V = \frac{4}{3}\pi\xi^3$

$\langle r^2 \rangle \sim \left(\frac{\xi}{2} \right)^2$

$$\simeq -\frac{\pi}{6} \frac{K_B T}{\phi_0^2} \xi(T) \simeq 10^{-7} \cdot \frac{1}{(t-1)^{1/2}}$$

Schmid. P.R.B. 180
527 (1969)

$\chi_{fluct} \ll -\frac{1}{4\pi}$

enhancement limited by
① 1-st order transition in H-field
② finite transition width
③ fluctuation effect → weakened or destroyed in a high field

Generalization of Schmid result.

(Valid only for a finite field)

$$\delta M \sim (T - T_{c_2})^{-\frac{1}{2}}$$

$$\frac{\delta M}{H^{1/2} T} = f(x)$$

$$x = \frac{T - T_c}{H} \left(\frac{dH_{c_2}}{dT} \right)_{T_c}$$: Prange, P.R.B. 1. 2349 (7?)

Not in good agreement with exp.

→ Short wavelength [$\leq \xi(0)$] fluctuations taken into account, dominating $T \gg T_c$

strong H

$$\frac{\delta M}{H^{1/2} T} = f_{PAW}(x, H/H_s)$$: Patton, Ambegaokar & Wilkins
Solid state Com.
7, 1287 (1969)

H_s: material dependent scaling field

The divergence near $T_{c_2}(H)$ in a finite field obscured by

① Superconducting and a sudden jump at $T > T_{c_2}(H)$ – type I

② finite transition width – type II

Fluctuation Diamagnetism in 2D

$$d \ll \xi \longrightarrow V \sim \pi \xi^2 d$$

$$\chi \approx - \frac{\pi^2 K_B T \xi^2 \langle r^2 \rangle}{\phi_0^2 V} = - \pi k_B T \langle r^2 \rangle / \phi_0^2 d$$

$\langle r^2 \rangle$?

$$\frac{\chi H^2}{8\pi} = \frac{\vec{J} \cdot \vec{A}}{2c} \propto A^2$$ in the London gauge

but

$$\oint \vec{A} \cdot d\vec{s} \approx B \quad (\text{area of the fluctuating region})$$

$$A \approx \mathcal{A} B / S$$ S: perimeter

$$\approx \mathcal{A} H / S$$

$$\chi \sim \left(\frac{A}{H} \right)^2 \approx \left(\frac{\mathcal{A}}{S} \right)^2$$

$$\rightarrow \begin{cases} - \langle r^2 \rangle = \left(\frac{\xi}{2} \right)^2 & \text{for a sphere} \\ \langle r^2 \rangle_{eff} \hat{=} \;\; \xi^2 : H_\perp & \\ \left(\frac{d}{2} \right)^2 : H_\parallel & \end{cases} \Big\} \text{ for a disk}$$

$$\chi_\perp^{2D} \approx - \frac{K_B T \xi^2}{\phi_0^2 d} \approx \frac{\xi}{d} \chi^{3D} \propto (t-1)^{-1}$$

$$\chi_\parallel^{2D} \approx - \frac{K_B T d}{\phi_0^2} \approx \frac{d}{\xi} \chi^{3D}$$ ~ almost observable constant.

$$\chi^{2D}_{\perp,\text{total}} = \chi^{2D}_{\perp} \cdot \text{Volume of the film}$$

$$\propto (\xi/d) \cdot \chi^{3D} \cdot d \sim \xi \chi^{3D}$$

$\rightarrow$ Very small

$\therefore$ $\chi^{2D}_{\perp}$, total observable only in layered structure

- artificially fabricated
- intrinsically formed

TaS_2

all the high-Tc. s.c. materials, etc

layered structure with Josephson links between layers

$\longrightarrow$ dimensional crossover with T, H

$\longrightarrow$ Lawrence - Doniach theory.

Time dependence of fluctuations Conductivity:

- nonequilibrium properties
- fluctuation lifetime T > Tc 에 비례한다.

  time limits the period available for acceleration in an applied field
- TDGL

linearized TDGL

$$\frac{\partial \psi}{\partial t} = -\frac{1}{\tau_{GL}} (1 - \xi^2 \nabla^2 \psi) \qquad T > T_c \quad \text{without e.m. interaction}$$

equil. GL eq

$$\left(-i\nabla - \frac{2\pi \vec{A}}{\Phi_0}\right)^2 \psi = -\frac{1}{\xi^2}\psi$$

$$\vec{A} = 0 \quad \Rightarrow \quad (1 - \xi^2 \nabla^2)\psi = 0$$

τ_{GL} = ~~temp~~ temp - dependent relaxation time of the uniform (k=0) mode

$$= \frac{\pi \hbar}{8 k_B (T - T_c)}$$

for $K \neq 0$

$$\psi = \sum_K \psi_K e^{i\vec{K}\cdot\vec{r}} e^{-t/\tau_K}$$

then $$\sum_K \psi_K e^{i\vec{K}\cdot\vec{r}} \left(-\frac{1}{\tau_K}\right) e^{-t/\tau_K} = -\frac{1}{\tau_{GL}} \sum_K (1 + \xi^2 K^2) \psi_K e^{i\vec{K}\cdot\vec{r}} e^{-t/\tau_K}$$

$$\therefore \tau_K = \frac{\tau_{GL}}{1 + \xi^2 K^2}$$

$K \neq 0$ mode (with higher energy) decays mode rapidly o maintain a nonzero $|\psi_k|^2$, the thermal average value,

$$\frac{\partial \psi_k}{\partial t} = -\frac{1}{\tau_0}(1-\xi^2\nabla^2)\,\psi_k + F_k$$

white spectrum driving force

$$\langle \psi_k^*(0)\,\psi_k(t)\rangle = \langle|\psi_k|^2\rangle\, e^{-t/\tau_k}$$

$$\langle|\psi_k|^2\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\langle|\psi_{k,\omega}|^2\rangle\, d\omega$$

$$\langle|\psi_{k,\omega}|^2\rangle = \langle|\psi_k|^2\rangle\,\frac{2\tau_k}{1+\omega^2\tau_k^2}$$

$$= \frac{2m^*}{\hbar^2}\,\frac{k_BT}{k^2+\frac{1}{\xi^2}}\,\frac{2\tau_k}{1+\omega^2\tau_k^2}$$

$$= \left(\frac{2m^*\xi^2}{\hbar^2}\right)\frac{k_BT}{1+k^2\xi^2}\,(\;''\;)$$

$$= \frac{1}{\alpha}\,\frac{\tau_k}{\tau_0}\left(\frac{2k_BT\,\tau_k}{1+\omega^2\tau_k^2}\right)$$

$$= \frac{16k_B}{\pi}\cdot\underset{\substack{\uparrow\\ \text{temp. independent}}}{\frac{T-T_c}{\hbar\alpha}}\;\frac{k_BT\,\tau_k^2}{1+\omega^2\tau_k^2}$$

Fluctuation - Enhanced Conductivity : $T > T_c$

1) Aslamasov -, Larkin conductivity

$$\sigma_{AL}' = \frac{e^{*2}}{m^*}\sum_k n_s(k)\,\tau_s(k)$$

$$= \frac{(2e)^2}{m^*}\sum_k \frac{\langle|\psi_k|^2\rangle}{2}\,\tau_s(k)$$

$$f = \sum_k\left(\alpha + \frac{\hbar^2k^2}{2m^*}\right)|\psi_k|^2$$

$$\langle|\psi_k|^2\rangle = |\psi_k|^2$$

$$= \frac{k_BT}{\alpha+\frac{\hbar^2k^2}{2m^*}} = \frac{2m^*\xi^2}{\hbar^2}\,\frac{k_BT}{1+\xi^2k^2}$$

$$= \frac{\pi(2e)^2\xi^2(0)}{16\hbar}\cdot\frac{T_c}{T-T_c}\,\underbrace{\sum_k\frac{1}{(1+\xi^2k^2)^2}}_{\frac{1}{(2\pi)^d}\int\frac{d^dk}{(1+\xi^2k^2)^2}\;\sim\;(T-T_c)^{-(2-d)/2}}$$

$$\sim (T-T_c)^{-(4-d)/2}$$

$$\sigma'_{AL} \sim \begin{cases} \frac{1}{32}\frac{e^2}{\hbar\xi(0)}\left(\frac{T}{T-T_c}\right)^{1/2} \leftrightarrow \frac{e^2}{32\hbar\xi(0)}\frac{1}{\epsilon^{1/2}} & : 3D \\ & \epsilon = \ln(T/T_c) \\ \frac{e^2}{16\hbar d}\frac{T}{T-T_c} \rightarrow \frac{e^2}{16\hbar d}\frac{1}{\epsilon} & : 2D \quad (d \ll \xi) \\ \frac{\pi e^2\xi(0)}{16\hbar A}\left(\frac{T}{T-T_c}\right)^{3/2} & : 1D \quad (A \ll \xi^2) \end{cases}$$

Notes :

i) 3D $\rightarrow$ $\sigma_0 \equiv \frac{e^2}{32\hbar\xi(0)}$

$$\sigma/\sigma_n \simeq [k_F^2 \ell \xi(0)]^{-1} \sim 10^{-5\pm2}$$

ii) Actual measurement on $\epsilon'_{AL} = \sigma'_{AL} d$

$\longrightarrow$ thickness not crucial

iii) Maki - Thompson Conductivity $\rightarrow$ due interaction between the quasi particles and fluctuation pairs

$$\epsilon'_{MT} = \frac{e^2}{m}\sum_K \underset{\uparrow}{n_s(K)}\ \underset{\uparrow}{\tau_D(K)}$$

Current carried by quasi particles

lifetime for diffusive decay of density fluctuations.

$$\tau_D(K) = \frac{1}{DK^2 + \frac{1}{\tau_n}}$$

τ_n : lifetime of quasi particles against condensing into fluctuation pairs

$$\text{OR} = \frac{1}{DK^2 + \frac{1}{\tau_i}}$$

$$G'_{MT} = \frac{e}{m}\frac{2m\xi^2}{\hbar^2}\underbrace{\sum_K \frac{K_B T_c}{1+\xi^2K^2}\frac{1}{DK^2+\frac{1}{\tau_i}}}_{}$$

$$\int_0^\infty \frac{2\pi K\,dK}{(2\pi)^2}\frac{1}{1+\xi^2K^2}\frac{1}{DK^2+\frac{1}{\tau_i}} \qquad 2D.$$

$$= \frac{e^2}{16\hbar}\frac{1}{\epsilon-\delta}\ln\left(\frac{\epsilon}{\delta}\right)$$

$$\epsilon \equiv 1 - \frac{T}{T_c} \quad \text{OR} \quad \ln\left(\frac{T}{T_c}\right)$$

$$\delta \equiv \frac{\xi^2(0)}{D\tau_i} = \frac{\pi\hbar}{8K_BT_c}\frac{1}{\tau_i} \overset{\text{OR}}{=} \frac{\tau_{GL}(0)}{\tau_i} \overset{\text{OR}}{=} \frac{\xi^2(0)}{\xi_i^2}$$

pair breaking strength.

Chapter 9.

The high Temperature Superconductors

9.1. Introduction

1911. Kramerlingh Onnes 의 수은에서의 초전도체 4k

1973 Gavaler 23K

1986 LBCO Bednorz, Müller 1987. 노벨상

이 발견 : Surprising and Exciting

1. T_c 가 매우 증가
2. Oxide 가 초전도 만든다 ?

90k YBCO

- United States, Japan, China
- Y 대신 La, Nd, Sm, Eu, Gd, Ho, Er, Lu
- BSCCO, TBCCO

공통점

1. CuO_2 plane 이 있다.

YBCO - 이외에도 chain 이 있다.

평면 위의 전자에 대한 저수지.

YBCO 93k, BSCCO 110K, TBCCO 130k

1. N_2 cooling
2. What is the mechanism responsible for the high T_c.,

BCS 의 Cooper pair와 같나?

적어도 G-L 이론은 成立

그러나 modified by the radically modified parameter values.

Mechanism은 모른다 쳐도 Magnetism에서 BCS/GL 벗어

Short coherence length,

Prominant fluctuation Effects.

T_c 높다.

3. decoupled superconducting film planes

9.2 에서 다룬다.

4. Resistive transition can be understood by a discussion of the elastic properties of the flux lattice.

Melting, Vortex glass transition 보인다.

5.

High frequencies losses.

6. Unconventional d-wave pairing.

9.2. The Lawrence - Doniach Model.

Layered structure 를 기술.

예 : layered transition metal dichalcogenides

Such as TaS_2 with organic molecules intercalated between the metallic layers

2D functions coupled together with Josephson tunneling between adjacent layers.

$$F = \sum_n \int \left[\alpha |\psi_n|^2 + \frac{1}{2}\beta|\psi_n|^4 + \frac{\hbar^2}{2m_{ab}} \left(\left|\frac{\partial \psi_n}{\partial x}\right|^2 + \left|\frac{\partial \psi_n}{\partial y}\right|^2 \right) + \frac{\hbar^2}{2m_s s^2} |\psi_n - \psi_{n-1}|^2 \right]$$

만약 $\psi_n = |\psi_n| e^{i\varphi_n}$

$$\frac{\hbar^2}{m_e s^2} |\psi_n|^2 \cdot [\, 1 - \cos(\varphi_n - \varphi_{n-1})]$$

이것을 넣어 ψ_n 에 대해 미분하자.

$$\alpha \psi_n + \beta |\psi_n|^2 \psi_n - \frac{\hbar^2}{2m_{ab}} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi_n - \frac{\hbar^2}{2m_c s^2} (\psi_{n+1} - 2\psi_n + \psi_{n-1}) = 0$$

만약 Vector Potential 까지 넣는다면

$$\alpha \psi_n + \beta |\psi_n|^2 \psi_n - \frac{\hbar^2}{2m_{ab}} \left(\nabla - i \frac{2e}{\hbar c} \vec{A} \right)^2 \psi_n - \frac{\hbar^2}{2m_c s^2} \left(\psi_{n+1} e^{-2ieA_z s/\hbar c} - 2\psi_n + \psi_{n-1} e^{2ieA_z s/\hbar c} \right) = 0$$

9.2.1. The Anisotropic Ginzburg - Landau limit.

만약 z 방향으로 smooth 하다면

$$\frac{\psi_n - \psi_{n+1}}{s} \longrightarrow \frac{\partial \psi}{\partial z}$$

$$\alpha \psi + \beta |\psi|^2 \psi - \frac{\hbar^2}{2} \left(\nabla - i \frac{2e}{\hbar c} \vec{A} \right) \cdot \left(\frac{1}{m} \right) \left(\nabla - i \frac{2e}{\hbar c} \vec{A} \right) \psi = 0$$

① If the interlayer coupling is weak

$$m_c \gg m_{ab}$$

만약 anisotropic 하다면 $\frac{1}{m}$ 이 reciprocal mass tensor 가 됩니다. The mass anisotropy - ξ to be anisotropic

$$\xi_i^2(T) = \frac{\hbar^2}{2m_i |\alpha(T)|}$$

② Since $\alpha(T) \sim (T - T_c)$, ξ_i scale with $\frac{1}{\sqrt{m_i}}$ and diverges as $|T - T_c|^{-\frac{1}{2}}$.

앞에서 $2\sqrt{2}\pi \, H_c(T) \, \xi_i(T) \, \lambda_i(T) = \Phi_0$

③ λ_i 는 ith axis 로 흐르는 전류를 Screening 한다.

not ith axis 로의 field screening

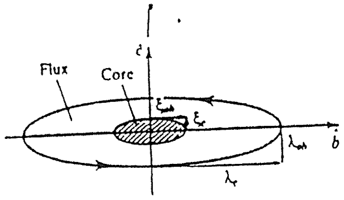


FIGURE 9.1
Schematic cross section of a vortex along the a axis in an anisotropic superconductor. The various dimensions are related by $\gamma = \lambda_c/\lambda_{ab} = \xi_{ab}/\xi_c = (m_c/m_{ab})^{1/2}$.

④ $$H_{c_2} = \frac{\Phi_0}{2\pi\xi^2}$$

$$H_{c_2\parallel c} = \frac{\Phi_0}{2\pi\xi_{ab}^2}$$

$$H_{c_2\parallel ab} = \frac{\Phi_0}{2\pi\xi_{ab}\xi_c}$$

So that $H_{c_2\parallel ab} \gg H_{c_2\parallel c}$

$\xi_{ab} \gg \xi_c$

Relation.

$$\gamma \equiv \left(\frac{m_c}{m_{ab}}\right)^{1/2} = \frac{\lambda_c}{\lambda_{ab}} = \frac{\xi_{ab}}{\xi_c} = \left(\frac{H_{c_2\parallel ab}}{H_{c_2\parallel c}}\right)$$

$$= \left(\frac{H_{c_1\parallel c}}{H_{c_1\parallel ab}}\right)$$

YBCO : $\frac{m_c}{m_{ab}} \simeq 50$ $\gamma \sim 7$

BSCCO : $\frac{m_c}{m_{ab}} \simeq 20{,}000$, $\gamma \gtrsim 150$

⑤ Angular dependence interpolation H_{c_2} ...

$$\left(\frac{H_{c_2}(\theta)\sin\theta}{H_{c_2\parallel c}}\right)^2 + \left(\frac{H_{c_2}(\theta)\cos\theta}{H_{c_2\parallel ab}}\right)^2 = 1$$

OR equally

$$H_{c_2} = \frac{H_{c_2\parallel ab}}{(\cos^2\theta + \gamma^2\sin^2\theta)^{1/2}}$$

Blatter, Geshkenbein, Larkin

So long as the continuum GL approximation is applicable, one begins by introducing related coordinates,

& $\vec{r} = (\tilde{x}, \tilde{y}, \tilde{z}/\gamma)$, $\vec{A} = (\tilde{A}_x, \tilde{A}_y, \gamma\tilde{A}_z)$

$\vec{B} = (\gamma\tilde{B}_x, \gamma\tilde{B}_y, \tilde{B}_z)$

which makes isotropic the Gauge-invariant derivative term, at the expense of introducing anisotropy in the magnetic energy terms.

만약 $B \gg H_{c_1}$, 이면 flux들은 서로 overlap 할것이고 macroscopic magnetic energy는 average field로 계산이 된다.

$$Q(\theta, H, T, \xi, \lambda, \gamma, \delta) = S_Q\,\tilde{Q}(\varepsilon_\theta H, \gamma T, \xi, \lambda, \gamma\delta)$$

θ: field와 ab plane 사이 각도

δ : scalar disorder strength

Q : desired quantity for which the isotropic result $\tilde{Q}$ is known,

$\varepsilon_\theta^2 = \gamma^{-2}\cos^2\theta + \sin^2\theta$

$S_Q = \frac{1}{\gamma}$ for volume, energy, temperature, action

$S_Q = \frac{1}{\varepsilon_\theta}$ for magnetic field.

9.2.2.

Crossover to two-dimensional Behavior.

만약 $T \to T_c$, $\xi_c \approx \xi_c(0)\,(1-t)^{-\frac{1}{2}} \to \infty$

$T \to$ ZERO $\xi_c \to$ a limiting value.

이 길이가 S보다 작을수 있다.

3D continuum approximation이

2D individual layer로 바뀐다.

Klem 등이 연구하다. - layered dichalcoginides

실험과 연관하여.

$\xi_c(T^*) = S/\sqrt{2}$

$T < T^*$ 이면 core 가 없어지고

$H_{c2} \to \infty$ 가 된다.

Unphysical infinity is eliminated by taking into account ① the infinite layer thickness,

② pair breaking due to Pauli Paramagnetism

③ Spin-orbit coupling effect.

Discrete 하므로

$$0 = \alpha\psi_n + \beta|\psi_n|^2\psi_n - \frac{\hbar^2}{2m_{ab}}\left(\nabla - i\frac{2e}{\hbar c}\vec{A}\right)^2\psi_n$$

$$- \frac{\hbar^2}{2m_c S^2}\left(\psi_{n+1}\,e^{-2ieA_zS/\hbar c} - 2\psi_n + \psi_{n-1}\,e^{2iA_zS/\hbar c}\right)$$

H_{c2}를 구하여 보자.

가정 ① $A_z = Hx$, $\vec{H} = H\hat{y}$

② lowest eigenvalue 가 H_{c2} 결정

linearized version of GL 에서

$\boxed{4\text{-}8}$에서 계산했던 것 처럼 ψ는 x 만의 함수가 된다.

$$-\frac{d^2\psi}{dx^2} + \frac{2m_{ab}}{m_c S^2}\left[1 - \cos\frac{2\pi HSx}{\Phi_0}\right]\psi = \frac{1}{\xi_{ab}^2(T)}\psi$$

Check point.

① $S \to 0$, $$\frac{d^2\psi}{dx^2} + \frac{m_{ab}}{m_c}\left(\frac{2\pi Hx}{\Phi_0}\right)^2\psi = \frac{1}{\xi_{ab}^2(T)}\psi$$

따라서 Critical field 는

$$H_{c2} = \frac{\Phi_0}{2\pi\xi_{ab}^2}\left(\frac{m_c}{m_{ab}}\right)^{1/2} = \frac{\Phi_0}{2\pi\xi_{ab}\xi_c}$$

② $m_c S^2 \to \infty$ 완전히 decoupled.

$$-\frac{d^2\psi}{dx^2} = \frac{1}{\xi_{ab}^2}\psi$$

Uniform ψ 이므로 $\xi_{ab} \to \infty$

③ $H \to \infty$ 이면 cosine 항은 average zero 이다.

$$-\frac{d^2\psi}{dx^2} = \left[\frac{1}{\xi^2_{ab}(T)} - \frac{2m_{ab}}{m_c s^2}\right]\psi$$

uniform한 Solution 에 대해

$$\xi_{ab}^2 = \frac{m_c s^2}{2m_{ab}} \quad \text{OR} \quad \xi_c(T) = \frac{s}{\sqrt{2}}$$

③ $H \neq \infty$, but large

$$H_{c2,ab}(T) \approx \frac{(\Phi_0/2\pi s^2)\left(\frac{m_{ab}}{m_c}\right)^{1/2}}{\left[1 - s^2/2\xi_c^2(T)\right]^{1/2}}$$

$T \to T^*$ 이면 $H_{c2,ab}(T) \sim (T - T^*)^{-\frac{1}{2}}$

This divergence is artifact.

① Pauli paramagnetism

② finite layer thickness

등 고려 안함

9.2.3. Discussion.

Lawrence - Doniach theory

Artificial layered material

1) Anisotropic 3D behavior

$H_{c2} \propto T_c - T$

2) 2D behavior $H_{c2} \propto (T_c - T)^{1/2}$

3) Cross over behavior 관찰

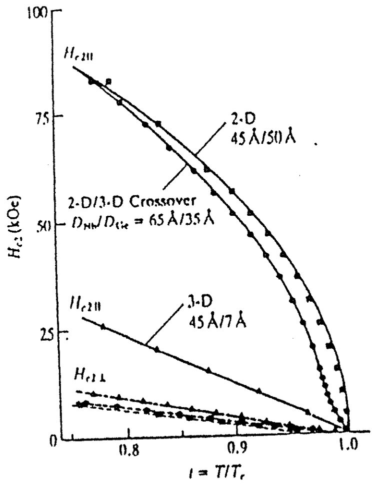


FIGURE 9.2
Upper critical fields of layered Nb/Ge composites with layer thickness D_{Nb} and D_{Ge} as indicated. Decreasing the Ge thickness effects the progression from anisotropic 3-D behavior $[H_{c2} \sim (T_c - T)]$ to "crossover" to "decoupled," or 2-D behavior with $H_{c2} \sim (T_c - T)^{1/2}$. The solid lines are from the Lawrence-Doniach model. ($H_{c2\perp}$ is essentially independent of the thickness parameters since it is determined solely by the coherence length in the plane.) [*After Ruggiero et al.*, Phys. Rev. Lett. 45, 1299 (1980).]

최근 실험

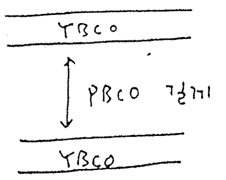
1. Multilayer samples of superconductors.

Monn Ge=3 Ge: Insulator.

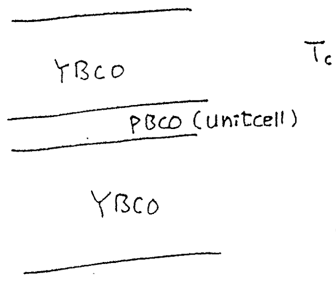
Mass ratio 조절하여 (YBCO, BSCO 비슷하게)

resistive transition이 " 와 비슷함 B 관찰

2. YBCO 와 PBCO 비교



$T_c \simeq 20K$



$T_c \sim 56K$

왜 T_c 낮아지나?

G.L 의 linearized version으로

이해가능.

YBCO, BSCCO

① LD 이론 과 실제 물질간의 문제

unit cell 속에 equivalent CuO_2 layer가 있다.

S 를 무슨값으로 넣어야 하나?

② $H_{c2}(T)$ 의 값이 fluctuation에 의해 poorly defined

③ $\frac{m_c}{m_{ab}} = 50, 20,000$

$\xi_c(0)$ 는 2.8Å, 0.1Å for YBCO, BSCCO

$\xi_c(T^*) = S/\sqrt{2}$, we find $\frac{T^*}{T_c} = 0.84$ and 0.999

YBCO 는 3D GL superconductor 90 — 78K 까지

BSCCO 는 T_c 보다 0.1K 아래이다.

④ 위의 차이에 의해 magnetic property 도 아주 다를것이다.

$$\left(\frac{H_{c_2}(\theta)\sin\theta}{H_{c_2\parallel c}}\right)^2 + \left(\frac{H_{c_2}(\theta)\cos\theta}{H_{c_2\parallel ab}}\right)^2 = 1$$

Bulk : YBCO

$$\left|\frac{H_c(\theta)\sin\theta}{H_{c\perp}}\right| + \left(\frac{H_c(\theta)\cos\theta}{H_{c\parallel}}\right)^2 = 1$$

thin film : BSCCO

torque measuremnt 에서도

YBCO : 3-D angular dependence holds down to 80K

BSCCO : results are Sample dependent, but it appears that discrete layers play an important role even very near T_c.

⑤ 매우 얇은 film (isolated)

$$H_{c\parallel} = 2\sqrt{6}\, H_c\, \lambda/d$$

$$= \left[\sqrt{3}\, \Phi_0/\pi d\, \xi_{ab}(0)\right] (1-t)^{1/2}$$

$d \sim 1$Å, $\xi_c \sim 0.4$Å 이라 하고 BSCCO에 적용하니 OK.

9-3. Magnetization of layered superconductors.

Magnetization 해석

9.3.1. The anisotropic Ginzburg - Landau Regime

Anisotropic 하기에 $\vec{H}, \vec{B}, \vec{M}$ 이 colinear 하지 않다.

(unless they lre along a principal axis)

fortunatoly,

$H_{c_1} \ll H \ll H_{c_2}$,

K 가 크면 Core 가 작고, Order parameter 가 경처 const. ⇒ London Assumption 이 가능

$$B \approx H \gg |M| \sim H_{c_1}$$

Kogan 등은 Anisotropic London 방정식 응용.

Mass anisotropy

Helmholtz free energy

$$F = \frac{B^2}{8\pi} + \frac{H^*}{4\pi} \left(B_{ab}^2 + \gamma^2 B_c^2 \right)^{1/2}$$

B_{ab} : B 의 ab 방향 Component

$$H^* \approx \frac{\Phi_0}{8\pi \gamma \lambda_{ab}^2} \cdot \ln \frac{\eta H_{c_2}(0)}{B}$$

$$\vec{H} = 4\pi \frac{\partial F}{\partial \vec{B}} = B - 4\pi \vec{M}$$

$$H_{ab} = B_{ab} + H^* \frac{B_{ab}}{\left(B_{ab}^2 + \gamma^2 B_c^2 \right)^{1/2}}$$

$$H_c = B_c + H^* \frac{\gamma^2 B_c}{}$$

Note ① $M = (B-H)/4\pi \sim H^* \ll H$

③ Magnetization $\not\parallel$ H

이것은 strongly tilted away from the plane.

$$\frac{M_{ab}}{M_c} = \gamma^{-2} \frac{B_{ab}}{B_c}$$

θ_M : angle made by M relative to the planes.

$$\tan\theta_M = \gamma^2 \tan\theta$$

$$\gamma^2 \sim 50 \sim 20{,}000$$

따라서 θ 가 작아도 M_c 가 크다.

prefer to form a stack of pancake vortices

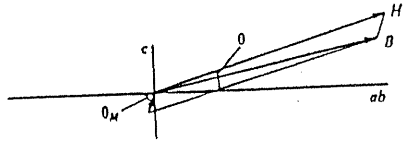


FIGURE 9.3
Schematic diagram illustrating the fact that M is not collinear with B unless B lies along a principal axis, and that |M| ≪ |B|, so that B ≈ H.

Tuominen : $\frac{M_T}{M_L}$ is a ft. only of γ, θ

These prediction has been tested.

Torque 측정후 $\vec{\tau} = \vec{M} \times \vec{H}$

$$\tau(\theta) = V (M_c H_{ab} - M_{ab} H_c)$$

$$= V (\gamma^2 - 1) \frac{H H^*(\theta)}{4\pi} \frac{\sin\theta \cdot \cos\theta}{(\cos^2\theta + \gamma^2 \sin^2\theta)^{\frac{1}{2}}}$$

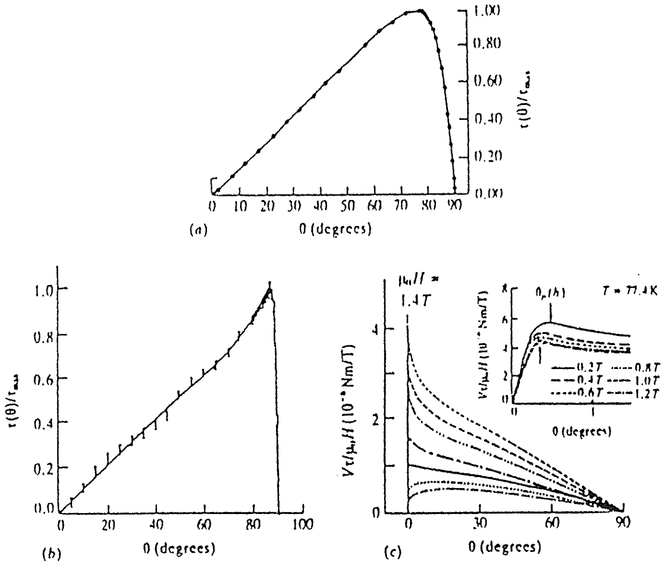


FIGURE 9.4
Comparison of measured normalized torque data (at B = 1T) with prediction of (9.19) for (*a*) YBCO and (*b*) BSCCO. [The data in (a) are taken from Farrell et al., *Phys. Rev. Lett. 64, 1573 (1990)*; they were taken at 80 K and fitting parameters are γ = 7.9 and $H_{c2\parallel}$ = 23 *T*. The data in (*b*) are taken from Farrell et al., *Phys. Rev. Lett. 63, 782 (1989)*; they were taken at 78.5 K and fitting parameters are γ = 55 and $H_{c2\parallel}$ = 35T]. (*c*) Angular dependence of scaled torque of a BSCCO crystal at 1.4T and various temperatures. [*After Martinez et al.*, Phys. Rev. Lett. *69, 2276 (1992)*.] From top to bottom, the plots correspond to T = 77.4, 78.6, 79.7, 81.5, 82.6, 83.1, and 84.4 K. The inset shows the behavior with H near the *ab* plane for various field values at 77.4 K, showing that the torque drops from maximum to zero in ~0.3° of rotation. [*Note*: In parts (*a*) and (*b*), θ is measured from the *c* direction and is equivalent to (90° − θ) in the text, whereas in part (*c*), θ is measured from the *ab* plane, as in the text.]

$H^*(\theta)$ 의존성 은 γ가 커지면서 커진다.

Small γ , $\tau(\theta) \sim \sin\theta \cdot \cos\theta$

$\gamma \gg 1$, Scale of the torque becomes independent of γ

$\tau(\theta) \sim \cos\theta$ as $\theta \to 0$

Maximum $\theta_m \sim \gamma^{-\frac{1}{2}}$

Half maximum − $\theta_{1/2} \sim \frac{1}{\sqrt{3}} \gamma^{-1} \ll 1$

실험 : Martinez 그림 (c)

이 drop은 0.3° 이내로 정확

일치 않는 부분

이 3D GL 이론은 highly anisotropic material에는 부적당하다.

- discreteness of the layer를 계산에 넣어야 한다. − lock-in transition 생각하자.

9-3-2 The Lock-in transition

$\xi < \frac{S}{\sqrt{2}}$ 이면 Core structure가 drastic change

특히 이경우 Vortex Core는 plane 사이에서 있으려 한다.

Feinberg, Villard

H 가 finite angle θ_H 으로 가해진 경우

$\theta_H < \theta_c$ 이라면 flux lines run strictly parallel to the planes, remaining "locked in" between the layers. — Transverse Meissner effect.

Bulaevskii, Ledvij, Kogan

An applied field first penetrates the planes when its perpendicular component $H \sin\theta$ exceeds a threshold value H_J, which is of the order of $H_{c_1 \parallel c}$.

$\theta_c \sim \frac{1}{H}$ at small angle.

Quantitative test는 demagnetization effect 때문에 어렵다.

Martinez : torque in BSCCO at 77K ($T/T_c \approx 0.92$) increase linearly with the internal perpendicular $H_z \sim 100\,Oe$

$$H_{c_1} = \frac{H_c}{\sqrt{2}k} \ln k$$

$k = 70$, $\lambda_{ab}(0) \simeq 1500 A^\circ$ 일때의

$H_{c_1 \parallel c}$

Organic layered Superconductor

$(BEDT-TTF)_2\ Cu(SCN)_2$

Mansky, Chaikin, Haddon

$H_{th} \sin\theta \simeq H_J$

— ac susceptibility Measurement

그림 9-4의 YBCO

No discontinuity of the angular dependence

Sharp lock-in phenomena is washed out by thermal fluctuation in the region of reversible magnetization near T_c

Low temp에서만 보이는 현상일지 모르다

— Mansky low temp data

Futher. theoretical work of B. L. K

fluctuation effect 고려

. Martinez 의 data에 잘 맞아들어간다.

Martinez conclude that the extreme anisotropy of BSCCO cannot be adequately described by the anisotropic GL model : Only a sample dependent lower bound $\underline{\underline{\gamma > 150}}$ can be given.

9.4. Flux Motion and the Resistive Transition:

An initial overview

equilibrium properties treated in G.L theory

만 생각했다 - anisotropy and planar structure

but ignoring fluctuations

extrinsic flux pinning

and things in the absence of

transport current.

Classically

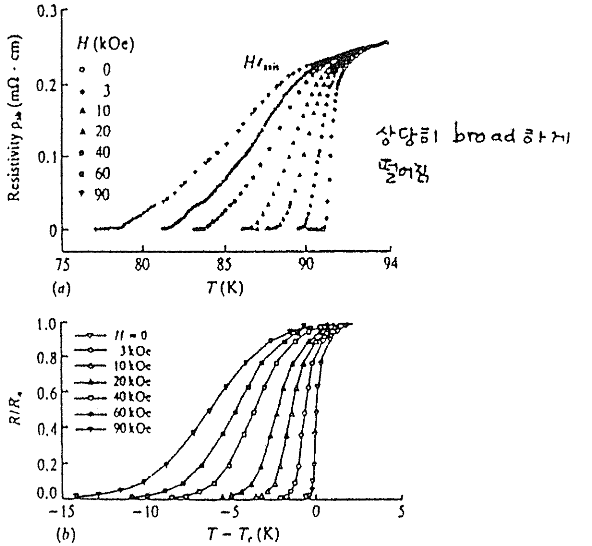
fluctuation : 중요하지 않고

pinning is reasonably effective

ΔT : 작다.

ΔH : 작다.

HTSC -



상당히 broad하게 떨어짐

FIGURE 9.5

(a) $\rho_{ab}(T)$ of YBCO crystal for various values of H (*reported by Iye et al.*) showing the broadening of the resistive transition by magnetic field. (*b*) Comparison curves computed by a model of Tinkham. [Phys. Rev. Lett. *61, 1658 (1988)*.] Although agreement is good over the top 90 percent of the resistive transition, the experimental resistance cuts off more sharply as $R \rightarrow 0$.

왜 그런가?

1. T_c is large

v_F " small

ξ " small — small coherence volume

$$\underbrace{\frac{H_c^2}{8\pi}}_{\text{크고}} \cdot \underbrace{\xi^3}_{\text{작고}} \longrightarrow \text{작다}$$

따라서 thermally activated processes

with rates proportional to $\exp(-F_0/kT)$

이것이 매우 중요하다

예. $T_p = 800K$, $\Omega = 10^8$

$F_0 = kT_p$

$\Omega e^{-F_0/kT}$ 10^3 per second at 77K

10^{-79} " at 4K

이 Rapid Relaxation에 의해 Magnetization이 넓은

온도 영역에서 Reversible 하다. - T_c 아래에서

또한 irreversible 한 부분이 나타난다

$$H_{irr} \propto (T_c - T)^{1.5}$$

irreversible line 이 있다

spin glass 라 하자 - Muller 가 제안

Yeshurun and Malozemoff 는 $(T_c - T)^{1.5}$ 로

$$U \propto (T - T_c)^{3/2}/B$$ 로 설명했다.

Tinkham : 이러한 activation energy를 Ambegaokar-Halperin의 formular에 넣어 thermally activated phase slippage in overdopped Josephson Junction

— field와 Temperature 의존성 설명했다.

이 Thermally activated flux flow model은 저항의 field dependent broadening에 잘 설명.

온도가 낮아지 $R \sim 0.1 R_n$ 이면 측정치가 훨씬 빨리 떨어진다.

— 이것은 freezing/melting temperature between vortex liquid to vortex solid.

First order 의 증거 – flux line solid (good Crystal)

– sharp jump in resistance

less perfect crystals

– transition maybe continuous

point pinning

Correlated pinning

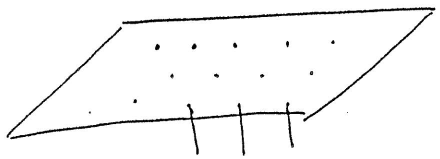
⋮

- 만약 layer가 weakly coupled 이면
  γ가 크면 3D Vortices tend to act like a string of 2D pancake vortices
- Correlated pinning on many layers leads to a collective pinning energy that is strong enough to resist thermal activation.

No simple universal explanation.

- fluctuation이 크다
- γ가 중요하다
- strength & geometric nature of the disorder appear to lead to different regime.

9.5. The Melting Transition.



이것이 온도 올리면 갑자기 깨어진다.

Mean field treatment of the classical superconductor

– Melting transition is obscure.

왜냐면 $|\psi|^2 \rightarrow 0$ near $T_{c2}(H)$ 이기에

Brezin : fluctuation effect를 생각하면 first-order nature가 맞는다.

pinning이 없으면

- highly resistive whether the flux lines are in a solid or liquid

pinning에 의해 Resistive transition이 길어진다. – in the presence of inhomogeneity and pinning

왜 Melting transition을 열역학적으로 측정하는것이 어려운가?

Vortex 자체가 mesoscopic size

따라서 number of degree of freedom

& associated entropy reduction

이것이 매우 작다

Specific heat measurement로는

매우 작다 – Relatively dramatic consequence for transport properties.

이 Section의 세가지 목적

1) Work out a simple model which predicts the melting line $B_M(T)$ in an anisotropic 3D superconductor

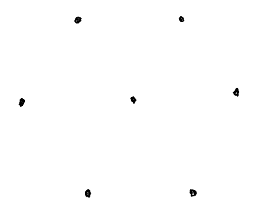
2) Summarize the experimental evidence showing that such a transition line is observed in YBCO

3) Work out the condition under which the melting becomes a 2-D process in BSCCO.

9.5.1. A Simple Model Calculation.

Model Calculation — Lindemann criterion

$$C_L = \square \times \text{interline spacing}$$



이것으로는 first order, 2nd order 인지는

force $f_{2x} = \frac{\Phi_0}{4\pi} \frac{\partial h_1(r_2)}{\partial x_2}$

가운데에서 force zero

약간 δx 움직이면 net fro $\neq 0$

$$\frac{\Phi_0}{4\pi} \sum \frac{\partial^2 h_i(\vec{r})}{\partial x^2} \delta x$$

low flux density

force $\sim e^{-r/\lambda}$

and flux lattice is very soft

$H_{c_1} \ll H \ll H_{c_2}$,

the vortices are overlapping but cores are not.

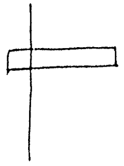
$$h(r) \sim \frac{\Phi_0}{2\pi\lambda^2} \ln\left(\frac{\lambda}{r}\right)$$

이것을 넣어 Component 별로 더하면

The restoring force constant per unit length

$$K \approx \left(\sqrt{3}\, \Phi_0 / 4\pi^2 \lambda^2\right) B \approx H_{c_1} B$$

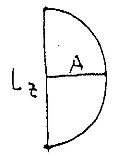
length of the displacement segment



. Sample thin : appropriate length ~ thickness

pancake ~ (" ~ s)

만약 Sample dimension 보다 appropriate length 가 작다면



elastic displacement energy

$\sim K A^2 L_z$

Stretching energy $\epsilon_1 A^2/L_z$

$\therefore \quad K A^2 L_z = \epsilon_1 A^2 / L_z$

Optimal length $L_z \sim (\epsilon_1/K)^{1/2} \sim \Phi_0/B$

그런데 $K = \sqrt{3}\,\Phi_0/4\pi^2\lambda^2$

$\epsilon_1 = (\Phi_0/4\pi\lambda)^2$ 代入 $\left(\sim \Phi_0/B\right)$

Mean vibration amplitude

$$A^2 \sim KT\,(K\epsilon_1)^{-\frac{1}{2}} = C_L^2 a_\Delta^2$$

Lindeman의 parameter $C_L = 0.15$ 라 하면

$$K T_m = C\, C_L^2\, \Phi_0^{5/2} \lambda^{-2} B^{-\frac{1}{2}}$$

where $C = \dfrac{1}{4\pi^2}$

이것으로 B_m을 구하면

$$B_m = C^2 \cdot C_L^4\, \Phi_0^5 (KT)^{-2} \cdot \lambda^{-4}$$

만약 Anisotropy를 넣는다면

Blatter :

$$B_m = \frac{C^2 C_L^4 \Phi_0^5}{(KT)^2 \cdot \lambda_{ab}^4\, \gamma\, (\cos^2\theta + \gamma^2 \sin^2\theta)^{1/2}}$$

$\lambda^{-2} \sim (T_c - T)$ 代入

$$B_m \propto (T_c - T)^2$$

The fit to data can be extended farther below T_c by using the two fluid temperature dependence

$\lambda^{-2} \sim (T_c^4 - T^4)$

. Note B_m과 H_{c2}의 각도 의존성은 같다.

$$\frac{B_m}{H_{c2}} = \frac{2\pi\, C^2 C_L^4 \Phi_0^4 \xi_{ab}^2}{(KT)^2 \lambda_{ab}^4 \gamma^2}$$

온도가 10배 커지고

$\frac{1}{\gamma^2} \sim 2 \times 10^{-2}$ for YBCO

10^{-4} for BSCCO

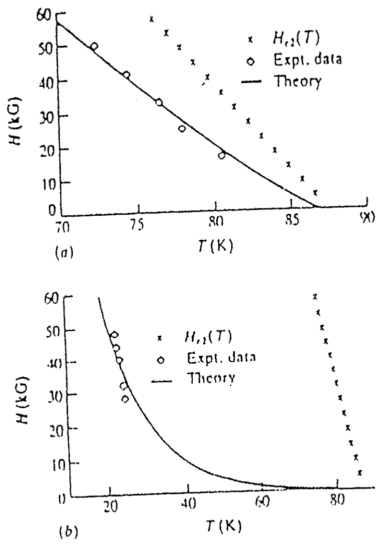


FIGURE 9.6
Phase diagrams for flux-line lattice melting in (a) YBCO and (b) BSCCO. The squares denote experimental estimates of the melting transition [From the vibrator data of Gammel et al., *Phys. Rev. Lett. 61, 1666 (1988)*.] The crosses indicate an estimate for $H_{c2}(T)$, namely, 440 kG × (1 − T/T_c). The solid curves [Calculated by Houghton et al., *Phys. Rev. B40, 6763 (1989)*] are in the framework of a 3-D anisotropic GL model, with c_L, γ, and κ as fitting parameters. As noted in Sec. 9.5.3, the nearly vertical rise of the melting field in BSCCO is probably better explained in terms of a 2-D melting temperature, as in (9.26).

실험 : Gammel이 처음으로 Vibration reed로 melting 측정하다.

Rather sharp peak in the damping of the oscillatory motion. at field dependent Temperature

$T_m(H)$. Qualitatively Similar to the irreversibility line in the H-T plane

Gamma − Melting이라 주장. (1988)

Houghton, Pelcovits − 이것이 frequency dependent crossover로도 설명이 가능하다.

$\tau(T,H) \sim \frac{1}{\omega}$ 와 비슷

Beck (1991.2)

Sharp melting transition in very low frequency.

0.1 Hz torsional Oscillator data on untwinned Crystal.

YBCO.

$\boxed{\text{실험 data}}$ $T_c - T_m = A B^{1/2} \epsilon^{1/2}$

A : constant

$\gamma^2 \epsilon^2 = \cos^2\theta + \gamma^2 \sin^2\theta$

이 angular dep. 는 이론의 예측과 일치

$H_m \sim (T_c - T_m)^\beta$ $\underline{\beta = 1.99 \pm 0.16}$

너무 잘 맞는다.

Safar − Melting의 further support

YBCO 에서 picovolt sensitivity , millikelvin temperature resolution in field up to 9 T.

Revealed reproducible hysteresis in the I-V curves whether sweeping the field or temperature.

Continuous phase transition 은 usually reversible, this <u>hysteresis</u> is evidence for a first-order transition.

Hysteresis 있는 것은 knee structure가 있다.

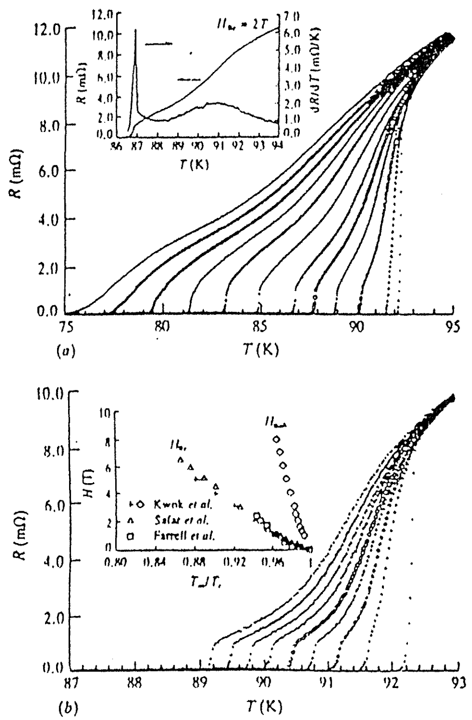


FIGURE 9.7
(*a*) Resistive transition in magnetic fields of 0, 0.1, 0.5, 1, 1.5, 2, 3, 4, 5, 6, 7, and 8 Tesla for $H_{\parallel c}$ in an untwinned YBCO crystal, showing a drop at the melting transition. (Inset) Determination of T_m from peak of dR/dT for $H = 2T$. (*b*) Similar data for $H_{\parallel ab}$, at $H = 0, 1, 2, 3, 4, 5, 6, 7$, and 8T. (Inset) Phase diagram of the melting transition for both field orientations, showing comparison with torsional oscillator data of Farrell et al., and hysteretic *I-V* criterion of Safar et al. The "zero resistance" data points of Iye et al. in Fig. 9.5*a* also coincide closely with these results. [*All other data plotted here are from W. K. Kwok et al.*, Phys. Rev. Lett. 69, 3370 (1992).]

① Kwak, Safar 의 결과 거의 비슷
Farrell

$(T_c - T)^2$ 보다는 slowly varying

질문 Beck은 $\boxed{2}$ 인데 왜 Kwok, Safar... 등등은 exponent 작나?

만약 Same physics 였다면 discrepancy appears puzzling

Two remarks concerning this issue.

1. q-22의 Melting 식은 $B_m \ll H_{c2}(T_m)$

$T \sim T_c$.

So far $B_m \sim (T_c - T)^2$ 낮은온도: 더 빨리 변한다.

$H_{c2} \sim (T_c - T)$

B approaches the 2nd order transition to the normal state at H_{c2}.

$\lambda^{-2} \sim n_s \sim |\psi|^2 \propto (1-b)$

where $b \equiv B/H_{c2}$.

그렇다면 $B_m \sim (T_c - T)$

2. $T \sim T_c$ 이면

Mean field temperature dependence break down.

Low T_c 10^{-6}K

HTSC 1 K

λ^{-2} 는 이곳에서 $(T_c - T)^{2/3}$ 이다.

We should expect $B_m \propto (T_c - T)^{4/3}$

B_m 은 $(T_c - T)^{4/3}$ $T \sim T_c$

에서 $(T_c - T)^2$ $T \ll T_c$ 로

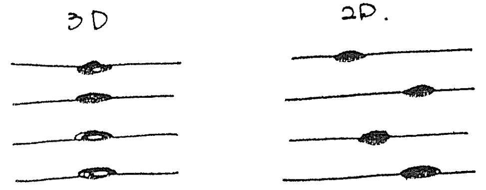
변하는 것이다.

두가지 모두 not reasonable.

9-5-3. Two dimensional vs. three dimensional Melting

YBCO : 3D

BSCCO : General Lawrence-Doniach Model recognizing the importance of discrete superconducting planes.



Josephson Coupling 과 magnetic Coupling 들이 없다.

$\gamma \to \infty$ 이면 $E_J \to 0$

Cross over $\gamma \le \frac{\lambda_{ab}}{s} \sim 100$

① Intraplane force constant

$$\kappa s \sim B \Phi_0 s / \lambda_{ab}^2$$

Josephson interplane term

$$E_J = \frac{\Phi_0^2}{16\pi^2\lambda_c^2 s}$$

a : intervortex separation

The coupling energy per vortex $\sim a^2 E_J$

② Intraplane force const. $\sim B$

$$B_{cr} \sim \Phi_0 (\lambda_{ab}/s\lambda_c)^2 \sim \Phi_0 / s^2\gamma^2$$

Fisher의 결과와 일치.

More detailed analysis,

Integrating over a Fourier Spectrum of fluctuations using the FLL elastic constants, Glazman, Koshelev

- same functional dependence on parameters with an estimated numerical const. of order 10

$$B_{cr} \approx (10^3 - 10^4 \text{ Tesla}) / \gamma^2$$

YBCO unobtainable field

BSCCO $B_{cr} \le 1$ Tesla

만약 $B \gg B_{cr}$

Vortex 사이의 interaction이 더 중요

2D Melting 온도를 구해보자.

$\frac{1}{2} \kappa s (\delta x)^2 \sim \frac{1}{2} kT$ and Lindemann melting criterions

이 방법으로

$$k T_m^{2D} = G\, c_L^2 \Phi_0^2 s / \lambda_{ab}^2$$

이것은 K-T expression 하고 같다

$$G\, c_L^2 = 1/128\pi^3\sqrt{3}$$

이 Melting 온도는 B에 무관하다. 왜냐하면 increasing stiffness with increasing B just compensate for smaller lattice spacing

만약 B_{cr} 보다 작은 B 는

interplane restoring forces are more important

이 Melting 온도는 3D 로 변화한다.

$(T-T_c)^2$ 에서 하늘로 올라가게 된다.

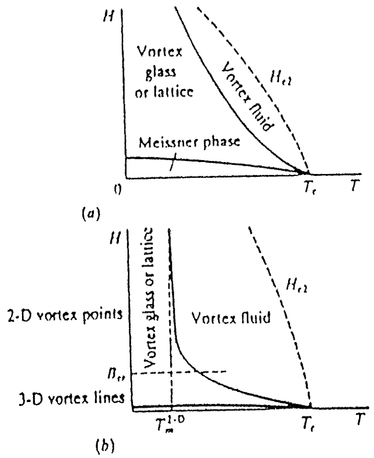


FIGURE 9.8
Schematic phase diagrams for melting of the flux-line solid for (a) 3-D material (e.g., YBCO) and (b) highly layered material (e.g., BSCCO). The latter shows the crossover from 3-D to 2-D melting at the crossover flux density B_{cr}, which is at inaccessibly high fields for the 3-D material in (a).

9.6. The effect of Pinning

flux lines in an ideal homogeneous material

Anisotropy

thermal vibration

여기에 Spatial inhomogeneity factor를 넣자.

atomic scale 에서의 stoichiometry 에서 벗어남

more extended defects

dislocations
grain boundaries
inclusions of 2nd phase
twin planes

Introduce sufficient pinning to raise $T_{melting}$

Many interesting Question

1. 저항이 온도 linear 하다
glass transition 지나면서
exponentially 낮아지나 ?

이 Section 에서는

brief discussion of pinning and flux-creep effect.

9.6.1. Pinning Mechanisms in HTSC.

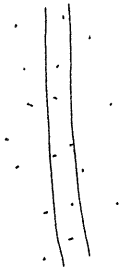
Larkin 과 Ovchinnikov 의 Collective pinning model.

- Effect of pinning by randomly distributed weak point defects.

- not correlated defects such as twin boundaries.

Point defects

departure from stoichiometry at even a single atomic site — 충분히 order parameter 줄일수 있다.



Thuneberg :

pinning force on a vortex

: initially rises linearly with the distance from a vacancy

ψ ~ 거의 Constant

Kes 가 estimate

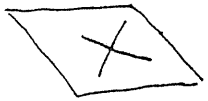
pancake vortex 당 8개의 vacancy가 있다.

BSCCO $J_c \sim 5 \times 10^6 \, A/cm^2$

pinning energy ~ 34 K

in temperature unit

Twin Planes and Other Extended defect.



(110), $(1\bar{1}0)$

point defect 이나 impurity 는 twin boundary에 걸쳐있게 된다.

pinning이 coherent 하다.

Larkin - Ovchinnikov collective pinning model is not appropriate.

실험: field가 plane 위의 1° 내외 에 있을때 J_c가 매우 증가한다

Stacking fault :

intergrowth of other phases with additional or fewer CuO planes per unit cell.

High Resolution electron microscope image에서 볼수있다.

Screw dislocation

: internal structure of the dislocation itself

Surface roughness which is induced by the growth pattern

Artificial defect :

J_c를 높이기 위해 Effective Pinning 중요

① Melt process, melt-quench heat treatment

: designed to introduce a high density of inclusions of off-stoichiometric second phase.

J_c가 매우 증가

② Effectiveness of radiation-induced pinning defects.

. Electron irradiation is relatively ineffective

. Neutron, Proton irradiation provides more Substantial deformations and more pinning.

. Most impressive

: bombardment by a high-energy beam of heavy ions.

High momentum 에 의해 ballistic trajectory

Create an extended set of correlated defects lying along a straight line.

이론 : Nelson과 Vinokur의 boson localization.

실험 : Krusin-Elbaum

randomly directed tracks of fission fragments also greatly increase critical currents and Significantly raise the irreversibility temperature.

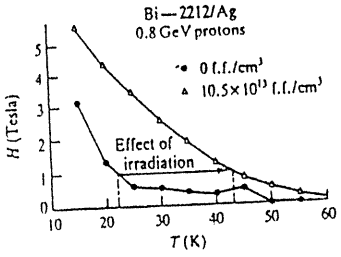


FIGURE 9.9
The irreversibility line in a BSCCO tape before and after introduction of defects by fission fragments induced by proton bombardment. The data at zero flux reflect the usual melting transition in a good sample. The data from the irradiated sample reflect a glass transition. The practical significance of the data is that the resistive transition in a field of 1T has been increased from ~22 to ~43 K, thus expanding the parameter range in which the material is effectively superconductive. (*After Krusin-Elbaum et al.*)

9.6.2.

Larkin - Ovchinnikov theory of Collective Pinning

FLL periodic & Rigid

- random collection of pinning site에 의해 효과적으로 pinning 되어 있지 않다.

$N = nV$ n : # of pinning center per volume

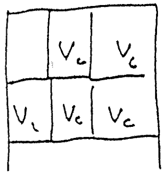
이때 net total force $\propto \sqrt{N}$

이 이론은 Larkin 과 Ovchinnikov 의 Collective Pinning theory라 불리운다.

Macroscopic volume & Correlated Volume V_c

with length $\sim L_c$ along the field direction

R_c transverse.



V_c 가 작다는 것은 FLL can adjust more frequently from region to region over the

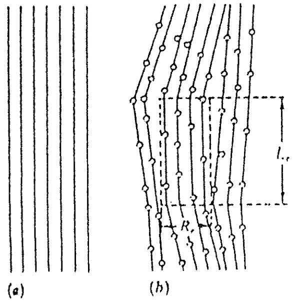


FIGURE 9.10
Schematic diagram illustrating the coherence volume concept of the Larkin-Ovchinnikov theory. (a) With no pinning, the flux-line lattice (FLL) is periodic and exactly parallel to the magnetic field. (b) With random attractive pinning sites, the local direction of the FLL is modulated slightly within each coherence volume (defined by R_c and L_c) so as to combine optimally the energy reduction from the pins with energy increase from distortion of the FLL.

Def. $a \doteq \left(\frac{\Phi_0}{B}\right)^{1/2}$

- lattice correlation is lost as soon as the distortion distance is a.
- distortion distance is only the range ξ of the pinning force, which is typically less than a.
- distortion 이 R_c, L_c 에 걸치어 있다.

Strains S_s, S_t of the order of ξ/R_c, ξ/L_c

shear tilt.

Elastic free energy per unit volume

$$\frac{1}{2}\left[C_{66} S_s^2 + C_{44} S_t^2 \right]$$

C_{66}, C_{44} 구함 계산

de Gennes and Maticon 1964
Brandt, Kogan, Campbell 1977 - 1989.

GL 이론 :

$\kappa \gg 1$, $H_{c2} \gg B \gg H_{c1}$

uniform shear of a triangular FLL in isotropic superconductor.

$$C_{66} \simeq \frac{H_c^2 \, b(1-b)^2}{16\pi}$$

$$\approx \frac{B H_{c1}}{16\pi}(1-b)^2 \qquad \left(b \equiv B/H_{c2}\right)$$

K와 거의 같은 order 이다

C_{44} 계산하자.

$S_t = \delta x / L$ 일때

line length의 증가는

$$(L^2 + \delta x^2)^{1/2} - L = (\delta x)^2 / 2L$$

flux의 갯수

길이당 에너지

$$\frac{1}{2} C_{44} S_t^2 = \frac{\Phi_0 H}{4\pi} \cdot \frac{(\delta x)^2}{2L} \cdot \left(\frac{B}{\Phi_0}\right) \frac{1}{L}$$

$$\cancel{\frac{1}{2}} C_{44} \cdot \frac{\cancel{(\delta x)^2}}{L^2} = \frac{\Phi_0 H}{4\pi} \cdot \frac{\cancel{(\delta x)^2}}{\cancel{2}L} \left(\frac{B}{\Phi_0}\right) \cdot \frac{1}{L}$$

$$\therefore C_{44} = \frac{HB}{4\pi}$$

Net free energy change.

$$\delta F = \frac{1}{2} C_{66} \left(\frac{\xi}{R_c} \right)^2 + \frac{1}{2} C_{44} \left(\frac{\xi}{L_c} \right)^2 - f\xi \frac{n^{1/2}}{V_c^{1/2}}$$

$$V_c = R_c^2 L_c$$

δF를 R_c, L_c를 변화하므로서 minimize 시키자.

$$L_c = \frac{2 C_{44} C_{66} \xi^2}{n f^2}, \quad R_c = \frac{2^{1/2} C_{44}^{1/2} C_{66}^{3/2} \xi^2}{n f^2}$$

$$V_c = \frac{4 C_{44}^2 C_{66}^4 \xi^6}{n^3 f^6}$$

1. Pinning이 더 크면 더 distort 된다.

2. Correlation volume is elongated along the field direction since $L_c/R_c = \sqrt{2} \, (C_{44}/C_{66})^{1/2} \gg 1$

3. 실험과 비교

Neutron diffraction - fairly good agreement.

$$\delta F_{min} = - n^2 f^4 / (8 C_{44} C_{66}^2 \xi^2)$$

pinning force per unit volume $\underline{f\,(n/V_c)^{1/2}}$ determines the maximum sustainable Lorentz density

$$J_c \frac{B}{c} = f \left(\frac{n}{V_c} \right)^{1/2}$$

$$= f \cdot \left(\frac{n^4 f^6}{{}^{2}\ \ {}^{4}\ \ {}^{6}} \right)^{1/2} = \frac{n^2 f^4}{2 \cdot C_{44} C_{66}^2 \xi^3}$$

Note

1. Melting 온도 근처에서는 $C_{66} \to 0$

   이 이론 틀린다.

   $\because$ based on linear elasticity theory

2. Single parameter $W = nf^2$ 에 의존.

   n : density of pin

   f : strength "

   Both L_c, R_c are proportional to W^{-1}

   while J_c scales with W^2.

Collective Pinning in 2D

$L_c = d$ the thickness of the film or layer.

$$V_c = R_c^2 d$$

$$\delta F = \frac{1}{2} C_{66} \left(\frac{\xi}{R_c} \right)^2 + \frac{1}{2} \overset{0}{\cancel{C_{44}}} \left(\frac{\xi}{L_c} \right)^2 - f\xi \frac{n^{1/2}}{V_c^{1/2}}$$

R_c에 대한 minimum

$$R_c = \frac{C_{66}\, a d^{1/2}}{n^{1/2} f}$$

$$\boxed{J_c \frac{B}{c} = \frac{n f^2}{C_{66}\, \xi\, d}}$$

$$R_c \sim W^{-1/2}, \quad J_c \sim W$$

Larkin - Ovchinnikov model은 Kes, Tsuei 가

Larkin - Ovchinnikov model

이것을 D dimension 으로 확장하자

1) $D = 4$ 가 Critical dimension 이다.

$D > 4$ 이면 physically meaningful minimum free energy 는 $L = \infty$, which implies that long-range order in the FLL is retained.

$D < 4$ long range order is destroyed by any density of small random defect.

이 이론의 문제점

1. 가정 - Correlation 이 elastic deformation of the FLL. 다른 dislocation 이 있어 R_c 보다 크면? Vacancies, Interstitial defect 등을 할때.

9. 6. 3. Giant Flux Creep in the Collective Pinning model

Thermal activation 이 중요하면 Collective Pinning 이론은 틀린다. - Giant flux creep in the high-temperature Superconductors.

Classi Anderson - Kim flux-creep theory

$\mathcal{U} = U_0 [1 - (J/J_c)]$ — Anderson Kim

$U = U_0 [1 - J/J_c]^{3/2}$ - tilted - washbord cosine potential.

일반적으로 $\mathcal{U} = U_0 [1 - \frac{J}{J_c}]^d$ 로 쓰자.

Pinning site가 많을때 - Collective pinning 일때

$$U(J) \approx U_0 (J_c/J)^{\mu} \quad \text{with} \quad \mu \leq 1$$

이 form 은 실험적으로 flux-creep measurements of Maley — 실험적으로 제안

차이점 - Anderson - Kim

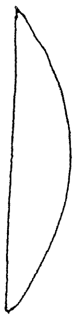
$J \to 0$ $\quad U(J) = U_0$ Nonzero linear resistance.

이 이론

$J \to 0$ $\quad U(J) \longrightarrow \infty$

$\boxed{정리}$.

$$\mathcal{U}(J) \sim U_0 (J_c/J)^{\mu}$$



두개의 힘 ① elasticity force

$$\epsilon_1 (\delta x)^2 L \longrightarrow \frac{1}{L^2} \cdot \left(\frac{L}{L_c}\right)^{2\xi} \cdot L \rightarrow L^{2\xi-1}$$

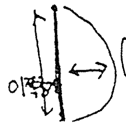
② Lorentz force

$$\underline{J_0 \left(\frac{\Phi_0}{c}\right) \frac{L_c}{c}} \times \underline{거리 \left(\frac{L}{L_c}\right)^{\xi} \cdot \left(\xi \frac{L}{L_c}\right)}$$

force

ξ 에서

$$= \left(\frac{J}{J_c}\right) \left(L/L_c\right)^{\xi+1}$$



$\left(\frac{L}{L_c}\right)^{\xi}$

$$\delta F(L) \approx U_0 (L/L_c)^{2\zeta-1} - J(\Phi_0/c) L_c \xi (L/L_c)^{\zeta+1}$$

$$= U_0 \left[\left(\frac{L}{L_c}\right)^{2\zeta-1} - \left(\frac{J}{J_c}\right)\left(\frac{L}{L_c}\right)^{\zeta+1} \right]$$

Minimize 시키자 $L = L_c (J_c/J)^{1/(2-\zeta)}$

이 activation energy

$$= U_0 (L/L_c)^{2\zeta-1}$$

$$= U_0 \left(\frac{J_c}{J}\right)^{\frac{2\zeta-1}{2-\zeta}}$$

$$= U_0 \left(\frac{J_c}{J}\right)^{\mu} \qquad \mu = \frac{2\zeta-1}{2-\zeta}$$

Numerical estimation

$$\mu = \frac{1}{7}, \quad \zeta = \frac{3}{5}$$

만약 flux가 bundle 이면

Scale as $B^{(2-\zeta)/2}$

Current Voltage Relation.

$$V \propto \exp\left[- \frac{U_0}{KT} \left(\frac{J_0}{J}\right)^{\mu} \right]$$

계속 unstable 해서 다른쪽으로 간다.

Anderson - kim theory

$$J(t) \approx J_c \left[1 - \frac{KT}{U_0} \ln(1 + t/t_0) \right]$$

$V \propto \exp\left[- \frac{U_0}{KT} \left(\frac{J_c}{J}\right)^{\mu} \right]$ 에서는

$$J(t) \approx J_c \left(\frac{KT}{U_0} \ln \frac{t}{t_0} \right)^{-\frac{1}{\mu}} \qquad \text{for} \quad J \ll J_c$$

이것을 $U(J) \sim U_0 (J_c/J)^{\mu}$ 속에 넣으면

$$\sim k_B T \ln(t/t_0)$$

이 이론을 $J \to J_c$ 에 까지 가져가면

아이- Anderson - Kim의 이론으로 환원된것이다.

$$J(t) \approx J_c \left[1 + \frac{\mu KT}{U_0} \ln\left(1 + \frac{t}{t_0}\right) \right]^{-\frac{1}{\mu}}$$

어떻게 측정하나.

$M(t) - M_{eq}$ 을 관찰한다.

9.6.4. The vortex - Glass Model.

Larkin, Ovchinnikov

pinning center 들이 있으면 아무리 weak 해도

crystalline 의 long range order 는 깨어진다.

$J \ll J_c$ highly nonlinear glassy response 보인다.

Anderson - Kim

linear - exponential -

질문: Well defined glass melting transition이 있는가.'

ZERO resistance 포함

M.P.A. Fisher, D.S. Fisher

- Vortex glass phase transition was proposed.

1) T_g 가정

$T \to T_g$, linear resistance is zero

2) $\xi_G \sim |T - T_g|^{-\nu}$

Vortex-glass phase correlation length.

$\tau_G \sim \xi_G^{z}$

Critical slow down을 기술하는 exponent

Scale 하자

$$\frac{E}{\text{length}\cdot\text{time}}, \quad \frac{J}{\left(\frac{1}{\text{length}}\right)^{D-1}}$$

$$\xi_G^{z+1} E \approx \mathcal{E}_\pm \left(\xi_G^{D-1} J \right)$$

$\mathcal{E}_\pm$: different scaling functions for temperature above +, below − the glass temperature T_g

$T < T_g$

$$E(J) \sim \exp\left[- \left(\frac{J_T}{J} \right)^{\mu} \right]$$

$J \to 0$ 저항 ZERO

$T = T_g$, Fisher predict a power law I-V characteristic

$$E \sim J^{(z+1)/(D-1)} = J^{\frac{z+1}{2}}$$

$T > T_g$ $E \sim J$

실험.

YBCO thin film — Koch

single X-tal — Gammel

$z \sim 4 \pm 1$.

Universal curve로 된다.

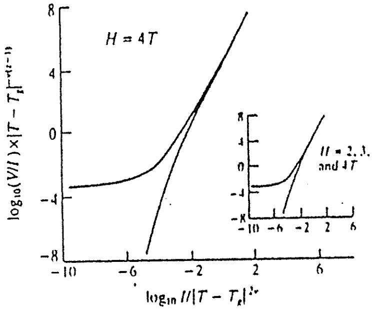


FIGURE 9.11
Empirical scaling functions for scaled nonlinear resistance vs. scaled current density for temperatures above and below the vortex-glass transition temperature T_g. This is a collapsed data plot of 119 I-V curves on a YBCO sample [*After Koch et al.*, Phys. Rev. Lett. *63*, *1511 (1989)*; *ibid*. *64*, *2586 (1990)*], with temperatures ranging from 84.5 to 72.7 K in 0.1 K intervals at $H = 4T$. The inset shows these data superimposed with similarly collapsed data at $H = 2$ and $3T$. In each case, $\nu = 1.7$ and $z = 4.8$ were used. The upper curve is for $T > T_g$; the flat part at the left corresponds to linear resistance at low current. The lower curve is for $T < T_g$ and shows no sign of approaching a nonzero linear resistance. The data points from all I-V curves superpose within the width of the plotted curves.

D=3 인 경우

$$\xi_G^{z-1} E/J \approx \mathcal{E}_{\pm}(\xi_G^2 J)/\xi_G^2 J$$

결론 : $E/J\,|T-T_g|^{\nu(z-1)}$ 은 $J/|T-T_g|^{2\nu}$ 의 Scale

함수이다.

$T > T_g$ 에서는

$R \sim (T-T_g)^{\nu(z-1)}$ 인 linear dependence가 보인다.

$\left(\dfrac{\partial \ln R}{\partial T}\right)^{-1}$ vs. T graph 에서 $\dfrac{1}{\nu(z-1)}$ 을

알아낼수 있다.

Gammel 은 $\nu(z-1) \approx 6.5$ 임을 발견하다.

Note.

① 가장 좋은 시료에서는 first-order phase transition이 보인다.

② Width of the vortex glass critical regime

Should increase with stronger disorder

Vortex Glass in two dimension.

lower Critical dimension D

$2 < D < 2$

따라서 2차원에서는 Glass transition으로 못간다.

$\therefore\ \xi_{2D} \propto T^{-2\nu_{2D}}$

이것을 고려하면

Note.

① 언제 J 의 non linearity 커지나?

$$J_{nl} \approx K_B T/\Phi_0 \xi_{2D} \propto T^{1+\nu D}$$

Dekker.

$$\nu_{2D} = 2.0 \pm 0.3$$

② linear resistivity 에서는

$$R_{lin} \propto \exp[-(T_0/T)^p]$$

T_0 : characteristic temp

Dekker : 3D scaling에는 잘 안맞고

2D 〃 에는 잘 맞더라.

9.6.5 Correlated Disorder and Bose Glass Model.

Correlated disorder - twin planes

Columnar defect

∴ add coherently

실험 Kwock - twinned single X-tal

Fisher의 … 이론.

이후 Nelson 과 Vinokur가 발전

만약 온도가 높으면

Vortex는 wander

⇒ Lorentz force of current에 의해 저항이 linear 하다.

low temp - linear resistance is zero

Bose Glass phase.라 부른다.

왜냐하면

flux line 이 Columnar defect에 붙잡혀 있는 것은 bosonic particle 이 2D potential 속에 잡혀 있는 것과 같다.

Free energy

1) $\left|\frac{d\vec{r}_i(z)}{dz}\right|^2$ 항 : increase of line energy due to meander away from the z-direction

2) defect pinning potential _ $U_D(r_i(z))$

3) an interaction potential depending on the separation of pairs of fluxons at the same height z.

$z \rightarrow t$ 로 바꾸면

The classical statistical mechanics of the fluxons in 3D is then equivalent to the quantum mechanics of interacting bosons in 2D random static potential $U_D(\vec{r})$

해법

1. tight binding approximation

세가지의 parameter가 있다.

① Chemical potential to control the number of particles. (fluxons)

② a repulsive energy for two fluxons on the same site

③ hopping matrix element.

이 fictitious quantum problem 의 ground state energy 계산하다.

sharp phase transition 인 Bose glass temperature T_{BG}.

결과

$T \rightarrow T_{BG}$

localization length $\ell_\perp \sim (T_{BG}-T)^{-\nu_\perp}$ with $\nu_\perp \gtrsim 1$

parallel correlation length along the z-axis $\ell_{\parallel} \sim \ell_\perp^2 / D_0$

이 길이는 fluxon을 옆으로는 $\ell_\perp$ diffuse 할때 위쪽으로의 길이이다.

$$\ell_{\parallel} \sim (T_{BG}-T)^{-\nu_{\parallel}}$$

$$\nu_{\parallel} = 2\nu_\perp$$

01 Model의 Prediction

① I-V 에서

$$V \sim \exp\left[-(J_0/J)^{\mu}\right]$$

$\mu = \frac{1}{3}$ at low current values

$\mu = 1$ at high "

② Scaling 이론에서는 두가지 scaling length 있다.

$\ell_\perp, \ell_\parallel$

$\nu_\perp , \nu_\parallel$

$\| \quad \|$

$\nu' \quad 2\nu_\perp$

$\|$

$2\nu'$

$$\tau \sim \ell_\perp^{z'}$$

$\rho \sim (T-T_{BG})^{\nu'(z'-2)}$	$\rho \sim (T-T_{VG})^{\nu(z-1)}$
$E \sim J^{(1+z')/3}$ at T_{BG}	$E \sim J^{\frac{1+z}{2}}$ at T_{VG}

Correlated Pinning인 경우에 대해

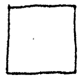
Sharp angular dependence in the position of the irreversibility line.

— isotropic vortex glass model에서는 없는 현상

9.7. Granular high temperature superconductors.

약점. inhomogeneities - isolated weak spots in , otherwise ideal crystalline material.

단결정 얻자



1mm 그나마도 twin plane 으로

1000Å

large-scale application

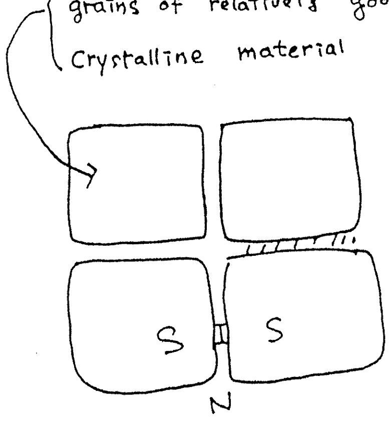
도대체 micron 밖에 안되다니

흑흑

Mixed oxide powder 의 pellet

grains of relatively good stoichiometric

Crystalline material



lower Tc 초전도

nonsuperconducting

but metallic

even semiconducting

Critical Current

I_c : Ambegaokar - Baratoff (AB) relation

Ideal AB junction

$I_c(0) \sim 10^{-2}$ A, $J_c(0) \sim 10^6$ A/cm^2

실험 $10^3 - 10^4$ A/cm^2

불행 : Magnetic field — G_1 낮아진다.

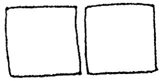
아마도 relatively few percolating path에 의해

q.7.1. Effective Medium Parameter

- Josephson Coupling으로 연결

Effective mass parameter in the anisotropic G-L model.

$J_{cJ} = I_c/a^2$



$$I = I_c \sin\left(\varphi_i - \varphi_j - \frac{2\pi}{\Phi_0}\int_i^j \vec{A}\cdot d\vec{s}\right)$$

London Gauge

$\varphi_i = 0$

$$\vec{J} = -2\pi \frac{J_{cJ}\, a}{\Phi_0}\vec{A}$$

Taking the curl.

$$\nabla^2 \vec{h} = \frac{8\pi^2 J_{cJ} a}{c\,\Phi_0}\vec{h}$$

where h is the local value of the magnetic flux density

$$\lambda_J = \left(c\Phi_0 / 8\pi^2 a J_{cJ}\right)^{1/2}$$

London penetration depth와 비교.

$$\lambda_L = \left(mc^2/4\pi n_s e^2\right)^{1/2}$$

Josephson penetration depth

$$\lambda_J = \left[\frac{c\,\Phi_0}{8\pi^2 J_c\,(2\lambda+d)}\right]^{1/2}$$

$J_c \leftrightarrow J_{cJ}$

$a \leftrightarrow 2\lambda+d$

$J_{cJ} = 10^3\,A/cm^2$, $a = 1\,\mu m$

$\lambda_J = 5\,\mu m$

$\lambda_J \doteq 5\cdot a$ 따라서 가정과 일치

This result is consistent with our assumption of slow spatial variations of the field on the scale of a

Note: GL 이론에서 $\xi(T)$

$T \ll T_c$

$\xi(T) \longrightarrow \xi_0$ if metal is clean

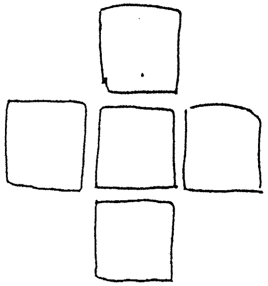
$\sqrt{\xi_0 l}$ " dirty

$\xrightarrow{T\to T_c} (1 - T/T_c)^{-\frac{1}{2}}$

Maximum phase gradient

$\nabla\varphi \sim 1/\sqrt{3}\,\xi$

$\sim \pi/2a$



$\xi_J = 2a/\sqrt{3}\pi \sim 0.4a$

$$\frac{H_{cJ}^2}{8\pi}\cdot a^3 = 3\cdot E_J$$

$$\therefore H_{cJ} = \left(\frac{12\Phi_0 J_{cJ}}{ca}\right)^{1/2} \sim 6\,a$$

$\kappa_J = \lambda_J / \xi_J = \left(3\Phi_0 c / 32 J_{cJ} a^3 \right)^{1/2}$

~ 10 HTSC

GL theory :

the field for first fluxon penetration in a high κ superconductor

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda^2} \ln\kappa$$

위의 λ_J, κ_J 를 代入 하자.

$$= \frac{2\pi a J_{cJ}}{c} \cdot \ln\left(\frac{\lambda_J}{\xi_J}\right)$$

$$\sim 0.5\ \mathrm{Oe}$$

$H > H_{c1J}$ 이므로

Josephson weak link 가 부서진다

그리고 flux 가 그속으로 들어간다.

Bean 의 Critical-state penetration occurs

for $H > H_{c1,J}$

flux 가 grain 사이로 들어가게 된다

Field gradient $\frac{4\pi J_{cJ}}{c} \sim 1000\ \mathrm{Oe/cm}$

H_{c2} for type II

$H_{c2} = \Phi_0 / 2\pi\xi^2$

위의 ξ 代入하자. $H_{c2J} = 3\pi\Phi_0 / 8a^2$

$\sim 24\ \mathrm{Oe}$ for $a = 1\mu m$

이 field는 flux quanta 형태가 a^2 속으로 들어간다.

결과

1. For $H < H_{c1J}$ (~ 0.5 Oe), the field is screened exponentially over a distance $\lambda_J \sim 5\mu m$

2. For $H_{c1J} < H < H_{c2J}$ (~ 25 Oe)

   the field penetrates a distance $\sim cH / 4\pi J_{cJ}$ in a Bean type critical state, leaving the grains as partially diamagnetic inclusions.

3. $H \longrightarrow H_{c2J}$

   J_{cJ} is reduced by a factor $\sim H_{c2}/H$ by the phase randomization, allowing even deeper field penetration

4. $H < H_{c1g}$ (~ 500 Oe), the field penetrates into each grain only to a depth $\lambda_g \sim 1500$ Å

5. When $H > H_{c1g}$, fluxons enter the grain set-up another " Bean model " screening within each grain, but with the penetration depth determined by J_{cg} instead of by J_{cJ}

6. Finally, at H_{c2g}, all superconductivity is extinquished.

9.7.2.

Relationship between Granular and Continuum Model.

9.8장

Fluxons and High frequency losses

High freq. electromagnetic properties

- London two fluid model
- BCS
- low - high freq.

Comprehensive and unified approach

Coffey and Clem

- ac field, dc field, surface normal anisotropy axis of the superconductor.
- frequency, temperature, pinning strength

Complex penetration depth

$$Z_s = R_s - iX_s$$

$$= -i \cdot \frac{4\pi\omega}{c} \tilde{\lambda}(\omega, B, T)$$

$\tilde{\lambda}(\omega, B, T)$: exponential decay and phase evolution of the rf magnetic field $\vec{b}$

$$\vec{b}(x,t) = \hat{z}\, b_0\, e^{-x/\tilde{\lambda}}\, e^{-i\omega t}$$

Clem과 Coffey의 일.

$$\nabla \times \vec{J}_s = -\frac{c}{4\pi\lambda^2}(\vec{B} - n\Phi_0 \hat{z})$$

$$\nabla \times \vec{J}_s = -\frac{c}{4\pi\lambda^2}(\vec{B} - n\Phi_0 \hat{z})$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}$$

$$\therefore \nabla \times \nabla \times \vec{B} = -\frac{1}{\lambda^2}(\vec{B} - n\Phi_0 \hat{z}) + \underbrace{\frac{4\pi\sigma_{nf}}{c^2}\dot{\vec{B}}}_{\text{Current } \sigma_{nf}\vec{E} \text{ of normal fluid driven by the induced electric field}}$$

$$\nabla^2 \vec{B} = \frac{4\pi\sigma_{nf}}{c^2}\dot{\vec{B}} + \frac{1}{\lambda^2}(\vec{B} - n\Phi_0 \hat{z}) \quad \cdots\cdots *$$

Equation of motion for the displacement u from equilibrium at pinning site is

$$\eta \dot{\vec{u}}(x,t) + k\vec{u}(x,t) = \vec{J}(x,t) \times \Phi_0 \hat{z}/c \quad \cdots *.$$

η : viscous drag coefficient

k : restoring force constant.

Self consistently 풀자.

$$\tilde{\lambda}(\omega, B, T) = \lambda\left(\frac{1 + i\,\tilde{\delta}_0^2/2\lambda^2}{1 - 2i\lambda^2/\delta_{nf}^2}\right)^{1/2}$$

$T \to T_c$, $B \to B_{c2}(T)$, $n_s \to 0$

Clem and Coffey assume the simple analytic dependence

$$\frac{1}{\lambda^2(B,T)} = \frac{(1-t^4)\left[1 - B/B_{c2}(T)\right]}{\lambda^2(0,0)}.$$

9.9. Anomalous properties of high temperature and Exotic superconductors.

Microscopic Nature of the superconducting state.

Antiferromagnetic fluctuation – D wave pairing

⋮

왜 BCS, GL 이론에 의해 설명되는 현상들은

High Tc, anisotropy, estimated Fermi Velocity

density of state

- ac Josephson effect frequency $2eV/\hbar$ 관찰

flux quantum $\frac{hc}{2e}$

- Andreev reflection along time-reversed trajectory

보였다.

- Knight shift is observed

$\Rightarrow$ Spin Singlet

Not compatable with conventional S-wave superconductivity.

with material anisotropy.

(1) Superconducting phenomena directly reflecting the symmetry of the paired state

2) phenomena reflecting the density of states for quasi-particle excitations.

– Clean energy gap or

state in the gap.

9.9.1. Unconventional Pairing

BCS theory in Chap. 3.

gap parameter in state k in BCS.

$$\Delta_k \propto \langle C_{-\vec{k}\downarrow} C_{\vec{k}\uparrow} \rangle \propto -\sum_{k'} V_{\vec{k}\vec{k}'} \langle C_{-\vec{k}'\downarrow} C_{\vec{k}'\uparrow} \rangle$$

- ∴ : Isotropic system

∴ Δ_k was independent of k.

(S wave pairing에서는 지극히 당연)

Δ_k – symmetry cannot be lower than tetragonal.

conventional

Symmetry lower than tetragonal

Unconventional pairing

Two-electron bound state wave function.

- BCS 에서는 Singlet

$$\sum_k g(\vec{k}) \cos \vec{k} \cdot (\vec{r}_1 - \vec{r}_2)$$

- $$V_{kk'} \equiv \Omega^{-1} \int V(\vec{r}_1 - \vec{r}_2)\, e^{i(\vec{k}-\vec{k}')\cdot(\vec{r}_1-\vec{r}_2)}\, d(\vec{r}_1 - \vec{r}_2)$$

- 일반적으로 $\vec{k}-\vec{k}'$의 방향에 의존한다.

$$g(\vec{k}) = \frac{\sum_{k'} V_{kk'} g_{k'}}{2\epsilon_k - E}$$

group theory

Annett - 연구

YBCO - tetragonal, Orthorhombic

Chain ...

만약 tetragonal 이라면

Several possible symmetries for the gap function

S wave : full tetragonal symmetry of the Crystal

d wave Symmetry - $x^2 - y^2$ symmetry를 가진다

This function is clearly of lower symmetry than tetragonal - change sign

pairing state

Strong repulsive core at short distance

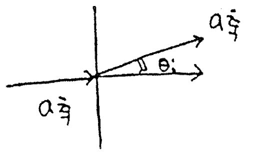
$r_{12} = 0$ 에서 probability amplitude goes to zero.

9.9.2. Pairing Symmetry and Flux quantization

$$\Delta(\vec{k}) \propto (k_x^2 - k_y^2) \propto \cos 2\theta$$

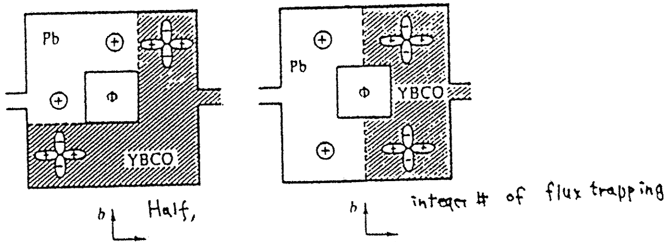
Sigrist and Rice

Josephson Current $\propto \cos 2\theta_i$



Sigrist and Rice 의 이론을

double-junction dc SQUID에 적용



(a) (b)

Woolman.

FIGURE 9.12
Superconducting SQUID rings combining films of Pb (with s-wave symmetry) and YBCO (with assumed $d_{x^2-y^2}$ symmetry). The dashed lines represent Josephson junctions between the two metals. In configuration (a), the intrinsic angle-dependent sign of the order parameter effectively introduces a phase difference of π between the two junctions, leading to a half-integral number of flux quanta trapped in the ring. In (b), both junctions are on the same face of the assumed d-wave superconductor, so that there is no such phase difference, and the quantization leads to integral numbers of trapped flux quanta, as expected with conventional superconductors.

$\therefore$ $\frac{\Phi}{2}$ trapped flux 일때만 ring이 lowest energy state 이다.

Woolman : $\frac{\Phi}{2}$ 실험으로 증명

Tsuei, Matha: : 만약 flux가 trapped 되어 있다면 어떤 일이 생기나?

- Scanning SQUID 이용.
- Scan the entire sample configuration for trapped flux or pinned vortices.

9.9.3. The energy gap

BCS 1) $\frac{2\Delta}{K_B T_c} = 3.52 K T_c$

2) No quasi particle states

3) density of state — [illegible] 가 singular peak in the density of states at the gap edge.

$\boxed{\text{실험}}$ Specific heat,

I-V for single-particle tunneling

Magnetic penetration depth λ

frequency-dependent electromagnetic absorption

$\frac{1}{T_1}$ for nuclear resonance

HTSC — clear deviations from BCS

아마도 d-wave 로 설명

The Gap width

$\frac{2\Delta}{KT_c}$: 4-7 정도

HTSC 는 not weak coupling BCS.

It is possible that very strong coupling could account for such a high $2\Delta/kT_c$ ratio

The Gap symmetry :

- Change of sign of Josephson coupling under rotation by $\pi/2$
- d-wave pairing

Angle resolved photoemission Spectroscopy on BSCCO (ARPES)

d-wave 가 something very similar.
gap like feature is found below T_c
along the K_x or K_y direction.
but not along direction rotated by $\frac{\pi}{4}$ from them in the ab plane

States in the gap.

NMR - ① No Hebel-Slichter peak in $\frac{1}{T_1}$

② At low temp $\frac{1}{T_1} \sim T^3$ or $T^{4.5}$

③ Cu 가 여러곳

Spatial variation of the superconductivity within the crystal unit cell.

해석

1. $\frac{1}{T_1}$ 의 exponential decay 없다

   - absence of clean energy gap
     power law variation is at least qualitatively consistent with the d-wave pairing.

2. absence of the Hebel-Slichter peak

   - Break down or modification of BCS coherence

Conclusion

1. 많은 inconsistencies 에 불구하고
   quasi particle excitation spectrum of the HTSC
   — BCS와는 상당히 변화되어야 한다.

2. Gap is wider relative to T_c, but is not
   Clean.

3. intrinsic density of state
   — zero energy 까지도 살아있다.
   expected if the pairing were
   d-wave instead of the BCS S wave.

9.9.4.
Heavy Fermion Superconductors

Heavy Fermion Superconductor
— exotic type of superconductor discovered
before the HTSC.

unconventional pairing

1. 이 경우 항상 f electron 이 포함된다.
   예 : $CeCu_2Si_2$ and UPt_3

2. Mass — electron mass 의 수백 배

3. Specific heat
   초전도는 Heavy Fermion 에 의해서이지
   다른 전자때문은 아니다.

existence of several distinct superconducting phase
in pressure - temperature plane.

— unconventional pairing with a complicated
order parameter.

$C_v \sim T$ or T^2 depending on the material.
quality of the sample

이러한 결과는 energy gap 이 line, 또는 point로
vanish 할 때다.
아마 P 또는 d로 설명가능.

$T_c \sim 1K$ 따라서 실험이 많이
되지는 않았다.

Great Scientific interest

— May provide one of the few
examples of unconventional pairing
in superconductors.

0. Special topics

Bogoliubov eq..

: Spectrum of excitations for spatially inhomogeneous Superconductors.

~~Andre~~
- Anderson dirty superconductor theory
- low lying excitations in a vortex core which make it quasi normal
- Magnetic impurity가 excitation spectrum을 어떻게 변화시키나.
- gapless superconductor

Time dependent GL theory for gapless superconductor

Chap 11.

Relaxation by inelastic electron-phonon processes when an energy gap does exist

10.1. The Bogoliubov Method:

Generalized Self-consistent field

BCS : $k\uparrow, k\downarrow$

k was a good quantum number

1959. Anderson

General prescription :

Applicable in dirty superconductor pair time reversed state.

Dirty Metal :

Electronic eigenfunctions are some functions $W_n(\vec{r})$ which are certainly far from plane waves.

$W_n(\vec{r})$ 은 $W_n^*(\vec{r})$ 과 degenerate

만약 ① time reversal noninvariant terms in the hamiltonian

② $W_n(\vec{r})$ 과 $W_n^*(\vec{r})$ 이 spin reverse 이면

결론 T_c, H_c, Δ 등은 independent of the electronic mean free path.

Bogoliubov eq.

Hartree-Fock equations of the many electron theory to include the effects of the superconducting pairing potential $\Delta(\vec{r})$ as well as the ordinary scalar potential $U(\vec{r})$

Bogolieubov - de Genne method

$$\psi(\vec{r},\uparrow) = \sum_n \{ \gamma_{n\uparrow} U_n(\vec{r}) - \gamma^+_{n\downarrow} V^*_n(\vec{r}) \}$$

$$[\ \gamma_{n\alpha}, \gamma_{m\beta}\ ] = 0 \qquad \text{anticommutation}$$

$$[\ \gamma_{n\alpha}, \gamma^+_{m\beta}\ ]_+ = \delta_{mn}\delta_{\alpha\beta}$$

$|V|^2$: Cooper pair가 occupy 될 확률

$|U|^2$: Cooper pair가 unoccupy 될 확률

$$\psi(\vec{r},\downarrow) = \sum_n \{ \gamma_{n\downarrow} U_n(\vec{r}) + \gamma^+_{n\uparrow} V^*_n(\vec{r}) \}$$

$$H_{eff} = \int d\vec{r} \{ \sum_\alpha \psi^+(\vec{r},\alpha) [\frac{1}{2m} (-i\hbar\nabla - \frac{e\vec{A}}{c})^2 + U(\vec{r}) - \mu] \psi(\vec{r},\alpha)$$

$$+ \Delta(\vec{r})\, \psi^+(\vec{r},\uparrow)\, \psi^*(\vec{r},\downarrow) + \Delta^*(\vec{r})\, \psi(\vec{r},\downarrow)\, \psi(\vec{r},\uparrow) \}$$

$$\Delta(\vec{r}) = V \langle \psi(\vec{r},\uparrow)\psi(\vec{r},\downarrow) \rangle$$

$$= V \sum_{n,n'} \{ \langle \gamma_{n\uparrow} \gamma^+_{n'\uparrow} \rangle U_n V^*_n - \langle \gamma^+_{n\downarrow} \gamma_{n'\downarrow} \rangle U_n V^*_{n'} \} \delta_{nn'}$$

$$= V \sum_n \langle 1 - \gamma^+_{n\uparrow}\gamma_{n\uparrow} - \gamma^+_{n\downarrow}\gamma_{n\downarrow} \rangle V^*_n(\vec{r}) U_n(\vec{r})$$

$$= V \sum_n (1-2f) V^*_n(\vec{r}) U_n(\vec{r})$$

$$H_{eff} \xRightarrow[\text{diagonalize 되었다고 하자}]{} E_g + \sum_{n\alpha} E_n \gamma^+_{n\alpha} \gamma_{n\alpha}$$

$$[\ H_{eff}, \gamma_{n\alpha}\ ] = \sum_{n',\alpha'} E_{n'} [\ \gamma^+_{n'\alpha'}\gamma_{n'\alpha'}, \gamma_{n\alpha}\ ]$$

$$= \sum_{n',\alpha'} E_{n'} [\ \gamma^+_{n'\alpha'}\gamma_{n'\alpha'}\gamma_{n\alpha} - \gamma_{n\alpha}\gamma^+_{n'\alpha'}\gamma_{n'\alpha'}\ ]$$

$$= -\sum_{n',\alpha'} E_{n'} \underbrace{(\gamma^+_{n'\alpha'}\gamma_{n\alpha} + \gamma_{n\alpha}\gamma^+_{n'\alpha'})}_{\delta_{nn'}\delta_{\alpha\alpha'}} \gamma_{n'\alpha'}$$

$$= -E_n \gamma_{n\alpha}$$

and

$$[\ H_{eff}, \gamma^+_{n\alpha}\ ] = E_n \gamma^+_{n\alpha}$$

$$[\ \psi(\vec{r},\uparrow), H_{eff}\ ]$$

$$= [\ \psi(\vec{r},\uparrow), \int d\vec{r}' \sum_\alpha \{ \psi^+(\vec{r}',\alpha') H_e \psi(\vec{r}',\alpha') + U(\vec{r}')\psi^*(\vec{r}',\alpha')\psi(\vec{r}',\alpha') \}$$

$$+ \Delta(\vec{r}')\psi^*(\vec{r}'\uparrow)\psi^+(\vec{r}'\downarrow) + \Delta^*(\vec{r}')\psi(\vec{r}'\downarrow)\psi(\vec{r}'\downarrow)\ ]$$

where $H_e \equiv \frac{1}{2e}(-i\hbar\nabla - \frac{e\vec{A}}{c})^2 - \mu$

$\Rightarrow$ 1 st term $= H_e(\vec{r})\,\psi(\vec{r},\uparrow)$

2nd term $= U(\vec{r})\,\psi(\vec{r},\uparrow)$

3rd term $= \Delta(\vec{r})\,\psi^+(\vec{r},\downarrow)$

4th term $= 0$

$\therefore$ $[\ \psi(\vec{r},\uparrow), H_{eff}\ ] = (H_e + U(\vec{r}))\,\psi(\vec{r},\uparrow) + \Delta(\vec{r})\,\psi^+(\vec{r},\downarrow)$

$[\ \psi(\vec{r},\downarrow), H_{eff}\ ] =$ 1st term $= H_e(\vec{r})\,\psi(\vec{r},\downarrow)$

2nd term $= U(\vec{r})\,\psi(\vec{r},\downarrow)$

3rd term $= -\Delta(\vec{r})\,\psi^+(\vec{r},\uparrow)$

$$\therefore\ [\Psi(\vec r,\downarrow),\ H_{eff}] = (H_e + U(\vec r))\,\Psi(\vec r,\downarrow) - \Delta(\vec r)\,\Psi^+(\vec r,\uparrow)$$

$$LHS = \sum_n \left[\gamma_{n\uparrow} U_n(\vec r) - \gamma^+_{n\downarrow} V^*_n(\vec r),\ H_{eff}\right]$$

$$= \sum_n [\gamma_{n\uparrow}, H_{eff}]\, U_n(\vec r) - [\gamma^+_{n\downarrow}, H_{eff}]\, V^*_n(\vec r)$$

$$= \sum_n \left[E_n U_n(\vec r)\gamma_{n\uparrow} + E_n V^*_n(\vec r)\,\gamma^+_{n\downarrow}\right]$$

$$i\hbar \frac{\partial U_n}{\partial t} = E_n U_n(\vec r) = (H_e + U(\vec r))\, U_n(\vec r) + \Delta(\vec r) V_n(\vec r)$$

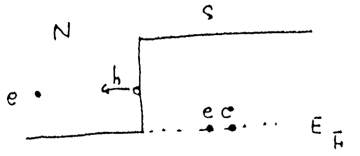
$$E_n V_n(\vec r) = -(H_e + U(\vec r))\, V^*_n(\vec r) + \Delta(\vec r)\, U^*_n(\vec r)$$

$$i\hbar \frac{\partial V_n}{\partial t} = E_n V_n(\vec r) = -(H^*_e + U(\vec r))\, V_n(\vec r) + \Delta^*(\vec r)\, U_n(\vec r)$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \begin{pmatrix} H_e + U, & \Delta \\ \Delta^*, & -(H^*_e + U) \end{pmatrix}\begin{pmatrix} U_n \\ V_n \end{pmatrix}$$

Bogolieubov - de Genne eq.

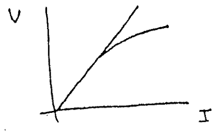
$$U_n^2 + V_n^2 = 1$$



Andreev - reflection.

electron이 들어와서 interface에 있는 다른 electron과 condensation이 일어나고, 마치 hole이 되돌아 가는 것 (현상)

. pair condensation : increase in the conductance



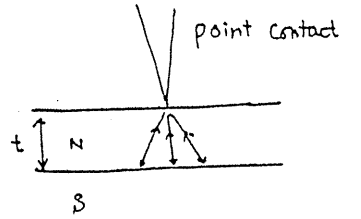
voltage 보다 Current가 크다.

· retroreflection

$$\vec v_e = \frac{1}{\hbar}\frac{dE_e}{dk_e},$$

$$\vec v_h = \frac{1}{\hbar}\frac{dE_h}{dk_h}$$

$$\vec v_h = -\vec v_e$$



ballastic Scattering

$t < l$

저항을 재면 non-ohmic

· Memory effect

$$S \rightarrow \Delta_0 e^{i\varphi}$$

hole이 S.C.의 wave function의 phase를 그대로 가지고 반사되므로 여기서 간섭 실험을 할수 있다.

온도 작게, System을 작게

즉 micro한 양이 macro한 것의 정보를 그대로 pick up.

In Bogolieubov - transf.

$\Psi(\vec{r}, t)$: spatial variations of Order parameter

$$i\hbar \frac{\partial}{\partial t} f = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x) - \mu(x) \right] f(x,t) + \Delta(x) g(x,t)$$

$$i\hbar \frac{\partial}{\partial t} g = -\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x) - \mu(x) \right] g(x,t) + \Delta(x) f(x,t)$$

f : electron branch

g : hole branch

Steady State

$$f = \hat{u} \quad e^{i(kx - Et/\hbar)}$$

$$g = \hat{v} \quad e^{i(kx - Et/\hbar)}$$

$V = 0$ free electron or hole

$$E\hat{u} = \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \hat{u} + \Delta \hat{v}$$

$$E\hat{v} = -\left(\frac{\hbar^2 k^2}{2m} - \mu \right) \hat{v} + \Delta^* \hat{u}$$

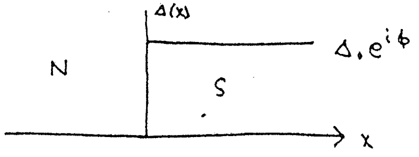
$$\begin{pmatrix} H, & \Delta(x) \\ \Delta^*(x), & -H \end{pmatrix} \Psi = E\Psi$$

$$\Psi = \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} e^{ikx}$$

$$H = \frac{\hbar^2 k^2}{2m} - \mu$$

Chemical potential

예1. NS interface



i) $x > 0$

$$\begin{pmatrix} H & \Delta_0 e^{i\phi} \\ \Delta_0 e^{-i\phi} & -H \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} e^{ikx} = E \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} e^{ikx}$$

$$\left(\frac{\hbar^2 k^2}{2m} - \mu \right) \hat{u} + \Delta_0 e^{i\phi} \hat{v} = E\hat{u}$$

$$\Delta_0 e^{-i\phi} \hat{u} - \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \hat{v} = E\hat{v}$$

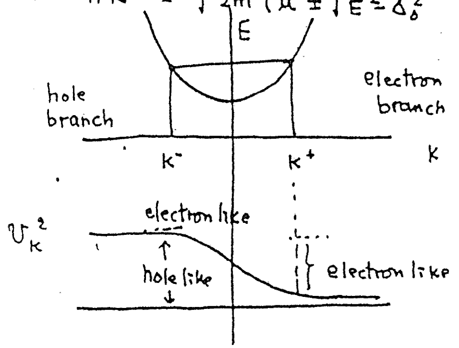
$$\left[E - \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \right] \hat{u} = \Delta_0 e^{i\phi} \hat{v}$$

$$\left[E + \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \right] \hat{v} = \Delta_0 e^{-i\phi} \hat{u}$$

이 둘을 곱하면

$$E^2 - \left(\frac{\hbar^2 k^2}{2m} - \mu \right)^2 = \Delta_0^2$$

$$\rightarrow \hbar k^{\pm} = \sqrt{2m\left(\mu \pm \sqrt{E^2 - \Delta_0^2}\right.}$$



for k^+

$$\left(E - \left(\frac{\hbar^2 k^2}{2m} - \mu\right)\right) \hat{u} = \Delta_0 e^{i\phi} \tilde{v}$$

$$\left(E - \sqrt{E^2 - \Delta^2}\right) \hat{u} = \Delta_0 e^{i\phi} \tilde{v}$$

Put $\hat{v} = v_0$

$$\hat{u} = \frac{\Delta_0}{E - \sqrt{E^2 - \Delta_0^2}} v_0 e^{i\phi} = \frac{E + \sqrt{E^2 - \Delta_0^2}}{\Delta_0} v_0 e^{i\phi}$$

$$= u_0 e^{i\phi}$$

$$u_0^2 + v_0^2 = 1$$

$$u_0^2 = \frac{1}{2}\left(1 + \frac{\sqrt{E^2 - \Delta_0^2}}{E}\right)$$

$$v_0^2 = \frac{1}{2}\left(1 - \frac{\sqrt{}}{E}\right)$$

K^+ : $\psi_e^{k^+}(x) = \begin{pmatrix} u_0 e^{i\phi} \\ v_0 \end{pmatrix} e^{ik^+ x}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ 이면 완전히 electron like

for k^- :

$$\left(E - \left(\frac{\hbar^2 k^{-2}}{2m} - \mu\right)\right) \hat{u} = \Delta_0 e^{i\phi} \hat{v}$$

$$\left(E + \sqrt{E^2 - \Delta_0^2}\right) \hat{u} = \Delta_0 e^{i\phi} \hat{v}$$

put $\hat{v} = v'$

$$\hat{u} = \frac{\Delta_0}{E + \sqrt{E^2 - \Delta_0^2}} v' e^{i\phi}$$

$$= \boxed{\frac{E - \sqrt{E^2 - \Delta_0^2}}{\Delta_0}} v' e^{i\phi}$$

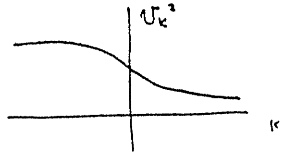
$$= u' e^{i\phi} . \; v' e^{i\phi}$$

$$u'^2 + v'^2 = 1$$

$$v'^2 = u_0^2 = \frac{1}{2}\left(1 + \frac{\sqrt{E^2 \Delta_0^2}}{E}\right)$$

$$u'^2 = v_0^2 = 1 - u_0^2 = \frac{1}{2}\left(1 - \frac{\sqrt{E^2 \Delta_0^2}}{E}\right)$$

K^- : $\psi_h^{k^-}(x) = \begin{pmatrix} v_0 e^{i\phi} \\ u_0 \end{pmatrix} e^{ik^- x}$



ii) $x < 0$

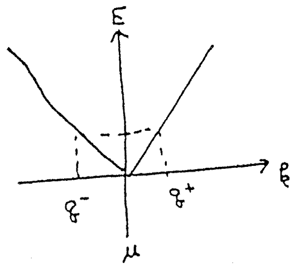
$$\begin{pmatrix} H & 0 \\ 0 & -H \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} e^{iqx} = E \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} e^{iqx}$$

$$\left(\frac{\hbar^2 q^2}{2m} - \mu\right) \hat{u} = E \hat{u}$$

$$\left(\frac{\hbar^2 q^2}{2m} - \mu\right) \hat{v} = E \hat{v}$$

$$\left[E - \left(\frac{\hbar^2 q^2}{2m} - \mu\right)\right] \hat{u} = 0$$

$$\left[E + \left(\frac{\hbar^2 q^2}{2m} - \mu\right)\right] \hat{v} = 0$$

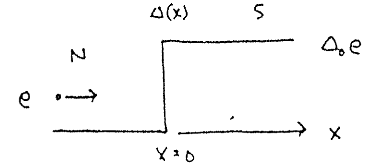


$$- \hbar q^{\pm} = \sqrt{2m(\mu \pm E)}$$

q^+ : $\hat{v} = 0$, $\psi_e^{q^+}(x) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq^+ x}$

q^- : $\hat{u} = 0$ $\quad \psi_e^{q^-}(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iq^- x}$

Summary

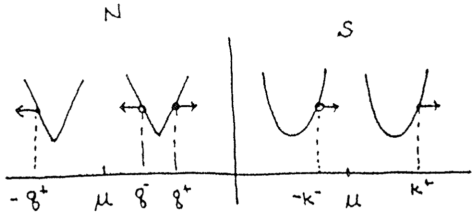


x<0

$$\Psi_{in} = \begin{pmatrix}1\\0\end{pmatrix} e^{iq^+x} + \underbrace{a\begin{pmatrix}0\\1\end{pmatrix} e^{iq^-x}}_{\text{Andreev reflection}} + b\begin{pmatrix}1\\0\end{pmatrix} e^{-iq^+x}$$

x>0

$$\Psi_{tr} = c\begin{pmatrix}u_0 e^{i\phi}\\ v_0\end{pmatrix} e^{ik^+x} + d\begin{pmatrix}v_0 e^{-i\phi}\\ u_0\end{pmatrix} e^{-ik^-x}$$



b.c $\Psi_{in}(0) = \Psi_{tr}(0)$

$\Psi_{in}'(0) = \Psi_{tr}'(0)$

$$\begin{pmatrix}1\\0\end{pmatrix} + a\begin{pmatrix}0\\1\end{pmatrix} + b\begin{pmatrix}1\\0\end{pmatrix} = c\begin{pmatrix}u_0 e^{i\phi}\\ v_0\end{pmatrix} + d\begin{pmatrix}v_0 e^{i\phi}\\ u_0\end{pmatrix}$$

$$q^+\begin{pmatrix}1\\0\end{pmatrix} + q^- a\begin{pmatrix}0\\1\end{pmatrix} - q^+ b\begin{pmatrix}1\\0\end{pmatrix} = ck^+\begin{pmatrix}u_0 e^{i\phi}\\ v_0\end{pmatrix} - dk^+\begin{pmatrix}v_0 e^{i\phi}\\ u_0\end{pmatrix}$$

이들 eqs 은 quasi particle에 대한 언급

$b = d = 0$

electron으로 들어와서 hole로 transform

→ mixed branch.

$1 = c u_0 e^{i\phi} \longrightarrow c = \frac{1}{u_0} e^{-i\phi}$

$a = c v_0 = \frac{v_0}{u_0} e^{-i\phi}$

$$\Psi_{in} = \begin{pmatrix}1\\0\end{pmatrix} e^{iq^+x} + \begin{pmatrix}0\\ \frac{v_0}{u_0} e^{-i\phi}\end{pmatrix} e^{iq^-x}$$

hole의 wave ft.이 $e^{-i\phi}$라는 phase를 pick up.

i.e, microscopic quantity가 macroscopic quantity를 pick up.

$$\Psi_{tr} = \begin{pmatrix}1\\ \frac{v_0}{u_0} e^{-i\phi}\end{pmatrix} e^{ik^+x}$$

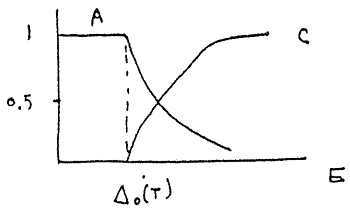
반사율 $A(E) = a^* a = \left|\frac{v_0}{u_0}\right|^2$

$= \left|\frac{E - \sqrt{E^2 - \Delta_0^2}}{E + \sqrt{E^2 - \Delta_0^2}}\right|$

투과율 $C(E) = c^* c\ (u_0^2 - v_0^2)$

$= 1 - \left|\frac{v_0}{u_0}\right|^2$

$= 1 - A(E)$



만약 계면이 깨끗 → Andreev reflection이 있다

" not. → $b \neq 0$ 가 되어 electron이 그냥 반사됨

10.1.1. Dirty Superconductors.

Anderson problem of a dirty, but nonmagnetic Superconductor.

the Normal state Eigen ft. w_n Satisfy

$$H_0 w_n = \xi_n w_n$$

where ξ_n is the eigenvalue measured from the chemical potential μ.

pure metal 이면

w_n 은 Bloch function이고 k 는 well defined 되어있다.

만약 impurity가 있으면

Metal이 ξ 안에서 homogeneous 하면

$\Delta(\vec{r})$ 이 계속 const.

$$u_n(\vec{r}) = u_n w_n(\vec{r})$$

$$v_n(\vec{r}) = v_n w_n(\vec{r})$$

where u_n, v_n are now simply numbers

$$(\xi_n - E_n)\, u_n + \Delta v_n = 0$$

$$(-\xi_n - E_n)\, v_n + \Delta^* u_n = 0$$

whose solution requires

$$E_n = (\xi_n^2 + \Delta^2)^{1/2}$$

Self-consistent Δ 찾자

$$\frac{1}{V} = \frac{1}{2}\sum_n \frac{|w_n(\vec{r})|^2}{E_n} \tanh\frac{\beta E_n}{2}$$

만약 Scattering이 density of state를 별로 변하게 하지 않는다면

T, Δ 는 별로 변하지 않는다.

10.1.2. Uniform Current in Pure Superconductors.

만약 전류가 흐른다면

gap이 깨지더라도 J_c 가 작아지더라도

초전도는 유지된다.

quasi particle의 energy는 $\vec{v}_s \cdot \vec{p}$ 만큼 증가한다.

$$\Delta = |\Delta|\, e^{i 2\vec{q}\cdot\vec{r}}$$

만약 impurity Scattering이 무시된다면

$u_n(\vec{r})$, $v_n(\vec{r})$ 은 아직도 간단한 plane wave

형태를 취한다.

$$\boxed{10-4} \quad H_0 u(\vec{r}) + \Delta(\vec{r}) v(\vec{r}) = E u(\vec{r})$$

$$- H_0^* v(\vec{r}) + \Delta^*(\vec{r}) u(\vec{r}) = E v(\vec{r}$$

$$\boxed{10-3} \quad \Delta(\vec{r}) = V \langle \psi(\vec{r},\uparrow)\, \psi(\vec{r},\downarrow$$

$$= V \sum_n v_n^*(\vec{r})\, u_n(\vec{r}) \cdot (1 - 2f_n)\rangle$$

$$v_k(\vec{r}) = v_k\, e^{i(\vec{k}-\vec{q})\cdot\vec{r}}$$

$$u_k(\vec{r}) = u_k\, e^{i(\vec{k}+\vec{q})\cdot\vec{r}}$$

$$(\xi_{k+q} - E_k)\, u_k + |\Delta|\, v_k = 0$$

$$(-\xi_{k-q} - E_k)\, v_k + |\Delta|\, u_k = 0$$

E 에 대해 풀면

$$E_k = \frac{\xi_{k+q} - \xi_{k-q}}{2} + \left[\left(\frac{\xi_{k+q} + \xi_{k-q}}{2} \right)^2 + |\Delta|^2 \right]^{1/2}$$

Since $\xi_k = \frac{\hbar^2 k^2}{2m} - \mu$

we have

$$\frac{1}{2}\left(\xi_{k+q} - \xi_{k-q} \right) = \frac{\hbar^2}{m} \vec{k} \cdot \vec{q} = \frac{\hbar}{m} \vec{P}_k \cdot \vec{q}$$

As long as $q \ll k_F$, $\xi_{k+q} + \xi_{k-q} \doteqdot 2\xi_k$

then $E_k = E_k^\circ + \vec{P}_k \cdot \vec{V}_s$

where $\vec{V}_s = \hbar\vec{q}/m$ is the velocity of the super current

$E_k^\circ = \left(\xi_k^2 + |\Delta|^2 \right)^{1/2}$ is the excitation energy in the absence of a current.

$$E_{min} = \Delta - P_F v$$

excitation 에 드는 에너지는 매우 작더라도 초전도 기여하는 다른 quasi particle 이 매우 많다.

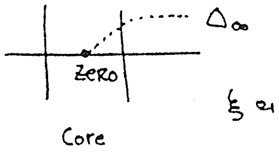
gapless superconductor.

We still expect the perfect conductivity properties.

10.1.3.

Excitations in Vortex

GL 이론 : $\Delta(z, r, \theta) \sim |\Delta(r)| e^{i\theta}$



ξ 의 간격으로 0 에서 Δ_∞로 변화한다

at T_c $\xi(T) \gg \xi(0)$ & G, L eq 는 exact.

GL 이론의 가정 만족

온도가 낮으면 Δ 가 매우 빨리 변화하는데.

What is the nature of the quasi-particle excitations?

Caroli, de Gennes, Matricon 이 처음 풀다.

$\left.\begin{array}{l}\text{Bogoliubov eq. 이용} \\ K \gg 1\end{array}\right] \Rightarrow$ 따라서 Core 내의 H field 무시.

Bardeen, Kümmel, Jacobs, Tewordt 의 논문

모든 K 에 대해 풀다.

Self-consistent solutions for $\Delta(r)$ and $h(r)$ using a variational expression for the free energy.

$r \gg \xi$

$$v_s = \frac{\hbar}{2mr} = \frac{\hbar}{m^* r}$$

Core 에서는

gap less superconductivity inside $r \approx \xi$

Bogoliubov eq. 을 풀어야 한다.

결과 : low lying excitations with the wave functions $u(r)$ and $v(r)$ localized near the vortex Core.

The lowest one lies at $\sim \frac{\hbar^2}{2m\xi^2} \sim \frac{\Delta_\infty^2}{E_F}$

Classic superconductor 인 경우

$10^{-4} \Delta_\infty << KT_c$

this is effectively gapless

실험 Hess

STM : spatially resolved density of states associated with a vortex in the layered Supercondactor 2H - Nb Se₂.

HTSC - $\xi \leq 20 \mathring{A}$

lowest level is comparable with the gap.

there may be only one bound state

FAR-IR spectroscopy Drew 와 그 애들
1992. P.R. L.

---

10. 2.
Magnetic Perturbations and Gapless Superconductivity.

전류가 매우 크면

— gapless 가 되면서 $J_c \neq 0$.

원인 $\vec{k}, -\vec{k}$ 에 drift momentum $\vec{q}$ 를 더하자.

$\xi_{\vec{k}+\vec{q}}$, $\xi_{-\vec{k}+\vec{q}}$, thus lifting the degeneracy of $\xi_{\vec{k}}$ and $\xi_{-\vec{k}}$, which has been exact because of time - reversal symmetry.

Anderson 의 dirty superconductivity 이론에 의하면

nonmagnetic alloys with mean free path $l < \xi_0$

Same Tc, Same BCS density of states as that for a pure superconductor

Abrikosov and Gor'kov 의 Magnetic impurity 있을 때 이론

- Strong depression of T_c
- modification of the BCS density of state

it becomes gapless for a finite range of concentration below the critical value which destroys superconductivity entirely

Maki, de Genne

: Abrikosov and Gor'kov for the density of states and the depression of T_c could be transcribed to describe the effect of many other pair breaking perturbation,

⇒ i.e, those which destroy the time reversal degeneracy of the paired states.

이런 예.

magnetic fields, currents, rotations.

spin exchange, hyperfine fields,

magnetic impurities.

Gapless superconductor의 다른 예

proximity effect 와 같은 spatial gradients in the order parameter 있는 경우

General formalism.

Green's function formalism of Gor'kov

자세한것은 Maki 와 de Gennes 의 논문 참조

10.2.1. Depression of T_c by Magnetic Perturbation.

AG 이론 및 Other magnetic Perturbation에 관한 이론

1. Energy difference 2α

- time reversed electron 에 의해

$k, -k$ 가 q의 drift momentum에 의해

depairing energy $\sim \vec{q} \cdot \vec{k}$

이런 Scattering 시간 동안의 일

$$\frac{d\varphi}{dt} = \left(\frac{2e}{\hbar c}\right) \vec{v_k} \cdot \vec{A}$$

만약 Scattering time τ 가 $\left(\frac{d\varphi}{d\tau}\right)^{-1}$ 보다 짧으면

phase change가 collision 할때 매우 작아

$\frac{d\varphi}{dt}$ 는 여러번의 collision에 의해 변화한다.

random work process 에 의해

Random work 에 의해

$$\frac{1}{\tau_k} = \tau \left\langle \left(\frac{d\varphi}{dt}\right)^2 \right\rangle$$

$$= \frac{1}{3} v_F^2 \tau \left(\frac{2e}{\hbar c}\right)^2 \langle A^2 \rangle$$

$$= D \left(\frac{2\pi}{\Phi_0}\right)^2 \langle A^2 \rangle$$

$$D = \frac{1}{3} v_F l = \frac{1}{3} v_F^2 \tau \quad , \quad 2\alpha = \hbar/\tau_K$$

Pair breaking에 의한 T_c 의 reduction.

$$\ln \frac{T_c}{T_{c_0}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\alpha}{2\pi k T_c}\right)$$

$\psi(z) = \Gamma'(z)/\Gamma(z)$ diagamma function.

$T_c = T_c(\alpha)$, $T_{c_0} = T_c(0)$

expansion.

$$k\,(T_{c_0} - T_c) = \frac{\pi\alpha}{4} = \frac{(\frac{\pi}{8})\hbar}{\tau_K}$$

Superconductivity is completely dectroyed

(i.e., $T_c = 0$) for

$$2\alpha = \frac{\hbar}{\tau_K} = 1.76\,kT_c = \Delta_{BCS}(0) \equiv \Delta_{00}$$

Temperature dependence of the Critical pair breaking strength α,

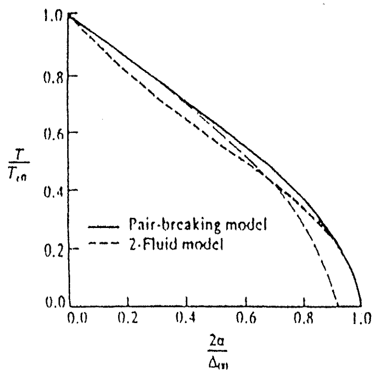


FIGURE 10.1
Universal functional relation between the pair-breaking parameter α and the reduced T_c is shown by solid curve. The shading depicts the gapless region. Values of 2α for various magnetic perturbations are given in (10.22a) to (10.22g). The dashed curve labeled two-fluid is a plot of $2\alpha/\Delta_{00} = (1-t^2)/(1+t^2)$, where $t = T/T_{c0}$. This relation reproduces the results of the GL critical-field calculations building in the two-fluid temperature dependences $\lambda(t) \propto (1-t^4)^{-1/2}$, $H_c(t) \propto (1-t^2)$, and hence $\alpha_c \propto \xi^{-2}(t) \propto \lambda^2 H_c^2 \propto (1-t^2)/(1+t^2)$. [See (10.23).]

Maki 와 de Genne

all pair breaking ergodic perturbations are equivalent to magnetic impurities in their effect on Tc.

This function $\alpha_c(T)$ should be a universal function,

$\alpha = \frac{DeH}{c}$ bulk type II in vortex state

$\alpha = 0.59\,\frac{DeH}{c}$ Surface sheath

$\alpha = \frac{1}{6}\,\frac{De^2H^2d^2}{\hbar c^2}$ thin film, parallel field

$\alpha = \frac{DeH}{c}$ thin film, perpendicular field

$\alpha = \frac{2De^2\langle A^2\rangle}{\hbar c^2}$ Small particles

AG는 magnetic impurity가 spin S,

Spin of the conduction electron s,

$$H \sim J(r)\,\vec{S}\cdot\vec{s}$$

pair breaking energy 는

$$2\alpha \simeq \frac{x J^2}{E_F}$$

x is the fractional impurity Concentration

J is an average over the atomic volume.

Effect of an external magnetic field on the electronic spin.

$$\frac{d\varphi}{dt} = \frac{2\mu_B H}{\hbar} \doteq \frac{eH}{mc}$$

appropriate Scattering time is τ_{so}

$$2\alpha \cong \frac{\tau_{so} e^2 \hbar H^2}{m^2 c^2}$$

만약 이것을 spin orbit scattering에 응용하면

pair breaking energy $2\alpha = 2\mu_B H$

zero energy excitation $\mu_B H = \Delta$

그러나 이 field 도착하기전에 first order transition to the normal state occurs

when $\mu_B H = \Delta_{00}/\sqrt{2}$

Clogston 과 Chandrasekhar 가 지적.

This limits the Critical field of material with negligible spin orbit scattering to a value

$H_P = \Delta_0/\sqrt{2}\,\mu_B$, with $\Delta_0 = 1.76\,kT_c$

$$\frac{H_P}{T_c} = 18,400\ G/K$$

대부분 type II superconductor 에서는 $H_{c_2} \geq H_P$,

importance of the randomizing ~~efect~~ of spin-orbit Scattering in reducing the pair breaking effect of the Zeeman energy of the spins.

10.2.2. Density of states.

Magnetic field 가하면 gap이 어떻게 되나?

thermal conductivity,
microwave absorption,
electron tunneling

Gap is fussing out with low-lying excitations coming in before the peak in the spectrum at the gap edge had entirely disappeared.

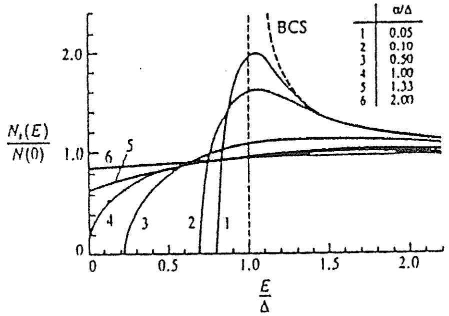


FIGURE 10.2
The density of states as a function of the reduced energy for several values of the reduced pair-breaking strength α/Δ. In this diagram Δ is understood as Δ(T, α). (*After Skalski et al.*)

Skalski 도 같을때 계산

gapless region 이 있다.

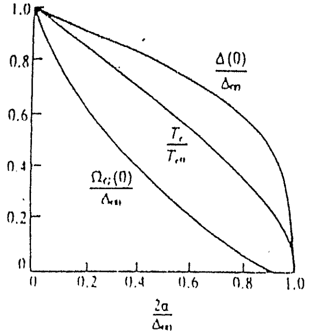


FIGURE 10.3
Decrease of spectral gap Ω_G and gap parameter Δ at $T = 0$ and of transition temperature T_c, with increasing pair-breaking strength α. (*After Skalski et al.*)

자세한 것 de - Gennes 책 참고

Excitation energies are changing from $|\xi|$ in the absence of Δ to

$$E = |\xi| \left(1 + \frac{1}{2} \frac{\Delta^2}{2\xi^2 + \alpha^2} \right) \qquad \Delta \ll \alpha$$

Note that for $|\xi| \gg \alpha$,

$$E \simeq |\xi| + \Delta^2 / 2|\xi| \simeq (\Delta^2 + \xi^2)^{1/2}$$

BCS results of the ordinary superconductors.

$|\xi| \ll \alpha$

$$E = |\xi| \left(1 + \frac{\Delta^2}{2\alpha^2} \right) \sim |\xi|$$

Hence shows No gap in the spectrum

$$\frac{N_s(E)}{N(0)} = \frac{d\xi}{dE} = 1 + \frac{\Delta^2}{2} \frac{E^2 - \alpha^2}{(E^2 + \alpha^2)^2} \qquad \Delta \ll \alpha$$

실험 : electron tunneling measurements of the density of states.

Woolf and Reif - lead and Indium films Containing magnetic impurities.

Gd 등 rare earth impurity 인 경우는 잘 맞음

Fe, Mn 등은 아니다.

gap이 더 빨리 zero로 간다

Gd - 4f electron

closely resemble the localized moments

: assumed in the AG theory

Fe, Mn

interact more strongly with the conduction electrons and thus are less localized.

Tunneling experiments using a thin film in a parallel magnetic field

— Cleaner test of the theory.

degree of localization of the magnetic moment에 관한 질문에 무관하므로

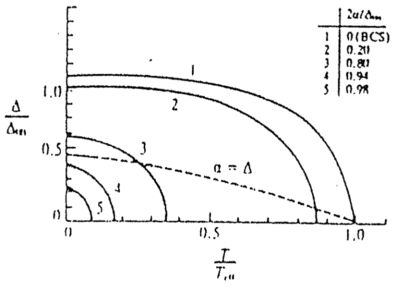


FIGURE 10.4
Temperature dependence of pair potential or gap parameter Δ for various pair-breaker strengths. The spectral gap Ω_G is zero in the shaded region defined by $\alpha > \Delta$. (*After Skalski et al.*).

실험이 매우 어려웠다. — 시료 많이 만들어야

Levine

Millstein and Tinkham

$d \sim 1000\,\text{Å}$

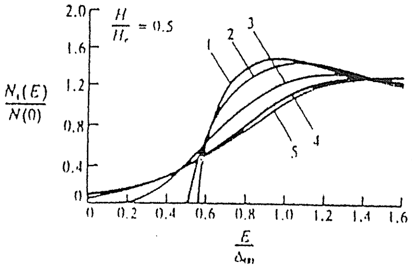
$\ell \sim 300\text{-}1200\,\text{Å}$

$\xi_0 \simeq 2300\,\text{Å}$

density of state 를 ^3He Cooling,

0.36K 에서

$T_c/10$ 까지



Abrikosov

& Gorkov 이론과

Semiquantitatively

agreement.

BCS 에 맞추는 것은 hopeless

FIGURE 10.5
The density of states as a function of energy E for several values of the mean free path and a magnetic field of half the critical field of the small spheres. [1 : $\ell = 0$; 2 : $\ell = (\pi/10)\xi_0$; 3 : $\ell = \pi\xi_0$; 4: $\ell = 10\pi\xi_0$; 5: $\ell = \infty$.] Curve 1 corresponds to the calculations of Skalski et al. (Fig. 10.2), based on the Abrikosov-Gor'kov theory. Curve 5 corresponds to the limit treated by Larkin. (*After Strässler and Wyder.*)

10.3. Time dependent Ginzburg-Landau theory

G.L 이론의 great sucess

time dependent generalization

Nonlinear time dependent GL equation을 얻자

Gor'kov and Eliashberg

- difficulties stem essentially from the singularity in the density of states at the Gap edge.
  - slowly decaying oscillatory response in the time domain

어떻게 해결하나?

presence of magnetic impurities or other pair breakers rounds off the singularity in the BCS density of states.

- 만약 pair-breaking strength가 충분하면 spectrum은 gap이 없어진다.

G.E 는 Rigorous version of a nonlinear

TDGL 방정식 얻다.

- 단 Gapless superconductor에 국한된다

Schmid : TDGL without restriction

But lack rigorous justification except in a gapless regime.

Hu and Thompson. 의 결과

$$D^{-1}\left(\frac{\partial}{\partial t} + i\frac{2e\Psi}{\hbar}\right)\Delta + \xi^{-2}(|\Delta|^2-1)\Delta + \left(\frac{\nabla}{i} - \frac{2e}{\hbar c}\right)^2\Delta = 0$$

$$J = \sigma\left(-\nabla\Psi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right) + \mathrm{Re}\left[\Delta^{*}\left(\frac{\nabla}{i} - \frac{2e}{\hbar c}\vec{A}\right)\Delta\right]\frac{1}{8\pi e\lambda^2}$$

$$\rho = \frac{\Psi - \varphi}{4\pi\lambda_{TF}^2}$$

⊕ Maxwell eq coupling the scalar and vector potential φ and $\vec{A}$ to the charge and current density ρ and $\vec{J}$.

D: normal state diffusion constant

Ψ: electro chemical potential devided by the electronic charge.

$\Delta_0 = \pi K\ [2(T_c^2 - T^2)]^{1/2}$

$\Delta = 1$ in the absence of fields.

$\xi = \hbar\ (6D/\tau_s)^{1/2}/\Delta.$

τ_s : Spin · flip scattering time

$\lambda = \hbar c[8\pi\sigma\tau_s]^{-\frac{1}{2}}/\Delta.$

temperature dependent magnetic penetration depth

Restriction T_c 바로 근처

∵ dissipation of energy by the time-varying field and current 무시해서

Few application

- paramagnetic alloy superconductor to a strong variable, magnetic field.

- τ near T_c

$$\tau = \tau_{GL} = \frac{\pi\hbar}{8K(T_c - T)}$$

- Most convincing experimental test of the TDGL theory is actually in the measurements of fluctuation conductivity above T_c.

  Above T_c, one automatically has a gapless superconductor, but even so there are complications as discussed there unless the so-called Maki terms are suppressed by residual pair-breaking effects of some sort.

10. 3. 1. Electron - Phonon Relaxation

Magnetic impurity는 없지만, relaxation process가 어떻게 되나.

Ġor'kov - Eliashberg theory is not directly applicable.

어떻게 - inelastic phonon-electron interaction to achieve between quasi- particles and condensate and within the quasi particle condensation.

inelastic phonon scattering time τ_E near T_c

어떻게 아나?

low temperature phonon-limited electronic thermal conductivity

으로 이시간 알아내자.

$$\tau_E(T_c) \sim \left(\frac{\Theta_D}{T_c}\right)^2 \sim 10^{-10}\ \text{sec}$$

much shorter in lead and much
longer in aluminum because of
their higher and lower T_c values

이것은 τ_{GL} 보다 매우 길다.

이것의 의미는

11 장에서 다룬다.

Nonequilibrium Superconductivity

Chapter 11

Nonequilibrium Superconductivity

11.1. 서론

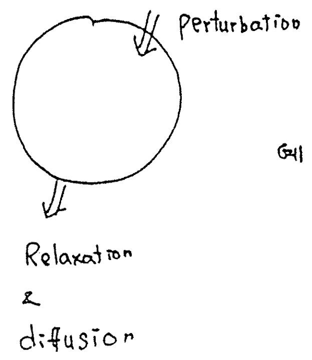
현재까지 Globally stable equilibrium
Metastable equilibrium

이 Chapter: electron population is driven out of thermal equilibrium

- Steady state dynamic equilibrium

OR
More general time-dependent regime

Dynamic equilibrium



예: ① Normal Current to a Supercurrent at a NS interface with associated resistive Voltage developed in the Superconductor.

② microwave irradiation.

$$\psi \sim |\Delta(r)|\, e^{i\varphi(\vec{r})}$$

Review Volume:

K.E. Gray, Nonequilibrium Superconductivity

11. 2.

Quasi - Particle disequilibrium

BCS .

$k\uparrow, -k\downarrow$ 의 Cooper pair, single particle excitation

from this state $E_k = (\Delta^2 + \xi_k^2)^{1/2}$

Thermal equilibrium 에서는 quasi particle state는

Fermi function $f_0(E_k/kT) = \dfrac{1}{1 + e^{E_k/kT}}$

만약 Non equilibrium region 에서는

Excitation의 성질

electronlike $\longrightarrow$ holelike 가 된다

$q_k = (u_k^2 - v_k^2)$ $\quad\quad q_k \approx -1$

$= \xi_k / E_k \approx 1$

-2Δ 에서 2Δ 까지 ξ_k 변할 때

Quasi - particle excitation form the normal electrons of two fluid model of superconductivity

density of state is $N_s(E) = N(0)E/|\xi|$

Actual occupation number in general

$$f_k \neq f_0(E_k/kT)$$

만약 Spacially uniform 하다면

$$\frac{2}{V} = \sum_k \frac{1-2f_k}{E_k} = \sum_k \frac{1-2f_k}{(\Delta^2 + \xi_k^2)^{1/2}}$$

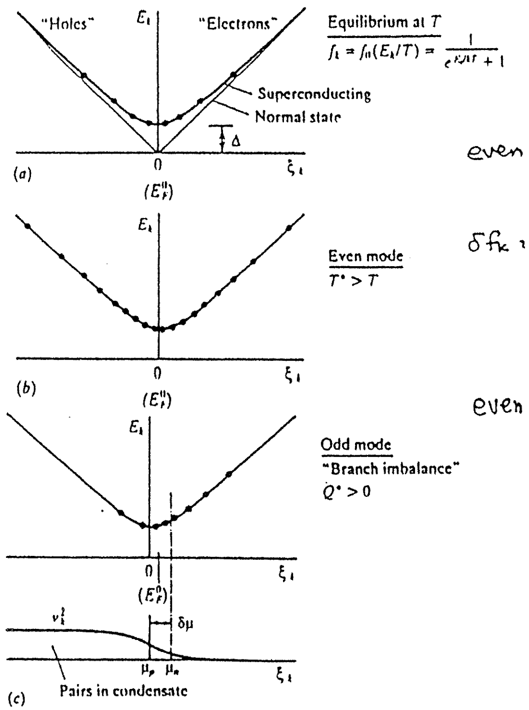
---

11. 2. 1.

Energy mode vs. Charge Mode. Disequilibrium

모든 경우 세세히 풀지말고 T^*, Q^* 를 도입하자.

$$\delta f_k \equiv f_k - f_0(E_k/kT)$$



even mode

δf_k 가 even or odd w.r.t. inversion through the local Fermi Surface

even: Symmetry correspond to a change in temperature

produce more quasi particles equally on both holelike and electronlike branch

Longitudinal 이라 불리운다

— Schmid, Schön

energy or temperature mode 라 불리운다.

FIGURE 11.1
(*a*) Dispersion curves of excitation energies in normal and superconducting states, with schematic indication of occupation numbers in thermal equilibrium. (*b*) Schematic indication of population with even (or energy) mode excited, with $T^* > T$. (*c*) Schematic indication of population with odd (or charge) mode excited, showing branch imbalance corresponding to $Q^* > 0$ and shift of μ_n and μ_p relative to the equilibrium value E_F^0.

이 mode는 phonon, photon 으로 excite 된다.

charged perturbation인 tunneling electron에 의해서도 excite 된다.

parameterize the strength of this longitudinal disequilibrium — T^* effective quasi particle temperature

Near T_c : $\Delta << kT_c$

$$\frac{\delta T^*}{T} \equiv \frac{T^* - T}{T} \approx \frac{1}{N(0)} \sum_k \frac{\delta f_k}{E_k}$$

$$= \int_{-\infty}^{\infty} \frac{\delta f_k}{E_k} \delta \xi_k$$

Schmid 의 방법

Aslamazov & Larkin 의 방법 과 같다.

방법 : G.L 방정식 쓴것이다

T^*, $\alpha + \delta\alpha^*$ 이용할것임

Odd Class

Charged perturbations

- electron injection, Conversion of normal Current to Supercurrent near an interface

$$Q^* \equiv \sum q_k \delta f_k = \sum_k \frac{\xi_k}{E_k} \delta f_k$$

Charge imbalance가 발생한다

Charge neutrality 만족하려면

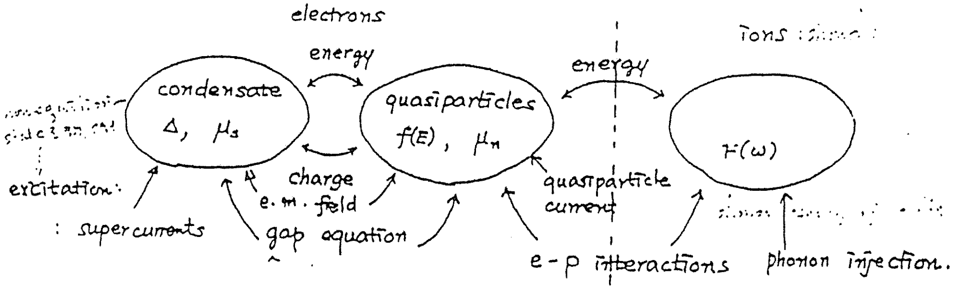
μ_n: electrochemical potential of the normal quasi particle

μ_p : Superconducting pairs

Nonequilibrium SC --- Charge Imbalance

I. Introduction.

- superconductors — electrons ⊕ phonon
  - electrons in SC. :
    : Cooperpairs
    quasiparticles
  - ions — represented by phonons



constraints → charge neutrality

interactions

- SC. state (order parameter)

$$\psi = |\psi| e^{i\phi}$$

A. perturbation modes

1) energy mode : amplitude 를 변화.
   - $\Delta(|\psi|)$
   - phonons 유입. ~ ("bolometric" perturbation)

2) charge mode : · phase 변화 유도
· μ_s 변화.

- external current
- quasiparticle injection (tunneling)

〈cf.〉 $[\phi, n] = i$

$\Delta n \Delta \phi \approx 2\pi$

B. generation modes (방법): injection,

1) steady-state (dc)

2) modulated generation method (ac)
   - electromagnetic absorption
   - ultrasonic attenuation.

3) transient (pulse mode) method

pico-second spectroscopy
femto-second spectroscopy } relaxation 관측.

C. detection.

$$\underbrace{\begin{array}{ccc} \delta\Delta & Q^* & J \\ \downarrow & \downarrow & \downarrow \\ \tau_\Delta & \tau_{Q^*} & \tau_J \end{array}}$$

· linked to the underlying microscopic process

· scattering times
· electron-phonon int.
· e-e int.
· spin orbit int.

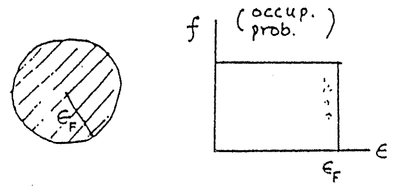
· spin-flip int. (mag. impurities)
· impurity scatterings (elastic scattering)

II. Basic theory of nonequilibrium.

1. quasiparticles & their distribution.

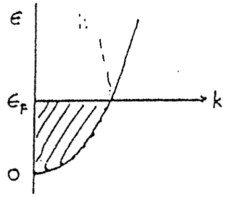
at T=0

〈normal metals〉

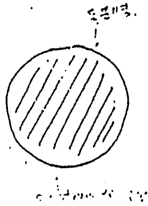


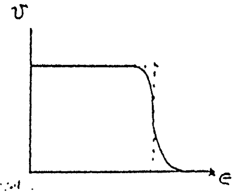
$\epsilon = \frac{\hbar^2 k^2}{2m}$

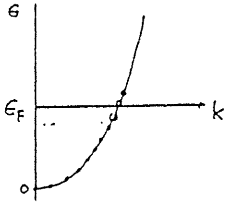
dispersion relation.

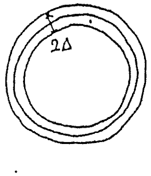


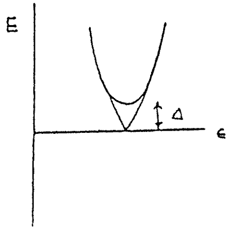
〈SC.〉

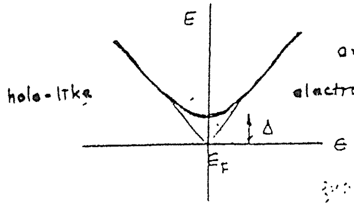












excitation energy [illegible]

[illegible] 100% [illegible]

[illegible] → [illegible]

$$E = \sqrt{(\epsilon - \epsilon_F)^2 + \Delta^2}$$

$$E = \sqrt{\xi^2 + \Delta^2}$$: Excitation energy.

BCS state $T = 0$

$$|\psi_G\rangle = \prod_{k=k_1, k_2 \cdots k_m} (u_k + v_k C^+_{k\uparrow} C_{-k\downarrow})|0\rangle$$

$|v_k|^2$ = pair $(k\uparrow, -k\downarrow)$ occupation prob.

$|u_k|^2 = 1 - |v_k|^2$ = empty state prob.

$$N = 2 \sum_k |v_k|^2$$

$$v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$$

$$u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$$

$$\xi_k = \frac{\hbar^2 k^2}{2m} - \epsilon_F$$

$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$

quasiparticle occupation

E_k

$$f(E_k) = \frac{1}{1 + e^{\beta E_k}}$$

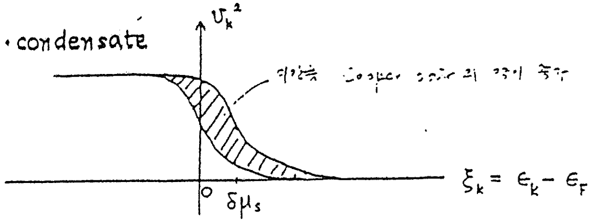
$\Uparrow$
[illegible]

※ normal metal

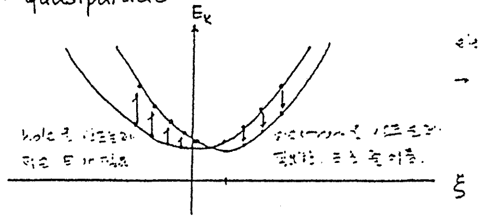
$$f(\xi_k) = \frac{1}{1 + e^{(\epsilon_k - \epsilon_F)/k_BT}}$$

- disequilibrium

- condensate

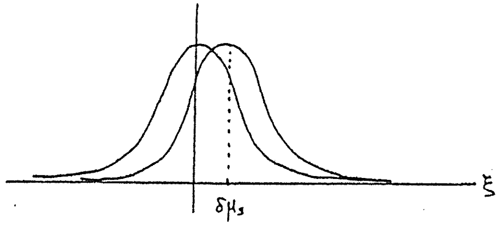


- quasiparticle



electron의 hole [illegible]
→ charge imbalance가 생기는 것

f(E) quasiptl [illegible]



quasiparticle charge distribution

$$Q^* = 2e \sum_{\delta\mu_s = 0} |v_k|^2 - 2e \sum_{\delta\mu_s \neq 0} |v_k|^2$$

$$= -2\,\underline{\underline{N(0)}}\, e\, \delta\mu_s$$

DOS of quasiptl.

$$2e \int_0^{\epsilon_F + \delta\mu_s} N(0) |v_\epsilon|^2 d\epsilon$$

2. Dynamical eqs and relaxation times

1) TDGL Time-Dependent Ginzburg Landau.

<u>GL</u> theory → [illegible] mean field theory.

$$f_{GL} = f_N + \alpha |\psi|^2 + \frac{1}{2}\beta |\psi|^4 - \frac{\hbar^2}{2m}\left(\nabla - 2ie\frac{\vec{A}}{\hbar}\right)^2 \psi$$

$$\tau_{GL}\frac{\partial \psi}{\partial t} = \psi - \frac{\beta}{|\alpha|}|\psi|^2 \psi - \xi^2 \nabla^2 \psi$$

[illegible]

문제점 . excitations X. [illegible]

2) quasiparticle relaxations

- recombination.
- scattering — inelastic scattering.

[illegible]

주원인: $\tau_{ep} \sim 10^{-8}$ sec Al near T_c [illegible]

electron-phonon scattering. [illegible]

$\tau_{ep} \sim 0.5\text{-}3 \times 10^{-10}$ sec Pb, In, Sn.

[illegible]

Theory of Charge Imbalance

1. Quasiparticle Charge and charge imbalance

BCS theory in sc, total charge per unit volume

$$Q_{tot} = \frac{2}{\Omega}\sum_k \left[u_k^2 f_k + v_k^2 (1-f_k)\right]$$

$$f_k(E_k) = \frac{1}{1+e^{\beta E_k}}$$

$u_k^2 f_k$ = (prob. of k state not filled by a pair)

x (prob. of being filled by quasiparticle)

$$\rightarrow Q_{tot} = \frac{2}{\Omega}\sum_k (u_k^2 - v_k^2) f_k + \frac{2}{\Omega}\sum_k v_k^2$$

$$= \underbrace{\frac{2}{\Omega}\sum_k q_k f_k}_{Q^*} + \underbrace{\frac{2}{\Omega}\sum_k v_k^2}_{Q_s}$$

Superfluid contribution

$$Q_s = \frac{2}{\Omega}\sum_k v_k^2$$

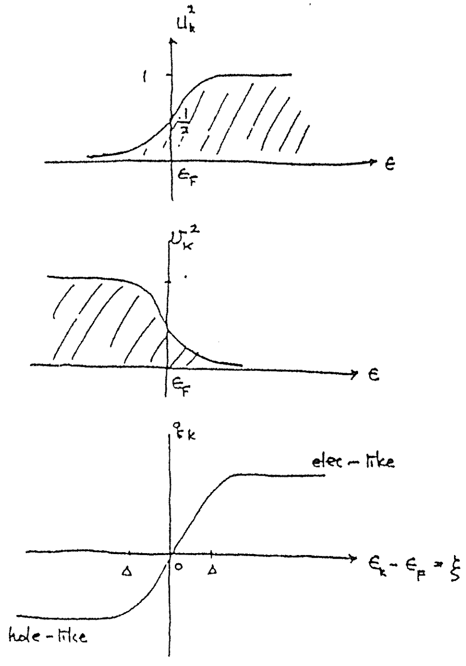
quasiptl contribution

$$Q^* = Q_{tot} - Q_s = \frac{2}{\Omega}\sum_k q_k f_k$$

$$q = u_k^2 - v_k^2$$

$$= \frac{1}{2}\left(1+\frac{\xi}{E}\right) + \frac{1}{2}\left(1-\frac{\xi}{E}\right)$$

$$= \frac{\xi}{E} \equiv \text{effective quasiptl charge}$$



$-1 \leq q_k \leq 1$

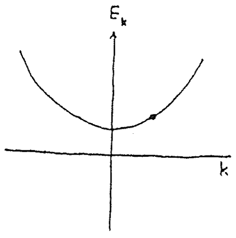
Some disequilibrium situations

qP charge $\quad Q^* = \frac{2}{\Omega} \sum_k q_k f_k$

charge imbalance

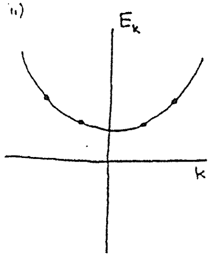
$$Q = \frac{2}{\Omega}\left(\sum_{k>k_F} f_k - \sum_{k<k_F} f_k\right)$$

i)



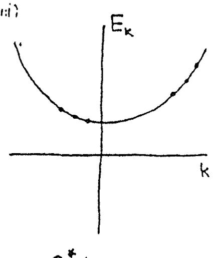
$Q^* \neq 0$
$Q \neq 0$

ii)



$Q^* = 0$
$Q = 0$

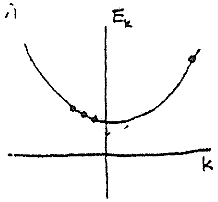
iii)



$Q^* \neq 0$
$Q = 0$

---

ii)



$Q^* = 0$
$Q \neq 0$

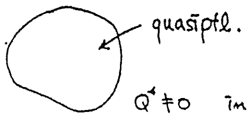
$$Q^* = \frac{2}{\Omega} \sum_k \underset{\substack{\uparrow \\ \text{odd}}}{q_k} \underset{\substack{\uparrow \\ \text{even fn}}}{f_k} = 0$$

in equil

$$Q_{tot} = \cancelto{0}{Q^*} + Q_s = \frac{2}{\Omega} \sum_k v_k^2$$

in inequilibrium

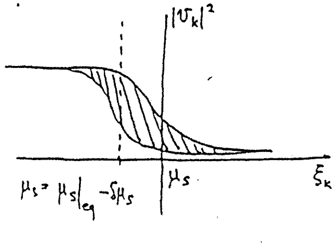
$f_k \neq$ Fermi function.

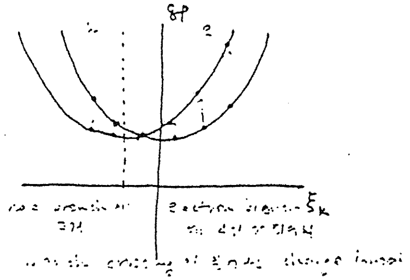


$Q^* \neq 0$ in inequil.

$Q^* > 0 \quad \rightarrow$ chemical pot. of change.

$$\delta\mu_s = \mu_s - \mu_s\big|_{eq} < 0$$

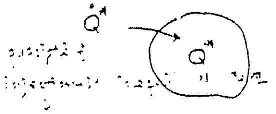




$$Q_{tot} = Q^* + Q_s\Big|_{\delta\mu_s \neq 0} = \frac{2}{\Omega}\sum_k v_k^2\Big|_{\delta\mu_s = 0}$$

$$\vee\ \boxed{Q^* = \frac{2}{\Omega}\sum_k{}' \left(v_k^2\Big|_{\delta\mu_s=0} - v_k^2\Big|_{\delta\mu_s\neq 0} \right)}$$

$$\xrightarrow{T=0} -2N(0)\,\delta\mu_s \longrightarrow eQ^* = -2eN(0)\,\delta\mu_s$$



[illegible]

$$\frac{1}{\tau_a^*}$$

$$\therefore\ \dot{Q}^* = \frac{Q^*}{\tau_a^*}$$

[illegible]

[illegible]

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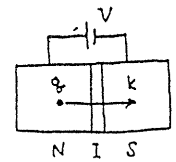
2. tunneling generation and detection of C.I.



eV : Kinetic Energy.

tunneling hamiltonian.

$$H' = \sum_{kq}\left(T_{kq}\, C_k^+ C_q + \overset{\text{hermitian conjugate}}{\text{H.C.}} \right)$$

$\Leftarrow$ Bogolieubov transf.

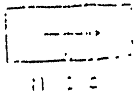
$$C_k^+ = u_k \gamma_{ek0}^+ + v_k \gamma_{hk1}.$$

$$H' = \sum_{kq} T_{kq}\left(u_k\gamma_{ek0}^+ + v_k\gamma_{hk1}\right)\left(u_q\gamma_{eq0} + v_q\gamma_{hq1}^+\right) + \text{h.c.}$$

$$\underline{\text{in N.}}\quad \left.\begin{array}{lll} q<q_F & u_q = 0\ , & v_q = 1 \\ q>q_F & u_q = 1\ , & v_q = 0 \end{array}\right).$$

$$H' = \sum_{k,\,q<q_F}\Big[T_{kq}\big(\overset{①}{u_k\gamma_{ek0}^+} + \overset{②}{v_k\gamma_{hk1}}\big)\gamma_{hq1}^+ + T_{qk}\,\gamma_{hq1}\big(\overset{③}{u_k\gamma_{ek0}} + \overset{④}{v_k\gamma_{hk1}^+}\big)\Big]$$

$$+ \sum_{k,\,q>q_F}\Big[T_{kq}\big(\overset{⑤}{u_k\gamma_{ek0}^+} + \overset{⑥}{v_k\gamma_{hk1}}\big)\gamma_{eq0} + T_{qk}\,\gamma_{eq0}^+\big(\overset{⑦}{u_k\gamma_{ek0}} + \overset{⑧}{v_k\gamma_{hk1}^+}\big)\Big]$$



= [illegible]

NIS junction :

the injection rate of qps into $\vec{k}$ in S (with a bias V)

→ Golden rule.

$$\dot{f}_k\Big|_{inj} = 2\frac{2\pi}{\hbar}|T|^2 N_n(0)\Big[\, u_k^2(1-f_k)\underbrace{(1-f_q)}_{f^\circ(E_k-eV)} - v_k^2 f_k(1-f^\circ(E_k+eV))$$

$$-u_k^2 f_k \underbrace{f^\circ(-E_k+eV)}_{[1-f^\circ(E_k-eV)]} + v_k^2(1-f_k)f^\circ(E_k+eV)\,\Big]$$

즉. 다시쓰면

$$\dot{f}_k\Big|_{inj} = 2\cdot\frac{2\pi}{\hbar}|T|^2 N_n(0)\Big[\, u_k^2(1-f_k)f^\circ(E_k-eV) - v_k^2 f_k(1-f^\circ(E_k+eV))$$

$$- u_k^2 f_k[1-f^\circ(E_k-eV)] + v_k^2(1-f_k)f^\circ(E_k+eV)\,\Big]$$

$$\because\ \left.\begin{aligned} u_k^2 + v_k^2 &= 1 \\ u_k^2 - v_k^2 &= q_k \end{aligned}\right\} \rightarrow \begin{aligned} u_k^2 &= \tfrac{1}{2}(1+q_k) \\ v_k^2 &= \tfrac{1}{2}(1-q_k) \end{aligned}$$

$$= \frac{4\pi}{\hbar}|T|^2 N_n(0)\Big\{\, \frac{1}{2}\big[f^\circ(E_k+eV) + f^\circ(E_k-eV)\big] - f_k$$

$$+ \frac{q_k}{2}\big[f^\circ(E_k-eV) - f^\circ(E_k+eV)\big]\,\Big\}$$

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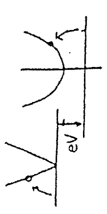
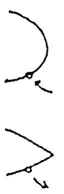
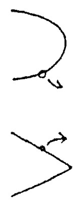
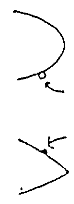
process #	particle change	prob.	electrons tunneling into S	excitations add in S	energy conservation	charge entering condensation
①		$u_k^2(1-f_k)(1-f_q)$	1	1	$E_q + E_k = eV$	$1 - q_k$
②		$v_k^2 f_k(1-f_q)$	1	-1	$E_q - E_k = eV$	$1 + q_k$
③		$u_k^2 f_k f_q$	-1	-1	$-E_q - E_k = -eV$	$-1 + q_k$
④		$v_k^2(1-f_k)f_q$	-1	+1	$-E_q + E_k = -eV$	$-1 - q_k$
⑤		$u_k^2(1-f_k)f_q$	+1	+1	$-E_q + E_k = eV$	$+1 - q_k$
⑥		$v_k^2 f_k f_q$	+1	-1	$-E_q - E_k = eV$	$+1 + q_k$
⑦		$u_k^2 f_k(1-f_q)$	-1	-1	$+E_q - E_k = -eV$	$-1 + q_k$
⑧		$v_k^2(1-f_k)(1-f_q)$	-1	+1	$+E_q + E_k = -eV$	$-1 - q_k$

charge Imbalance injection.

$$\dot{Q}^*_{inj} = \frac{2}{\Omega} \sum_k \xi_k \dot{f}_k \Big|_{inj}$$

$$= \frac{4\pi}{\hbar\Omega} |T|^2 N_n(0) \sum_k \xi_k^2 \left[f^\circ(E_k - eV) - f^\circ(E_k + eV) \right]$$

$$= \frac{4\pi}{\hbar\Omega} |T|^2 N_n(0)\, N(0) \int_\Delta^\infty \frac{N_s(E)}{N_s^2(E)} \left[f^\circ(E - eV) - f^\circ(E + eV) \right] dE$$

$$\frac{N_s(E)}{N(0)} = \frac{E}{\sqrt{E^2 - \Delta^2}} = \frac{E}{|\xi|} = \left| \xi_k^{-1} \right|$$ ⇒ $N_s(E)$ 로 이 note 에서 이용.
i.e. normalized 되었으므로

$$\sum_k f(\;) \longrightarrow N(0) \int_\Delta^\infty \frac{N_s(E)}{N(0)} f(\;)\, dE$$

$$G_{NN} = \frac{4\pi e^2}{\hbar} |T|^2 N_n(0) N(0)$$

∴ the injection rate of qps into k in S per unit vol.

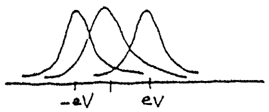
$$\dot{Q}^*_{inj} = \frac{G_{NN} V}{e\Omega} \int_\Delta^\infty \frac{1}{N_s(E)} \left(-\frac{\partial f^\circ}{\partial E} \right) dE. \qquad eV \ll k_B T,\ \Delta$$

$$I = \frac{2\pi e}{\hbar} |T|^2 N_n(0) \Big\{ u_k^2 (1 - f_k) f^\circ(E_k - eV) + v_k^2 f_k [1 - f^\circ(E_k + eV)]$$
$$- u_k^2 f_k [1 - f^\circ(E_k - eV)] - v_k^2 (1 - f_k) f^\circ(E + eV) \Big\}$$

$$= \frac{4\pi e}{\hbar} |T|^2 N_n(0) \sum_k \left[u_k^2 f^\circ(E_k - eV) - v_k^2 f^\circ(E_k + eV) - \xi_k f_k \right]$$

$$\therefore I = \frac{2\pi e}{\hbar} |T|^2 N_n(0) \Big\{ \sum_k \big[f^\circ(E_k - eV) - f^\circ(E_k + eV) - 2\xi_k \delta f_k \big]$$
$$+ \sum_k \tfrac{1}{2} f^\circ(E_k + eV) + \tfrac{1}{2} f^\circ(E_k - eV) - f_k^\circ \Big]$$

$$f_k \rightarrow f_k^\circ + \delta f_k$$



$$I = \frac{2\pi e}{\hbar} |T|^2 N_n(0) \sum_k \left[f^\circ(E_k - eV) - f^\circ(E_k + eV) - 2\xi_k \delta f_k \right]$$

$$= V G_{NN}\, g_{NS} - \frac{G_{NN}}{2N(0)e} Q^* = I_{eq} + I_{Q^*}$$

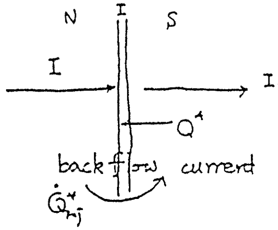
$$g_{NS} \equiv 2 \int_\Delta^\infty \frac{1}{N_s(E)} \left(\frac{\partial f^\circ}{\partial E} \right) dE$$

$$2 \sum_k \xi_k \delta f_k = 2 \sum_k \xi_k f_k = \Omega Q^*$$

$$\sum_k \xi_k f_k^\circ = 0$$

$$\dot{Q}^*_{inj} = \frac{G_{NN} V}{e\Omega} \int_\Delta^\infty \frac{1}{N_s(E)} \left(\frac{\partial f^\circ}{\partial E} \right) dE = \frac{2 G_{NN} V}{e\Omega} g_{NS}$$

$$= \frac{4\pi}{\hbar\Omega} |T|^2 N_n(0) N(0) \int_\Delta^\infty \frac{1}{N_s} \left[f_\circ(E - eV) - f^\circ(E + eV) \right] dE.$$



current

tunneling rate

$$\frac{1}{\tau_{tun}} \equiv \frac{G_{NN}}{2N(0)\, e^2 \Omega}$$

$$\frac{1}{\tau_{Q^*}} = \frac{\dot{Q}^*_{inj}}{Q^*} = \frac{F^*}{\tau_{tun}} \left(\frac{2N(0)\, eV}{Q^*}\, g_{NS} - 1 \right)$$

$$F^* = \frac{e\Omega\, \dot{Q}^*_{inj}}{I}$$

$$\frac{V}{I} = R_j = R_{eq}(T) + R_{Q^*}(T) = \frac{1}{G_{NN}\, g_{NS}} \left(1 + \frac{1}{\tau_{tun} / (\tau_{Q^*} F^*)} \right)$$

[illegible]

$$\therefore R_j = R_{eq} + R_{Q^*}$$

$$= \frac{1}{G_{NN}\, g_{NS}} + R_{eq} \cdot \frac{\tau_{Q^*} F^*}{\tau_{tun}}$$

[illegible]

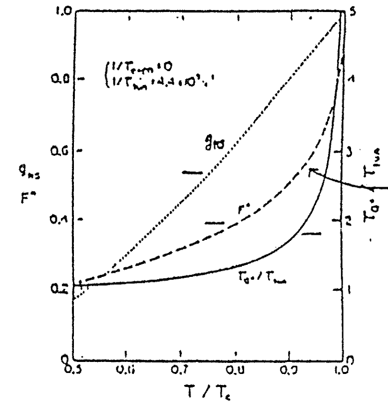
relaxation.

$T^3 \propto$ electron - phonon scatt.

$T \propto$ electron - electron scatt. ⟷ disordered system.

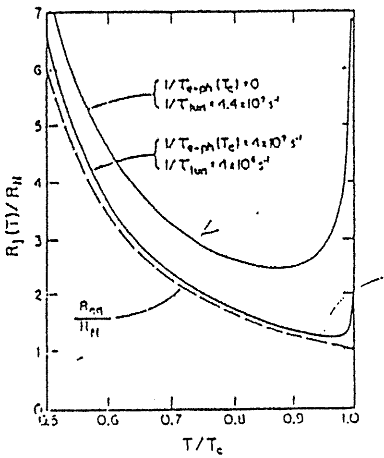
localization. [illegible]

[illegible]



g_{NS} : universal curve.

T_c 에 접근할수록 많은수가 quasiparticle로 되어있다.



charge imbalance 효과가 크다.

charge imbalance 효과가 적어지고 아주 T_c 근처 에서만 [illegible]

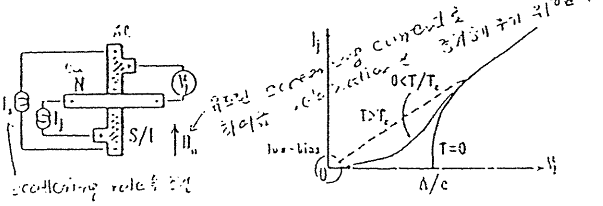
junction resistance 가 10 mA 정도 되어야 injection 이 [illegible] (Q^*) 일어나고, non-eq 효과를 볼수 있다

즉, 보통 junction 에서는 charge imbalance 효과가 [illegible]

Experiments on Aluminum Films

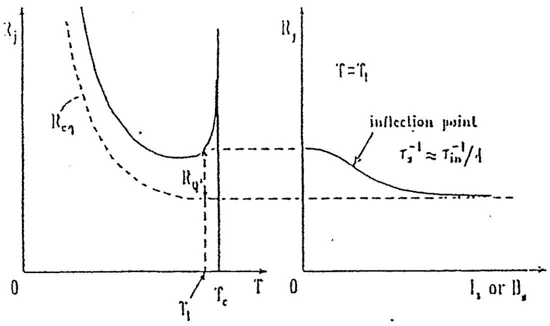
(ref. Lee et al., PRB 33,750 (?); PRB 40, 10111(?))

SIN Junction



유도된 screening current 를 차이가 relaxation 을 줄여 주기 위함.

scattering rate를 측정



$R_j = R_{eq} + R_{Q^*}$ $\qquad$ $R_{Q^*} \simeq R_{eq}\,\tau_{Q^*}/\tau_{in}$ near T_c $\quad \left(\begin{matrix} F^*\simeq 1 \\ g_{ns}=1 \end{matrix}\right)$

$$\tau_{Q^*}^{-1} \simeq \frac{\pi\Delta}{4k_BT}\,\tau_{in}^{-1}\left(\tau_{in}^{-1}+2\tau_s^{-1}\right)^{1/2}$$

$$\tau_s^{-1} \propto \frac{R_\square}{d}I_s^2, \quad \frac{d}{R_\square}B_\parallel^2$$

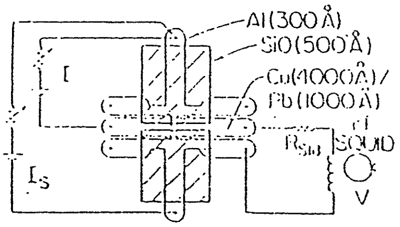
$$(I_s) \qquad (B_\parallel)$$

$T \sim T_c$

$$R_{eq} \cong \frac{g_{ns}^{-1}}{2N(0)e^2\Omega}\,\frac{1}{\tau_{in}^{-1}}$$

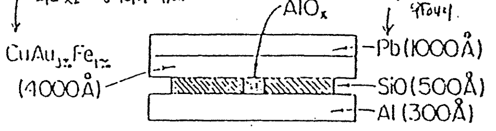
$$R_{Q^*} \cong \frac{g_{ns}^{-1}}{2N(0)e^2\Omega}\,\frac{1}{\tau_{Q^*}^{-1}}$$

Small junction resistance → relatively large nonequilibrium



proximity 효과로 의해 S.C가 되는것을 방지하기 위해

Normal은 equipotential 이어서 equipotential을 갖게하기 위해서.



In-situ fabrication with aperture masks.

Table 1. Sample parameters.

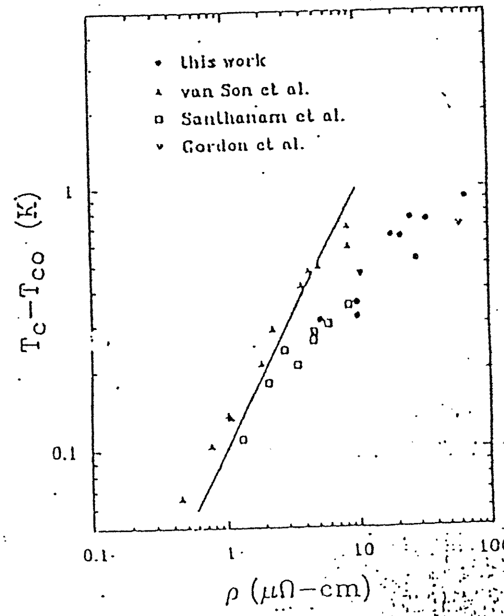
Al의 S.C. transition temp.

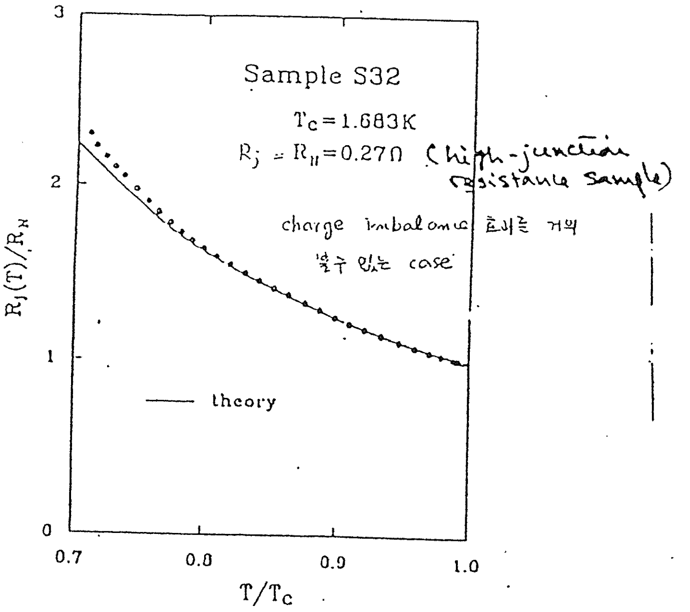
junction resistance → 이것을 다르게 하는 이유 Q*의 term 을 조절하기 위해서.

Sample	d (Å)	w* (μm)	ℓ** (Å)	$R_\square$† (Ω)	T_c (K)	R_J (mΩ)	$1/\tau_{exp}$ ($10^8 s^{-1}$)
S32	300	315	13.8	4.6	1.683	270	0.0023
SH13	343	300	49.3	2.37	1.440	1.0	0.58
SH17	422	300	41	2.31	1.519	0.16	3.1
SH24	303	206	18	7.2	1.800	3.25	0.18
SH25	300	315	11.6	11.5	1.905	4.44	0.12
SH30	303	314	15.3	8.62	1.920	1.25	0.43
SH31	300	206	77.5	1.72	1.470	1.67	0.35
SH33	300	300	6.0	22.3	2.070	1.25	0.47
SH34	301	300	21.6	6.15	1.808	0.165	3.2

*w is width of superconducting Al film.
**ℓ is electron mean free path calculated from $\rho\ell = 4\times10^{-16}\,\Omega\text{-m}^2/+67/$.
†$R_\square$ is sheet resistance at 4.2K.

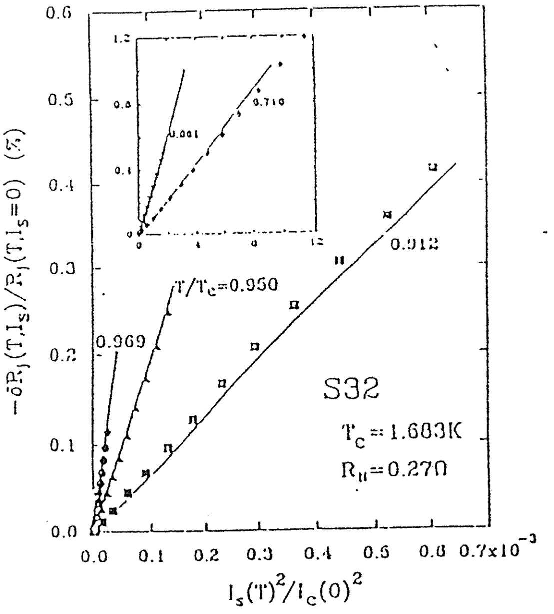
ℓ, 시료 의



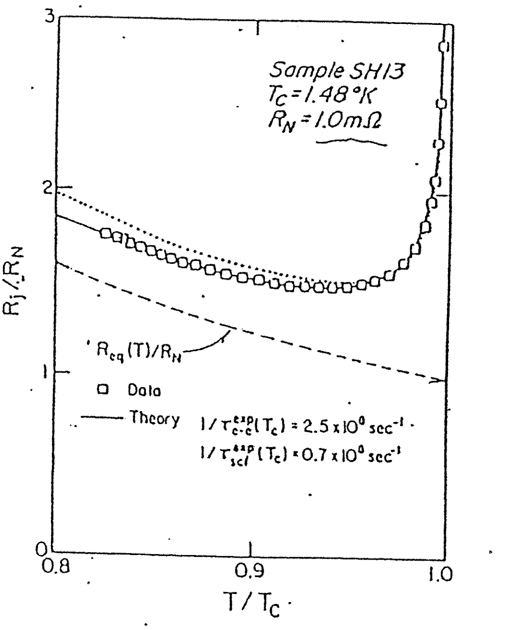


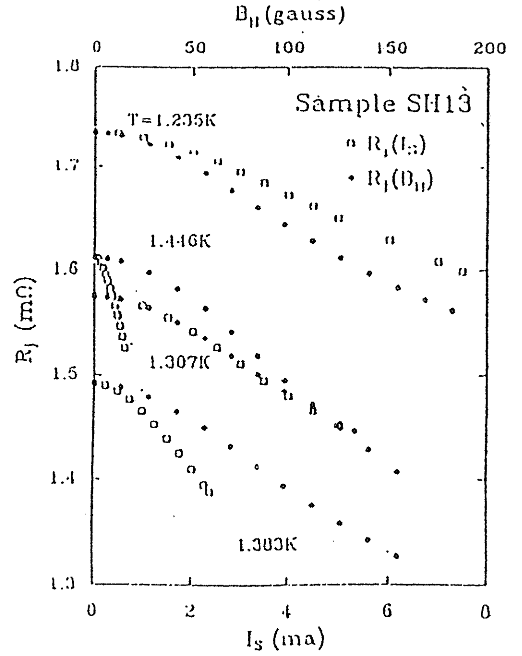
(high-junction resistance sample)

charge imbalance 효과를 거의 볼 수 없는 case

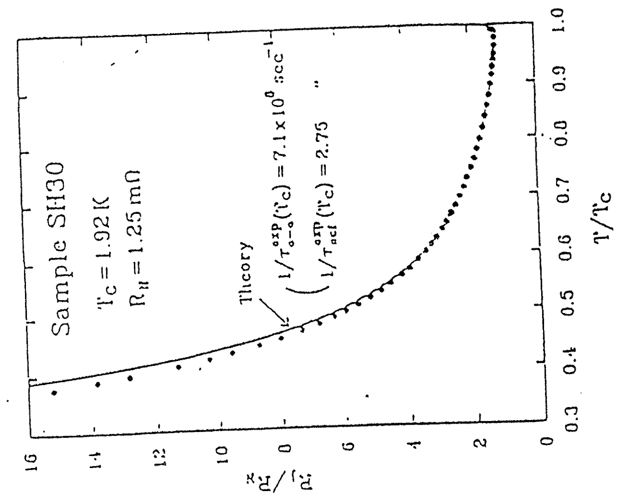


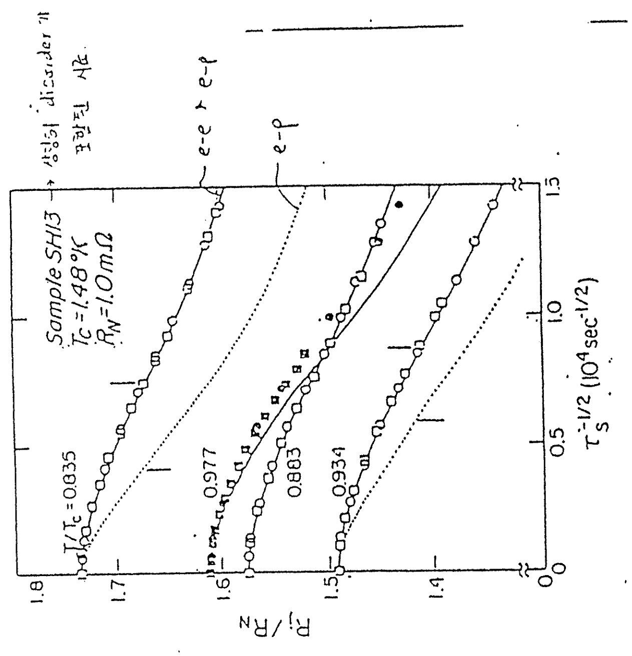
current에 의한 relaxation rate 의 증가
→ junction resistance 감소





· field에 의한 relaxation rate도 [illegible]
의존성이 있다.





11. 2. 2. Relaxation Times

disequilibrium caused by a given perturbation

: 이를 이해하기 위해 relaxation을 알아야 한다

Schmid 와 Schön 에 의하면 두개의 Relaxation Mode 있어

$$\tau_R^{(L)} = \tau_\Delta = \tau_T \approx 3.7\,\tau_E\, kT_c/\Delta$$

$$\tau_R^{(T)} = \tau_{Q^*} = \frac{4}{\pi}\,\tau_E\, kT_c/\Delta$$

τ_E : energy relaxation or inelastic scattering time for an electron at the Fermi Surface.

Kaplan이 많이 계산.

characteristic time for f_k to approach the Fermi function, 즉 δf_k를 zero로 relax 시킨다

이 시간은 inelastic scattering에 의해 δf_k의 변화를 set 시킨다. portion Δ/kT_c of thermally occupied state.

- Inelastic electron-phonon scattering

  : relax T^* effectively

∘ Q^* can be effectively relaxed by elastic scattering in the presence of gap anisotropy

11. 3. Energy-Mode Disequilibrium :

Steady State Enhancement of Superconductivity

Electron 을 초전도에 더 넣으니 T^* 작아지며

따라서 초전도 현상이 증가한다.

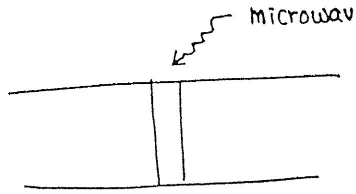
dynamic regime in the cooling (or heating)

— Order parameter의 시간 의존성

11 3.1.

Enhancement by Microwaves

Wyatt - Dayem Effect.



그러므로 dc critical current 가 커진다

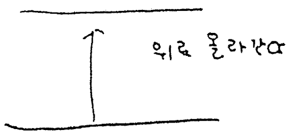
Klapwijk, Mooij

Enhancement : long narrow strips
bridge

△ 증가 Kommers and Clarke

$\hbar\omega < 2\Delta$ 인 빛으로 때리다

So that no new quasi-particles are generated.



$\delta f_k < 0$ for the low-lying states

$\delta f_k > 0$ 〃 higher 〃

예1 왜 quasi particle의 수가 주나

왜냐면 higher energy의 recombination 이 빠르기에

예2. Minimum freq. above 에서는 초전도 잘 되고

$f < f_c$ rf current 가 pair breaker로 작용

τ_E 는 aluminum film의 Sheet resistance 에 비교

언제 enhancement 가 생기나

$\delta T^* \sim -0.02 T_c$ in Alumina film

만약 T_c 위이면 two solutions