Chapter 15.

Ferromagnetism and Antiferromagnetism

spontaneous magnetic moment electron의 spin과 magnetic moment가 regular manner로 arrange

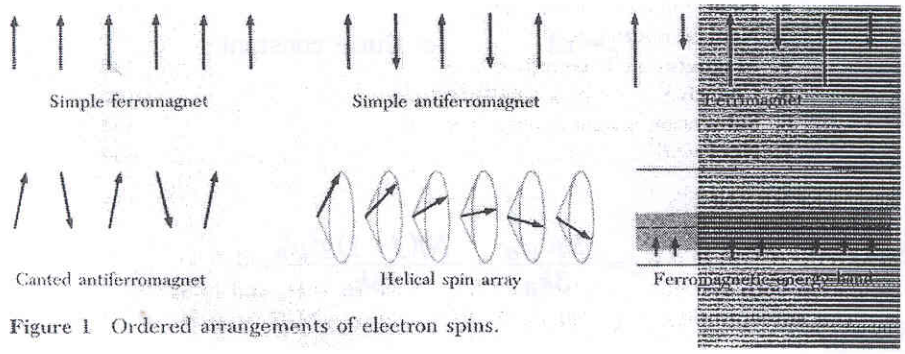


Figure 1 Ordered arrangements of electron spins.

Ferri magnet

Ferromagnetic energy band

◎ Ferromagnetic moment
◎ Spin wave, magnon
◎ Mean field theory
  - Spontaneous magnetization
◎ Neutron scattering
◎ Ferrimagnetic order
◎ Curie temperature
◎ Neel temperature
◎ Antiferromagnetic magnon

Curie point and Exchange Integral

N ions of spin S

exchange field

내부의 interaction에 의해 magnetic moment가 서로 평행이다

온도에 의해 깨질 수 있다.

exchange field를 magnetic field와 equivalent하게 표시

exchange field : B = 10^7 gauss 까지 올라간다.

Mean field approximation

$$\overrightarrow{B_E} = \lambda \overrightarrow{M}$$

Def. Curie Temp $T > T_c$ disordered parameteic phase

$T < T_c$ disordered ferrometeic phase

외부에서 Ba (applied field)가하다

$$\overrightarrow{M} = \chi_p (B_a + B)\ ,\quad \chi_P = \frac{C}{T},\qquad \overrightarrow{M} = \frac{C}{T}(B_a + \lambda \overrightarrow{M})$$

$$\chi = \frac{M}{B_a} = \frac{\frac{C}{T}B_a}{1-\frac{\lambda C}{T}} = \frac{C}{T-\lambda C} \qquad \vec{M}\cdot\left(1-\frac{\lambda C}{T}\right) = \frac{C}{T}B_a$$

Let λc = Tc

$$\therefore \chi = \frac{C}{T-T_c} \qquad T_c = c\lambda \qquad \text{c: Curie constant}$$

Detailed calculation

$$\chi \propto \frac{1}{(T-T_c)^{1.33}}$$

$$\lambda = \frac{T_c}{c} \qquad c = \frac{Np^2\mu_B^2}{3k_B} = \frac{NJ(J+1)g^2\mu_B^2}{3k_B}$$

$$= \frac{3k_BT_c}{Np^2\mu_B^2}$$

$$= \frac{3k_BT_c}{NJ(J+1)g^2\mu_B^2}$$

J=S

$$= \frac{3k_BT_c}{NS(S+1)g^2\mu_B^2}$$

철[Fe] : $T_c \simeq 1000K$, g=2, $S \simeq 1$, $\lambda \simeq 5000$

$M_s \simeq 1700$

$B_E = \lambda M = (5000)(1700) = 10^7G = 10^3T$

The exchange field는 quantum exchange field

i, j 에 S_i, S_j의 spin이 있다.

$u = -2J\, \vec{s_i} \cdot \vec{s_j}$

위 식은 Heisenberg model이라 불리 운다.

왜 이렇게 나오나 Pauli exclusion principal 때문이다.

exchange energy는 $-2J\, \vec{s_i} \cdot \vec{s_j}$이다.

exchange integral과 T_c 사이의 관계를 알아보자

Mean field 결과에 의하면

$$J = \frac{3k_BT_c}{2zs(s+1)} \qquad z : \# \text{ of } NN_b$$

$$s = \frac{1}{2}$$

철은 s=1 Curie temp corresponds to J= 11.9meV

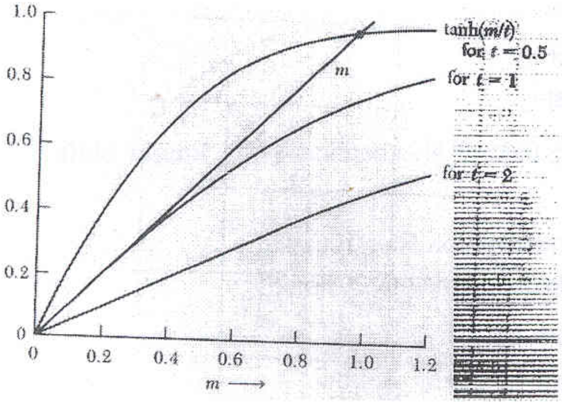
Temperature dependence of saturation Magnetization spin $\frac{1}{2}$ particle

$$M = N\mu \tanh(\mu B/k_B T)$$

$$\frac{M}{N\mu} = \tanh\left(\frac{M}{N}\cdot\frac{k_B T}{\lambda\mu N\mu}\right)$$

let $m = \frac{M}{N\mu}$ and $t = \frac{k_B T}{\lambda\mu^2 N}$

then $m = \tanh(m/t)$



t<1이어야 서로 만난다.

만나기 시작하는 점

$$T_c = \frac{Nu^2\lambda}{k_B}$$

2nd order ferromagnetic / paramagnetic transition

온도가 낮을 때 위 의식을 전개

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1-e^{-2x}}{1+e^{-2x}} = 1 - 2e^{-2x}$$

x가 매우 크다

$$m = 1 - 2\exp\left(-\frac{2m}{t}\right)$$

$$\frac{M}{N\mu} = 1 - 2\exp\left(-\frac{2M}{N\mu}\cdot\frac{N\mu^2\lambda}{k_B T}\right)$$

$$\therefore\ M = N\mu - 2N\mu\exp\left(-\frac{2M\mu\lambda}{k_B T}\right)$$

If T=0 M=Nμ

$$\therefore \triangle M \equiv M(0) - M(T)$$

$$= 2N\mu\exp\left(-\frac{2M\mu\lambda}{k_B T}\right) = 2N\mu\exp\left(-\frac{2T_c}{T}\right)$$

T=0.1Tc $\qquad \frac{2T_c}{T} = \frac{2}{0.1} = 20$

$$\triangle M = 2N\mu\exp(-20)$$

$$\frac{\triangle M}{N\mu} = 2\exp(-20) = 4\times 10^{-9}$$

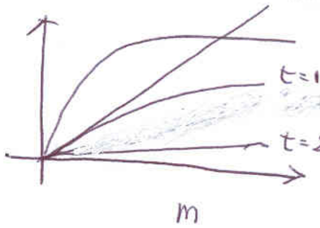
$M = N\mu \tanh(\mu B / k_B T)$

만약 $B_E = \lambda M$ 이면

$M = N\mu \tanh(\lambda \mu M / k_B T)$

Let $t \equiv k_B T / N\mu^2 \lambda$

$m = \tanh(m/t)$ 실험



$\tanh(m/t)$ $t = 0.5$

$t = 1$ Critical

or $T_c = N\mu^2\lambda / k_B$

For $T \ll T_c$

$\tanh \xi \cong 1 - 2e^{-2\xi}$ $\Delta M = 2N\mu \exp\left(-\frac{2N\lambda\mu^2}{k_B T}\right.$

$$\frac{\triangle M}{M(0)} = AT^{3/2}$$ spin wave theory로 해결

Kittel 책에 table 2 가 있는데

saturation magnetization M_c

ferromagnetic Curie temp T_c

magneton number N 가 표시되어 있다.

아닐수 있다 이유

위에서의 n_B가 integer가 .

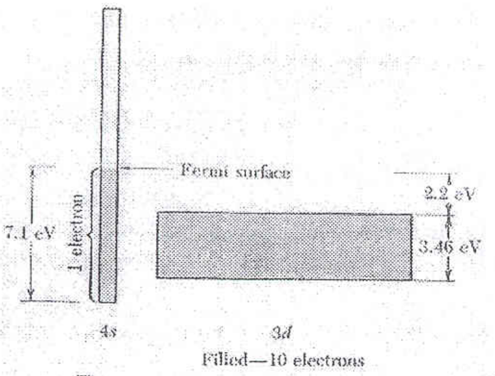
어떻게 n_B가 integer 아닐수있나

1. Spin -orbit interaction which adds or subtracts some orbital magnetic moment
2. Conduction electron magnetization
3. spin arrangement in ferromagnet

한 개는↓ 두 개는↑

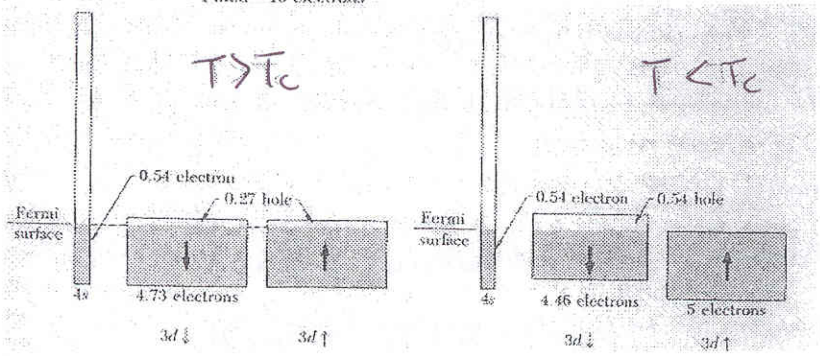
Itinerant electron model

C외는 Ferromagnetic이 아니다.



∴ Ferromagnetic이 아니다.

Ni는 Ferromagnetic이다.



Ferromagnet 이다

Magnons : Quantized spin wave

Heisenberg Interaction

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1}$$

Sp를 classical vector로 취급하자

ground state는

$$\vec{S}_p \cdot \vec{S}_{p+1} = S^2$$

exchange energy는 $U_0 = -2NJs^2$

First excited state Energy 는?

↑ ↑ ↑ ↓ ↑ ↑

$U = -2NJS^2 + 8JS^2$ 인가?

아님 spin wave를 생각하자.

Classical derivation of the magneon dispersion relation

$$-2J\vec{S}_p \cdot (\vec{S}_{p-1} + S_{p+1}) = 2J\frac{\vec{\mu}}{g\mu_B} \cdot (\vec{S}_{p-1} + S_{p+1})$$

$$\vec{\mu}_p = -gu_B\vec{S}_p$$

$-\vec{\mu}\cdot\vec{B}_p$ 모양이 되려면

$$\vec{B}_p = -\frac{2J}{g\mu_B}(\vec{S}_{p-1} + \vec{S}_{p+1})$$

Classical Mechanics에서

$\frac{d}{dt}(\hbar\vec{S}_p) = \vec{\mu}_p \times \vec{B}_p$이다

$$\therefore \quad \frac{d\vec{S}_p}{dt} = \frac{1}{\hbar}\vec{\mu}_p \times \vec{B}_p$$

$$= \frac{1}{\hbar}\vec{\mu}_p \times \left(-\frac{2J}{g\mu_B}\right)(\vec{S}_{p-1} + \vec{S}_{p+1})$$

$$= \frac{1}{\hbar}(-g\mu_B\vec{S}_p) \times \left(-\frac{2J}{g\mu_B}\right)(\vec{S}_{p-1} + \vec{S}_{p+1})$$

$$= \frac{2J}{\hbar}\vec{S}_p \times (\vec{S}_{p-1} + \vec{S}_{p+1})$$

$$= \frac{2J}{\hbar}(\vec{S}_p \times \vec{S}_{p-1} + \vec{S}_p \times \vec{S}_{p+1})$$

만약 excitation의 amplitude 가 작다. 즉 $(S_p^x,\ S_p^y \ll S)$

x, y ,z component로 나누자.

$$\frac{dS_{px}}{dt} = \frac{2J}{\hbar}(S_{py}S_{(p-1)z} - S_{pz}S_{(p-1)y} + S_{py}S_{(p+1)z} - S_{pz}S_{(p+1)y})$$

$$= \frac{2J}{\hbar}[\ S_{py}{}^y(\ S_{p-1}{}^z + S_{p+1}{}^z) - S_p{}^z(\ S_{p-1}{}^y + S_{p+1}{}^y)]$$

$$\frac{dS_{py}}{dt} = \frac{2J}{\hbar}[s_p^z(s_{p-1}^x + s_{p+1}^x) - s_p^x(s_{p-1}^z + s_{p+1}^z)]$$

$\frac{dS_{py}}{dt}$ = similar

$dS_p^z = S$라 하자

$$\frac{dS_{px}}{dt} = \frac{2J}{\hbar}\left[2s\ s_p^y - s(s_{p-1}^y + s_{p+1}^y)\right] = \frac{2J}{\hbar}\left[2s_p^y - (s_{p-1}^y + s_{p+1}^y)\right]$$

$$\frac{dS_{py}}{dt} = \frac{2J}{\hbar}\left[s(s_{p-1}^x + s_{p+1}^x - 2s\ s_p^x)\right] = \frac{2Js}{\hbar}\left[s_{p-1}^x + s_{p+1}^x - 2s_p^x\right]$$

$$\frac{dS_{pz}}{dt} = 0$$

Let $S_p^x = u\exp[i(pka - \omega t)]$, $S_p^y = u\exp[i(pka - \omega t)]$

$$-i\omega u = \frac{2Js}{\hbar}(2 - e^{-ika} - e^{ika})v = \frac{4Js}{\hbar}(1 - \cos ka)v$$

$$-i\omega v = +\frac{2Js}{\hbar}(-e^{-ika} - e^{ika} - 2)u = -\frac{4Js}{\hbar}(1 - \cos ka)u$$

$$\begin{vmatrix} i\omega & \frac{4Js}{\hbar}(1 - \cos ka) \\ -\frac{4Js}{\hbar}(1 - \cos ka) & i\omega \end{vmatrix} = 0 \quad \text{Hen} \quad \begin{vmatrix} i\omega(1 - \cos ka) & (1 - \cos ka) \\ & \end{vmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = 0$$

$$\therefore \hbar\omega = 4JS(1 - \cos ka)$$

$$\therefore \quad iu + v = 0$$

Take real part

$$s_p^x = u\cos(pka - \omega t) \quad ; \quad s_p^y = u\sin(pka - \omega t)$$

$$\hbar\omega = 4Js(1 - \cos ka)$$

spin wave in one dimension with hearest nbd interactions

$\left|ka\right| \ll 1$이면 $\quad \hbar\omega = 4Js \cdot \frac{(ka)^2}{2} = (2Jsa^2)k^2$

dispersion relation for a ferromagnetic cubic lattice

with nearest interaction

$$\hbar\omega = 2JS[Z - \sum_{\delta}\cos(\vec{k}\cdot\vec{\delta})]$$

summation is over the Z vector

for $k_a \ll 1$

$$\hbar\omega = (2JSa^2)k^2$$

Quantization of spin waves

spin wave를 Quantize하면

phonon, photon과 마찬가지로

$$\varepsilon_k = \left(n_k + \frac{1}{2}\right)\hbar\,\omega_k$$

Thermal excitation of Magnons

$$\langle n_k \rangle = \frac{1}{\exp\left(\frac{\hbar\,\omega}{kT}\right) - 1}$$

Total number of magnon excited at a temperature T

$$\sum n_k = \int d\omega D(\omega)\langle n(\omega)\rangle \qquad D(\omega)d\omega = \left(\frac{L}{2\pi}\right)^3 \int_{shell} d^3k$$

$$D(\omega)d\omega = \frac{1}{(2\pi)^3}\,4\pi k^2 dk \qquad \text{per unit volume}$$

$$= \frac{1}{(2\pi)^3}\,4\pi k^2 \frac{dk}{d\omega}\,d\omega$$

since $\hbar\,\omega = 2JSa^2k^2$

$$\hbar\,\frac{d\omega}{dk} = 2Jsa^2 \cdot 2k$$

$$\frac{d\omega}{dk} = \frac{4JSa^2k}{\hbar}$$

$$D(\omega)d\omega = \frac{1}{(2\pi)^3}\,4\pi k^2 \frac{\hbar}{4JSa^2 k}\,d\omega$$

$$k^2 = \frac{\hbar\,\omega}{2JSa^2}$$

$$= \frac{1}{4\pi^2}\,\frac{\hbar}{2JSa^2}\sqrt{\frac{\hbar\,\omega}{2JSa^2}}\,d\omega$$

$$= \frac{1}{4\pi^2}\left(\frac{\hbar}{Jsa^2}\right)^{\frac{3}{2}} \omega^{\frac{1}{2}}\,d\omega$$

$$\therefore \quad \sum_k n_k = \frac{1}{4\pi^2}\left(\frac{\hbar}{JSa^2}\right)^{\frac{3}{2}} \int_0^\infty \frac{\omega^{\frac{1}{2}}\,d\omega}{e^{\beta\hbar\,\omega} - 1}$$

$$\beta\hbar\,\omega = x \qquad \omega^{\frac{1}{2}} = \sqrt{\frac{x}{\beta\hbar}}$$

$$d\omega = \frac{1}{\beta\hbar}\,dx$$

$$= \frac{1}{4\pi^2}\left(\frac{\hbar}{JSa^2}\right)^{\frac{3}{2}} \cdot \int_0^\infty \frac{x^{\frac{1}{2}}\,dx}{e^x - 1}\sqrt{\frac{1}{\beta\hbar}}\,\frac{1}{\beta\hbar}$$

$$= \frac{1}{4\pi^2}\left(\frac{k_B T}{2JSa^2}\right)^{\frac{3}{2}} \cdot \underbrace{\int_0^\infty \cdot \frac{x^{\frac{1}{2}}dx}{e^x - 1}}_{0.0587\times 4\pi^2}$$

$$\frac{\sum n_s}{N\cdot S} = \frac{\Delta M}{M(0)} = \frac{a^3}{Qs}\sum n_s$$

$N = \frac{Q}{a^3}$, $Q = 1, 2, 4$ for sc, bcc, fcc lattice

ㄴ unit volume 내의 atom $= \frac{a^3}{Qs}\frac{1}{4\pi^2}\left(\frac{k_B T}{2JSa^2}\right)^{\frac{3}{2}} \cdot (0.0587) \cdot 4\pi^2$

$$\frac{\sum n_s}{\frac{Q}{Q^3}\cdot s} \qquad \therefore \frac{\Delta M}{M(0)} = \frac{0.0587}{Qs}\left(\frac{k_B T}{2JS}\right)^{\frac{3}{2}}$$

Bloch $T^{\frac{3}{2}}$ law : neutron scattering 으로 측정

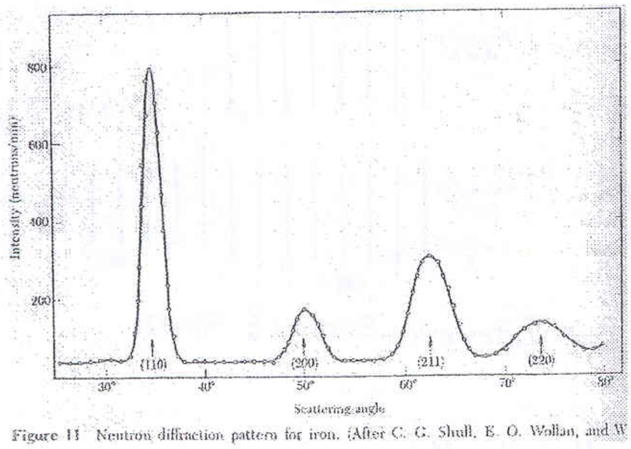
T_e아래 또는 위에서도 측정된다.

Neutron Magnetic Scattering

X-ray는 electronic charge의 spatial distribution을 본다.

Neutron : distribution nuclei

magnetization



Fe-Co alloy

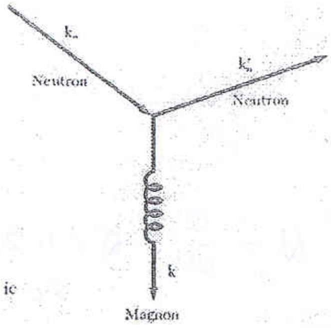
Figure 11 Neutron diffraction pattern for iron. (After C. G. Shull, E. O. Wollan, and W

만약 Incident neutron이 wave vector kn으로 들어와 kn′ 로 나가면

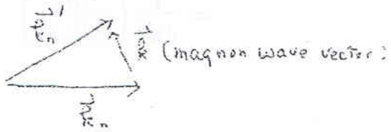
$$\vec{k}_n' = \vec{k}_n = \vec{k} + \vec{G}$$

$\vec{G}$: reciprocal lattice vector이다.

초전도연구단
단장 이 성 익



(magnon wave vector)



에너지 conservation에 의해

$$\frac{\hbar^2 k_n^2}{2Mn} = \frac{\hbar^2 k_n^{2\prime}}{2Mn} + \hbar\,\omega_k$$

Ferromagnetic Order

T=0일 때 Ferromagnetic Crystal은 fully saturated 되어있지 않다.

familiar example

예 : Fe_3O_4 or $FeO \cdot Fe_2O_3$ $\qquad g\mu_B s = 2 \cdot \mu_B \frac{5}{2} = 5$

2개 Fe^{3+} : $s = \frac{5}{2}$ and zero orbital momentum $\qquad 5\mu_B$

1개 Fe^{2+} $\quad s = 2$ $\qquad 4\mu_B$

따라서 Fe_3O_4는 $2 \cdot 5\mu_B + 4\mu_B = 14$ 가

expectation value all spin were parallel

문제 : Fe^{3+}에서는 서로 반대이다.

∴ Fe^{2+} ion 만이 기여한다.

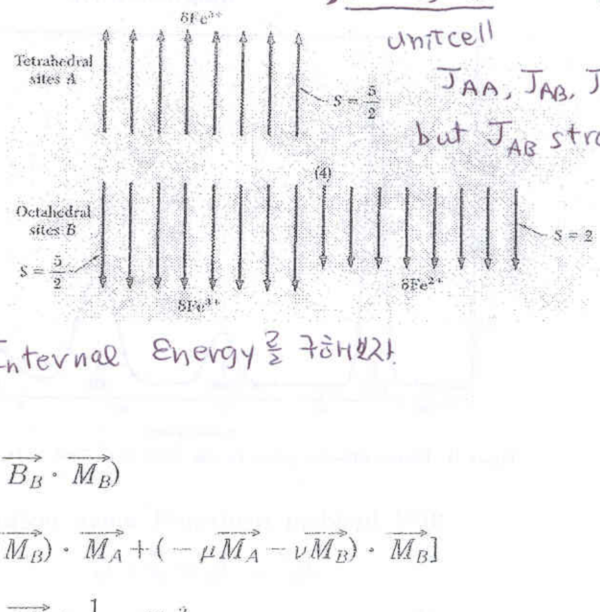
neutron diffraction이 이를 증명한다.

ferrites : $MO \cdot Fe_2O_3$

M : divalent cation Zn, Cd,
Fe, Ni, Cu, Co, Mg
ferrimagnet. poor conductor

Cubic ferrite의 (Fe_3O_4) 의 경우

unitcell



$J_{AA}, J_{AB}, J_{BB} < 0$

but J_{AB} strongest

Anti ferromagnetic interaction ⇒

Internal Energy를 구해보자

$$\vec{B_A} = -\lambda \vec{M_A} - \mu \vec{M_B}$$

$$\vec{B_B} = -\mu \vec{M_A} - \nu \vec{M_B}$$

interaction energy density

$$U = -\frac{1}{2}(\vec{B_A} \cdot \vec{M_A} + \vec{B_B} \cdot \vec{M_B})$$

$$= -\frac{1}{2}[(-\lambda\vec{M_A} - \mu\vec{M_B}) \cdot \vec{M_A} + (-\mu\vec{M_A} - \nu\vec{M_B}) \cdot \vec{M_B}]$$

$$= \frac{1}{2}\lambda M_A^2 + \mu \vec{M_A} \cdot \vec{M_B} + \frac{1}{2}\nu M_B^2$$

antiparallel 하고 $\mu M_A M_B > \frac{1}{2}(\lambda M_A^2 + \nu M_B^2)$ 이면

antiferroic 함

Curie Temperature and Susceptibility of Ferrimagnets

모든 interaction을 무시하고 A, B 사이의 interaction만 생각하자. (A,B는 anti ferromagnetic interaction 생각)

$M_A T = c_A(B_a - \mu M_B)$, $M_B T = c_B(B_a - \mu M_A)$

Nonzero solution of M_A and M_B

$$TM_A + \mu c_A M_B = c_A B_a$$

$$TM_B + \mu c_B M_A = c_B B_a$$

$$\therefore \begin{vmatrix} T & \mu c_A \\ \mu c_B & T \end{vmatrix} = 0$$

$$TM_A + \mu c_A M_B = c_A B_a$$

$$TM_B + \mu c_B M_A = c_B B_a$$

$$M_A = \frac{\begin{vmatrix} C_A B_a & \mu c_A \\ C_A B_a & T \end{vmatrix}}{\begin{vmatrix} T & \mu c_A \\ \mu c_B & T \end{vmatrix}}$$

$$T_c = \mu(c_A c_B)^{\frac{1}{2}}$$

$T > T_c$

Total은 ferromagnetic 하다.

$$\chi = \frac{M_A + M_B}{B_a} = \frac{1}{B_a}\left[\frac{c_A T B_a - \mu c_A c_B b_a + T c_B B_a - c - A B_a \mu c_B}{T^2 - \mu^2 c_A c_B}\right]$$

$$= \frac{(C_A + C_{B)T} - 2\mu C_A C_B}{T^2 - T_c^2}$$

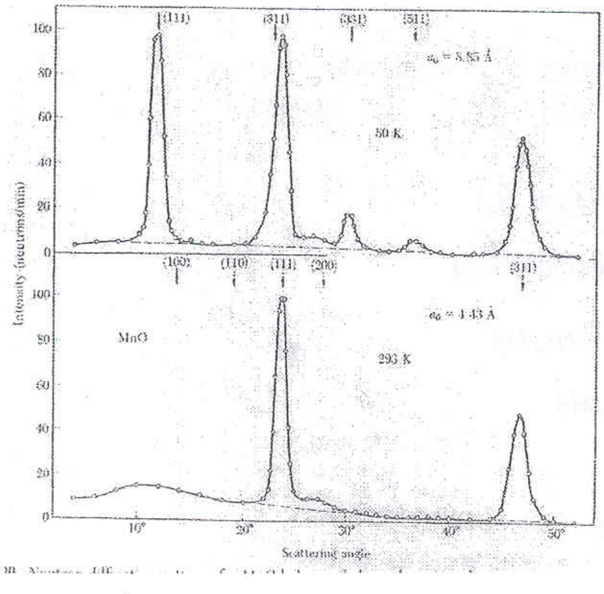
Iron Garnet

$M_3Fe_5O_{12}$: M trivalent metalion

Fe trivalent Fermion

Anti ferromagnetic ordering

예 MnO (NaCl 구조를 갖는다.)

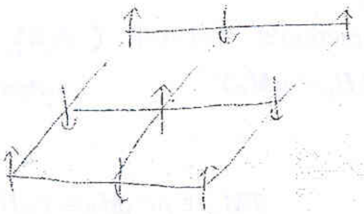


80K

a_0 = 8.85Å

293K

a_0 = 4.43Å



그러므로 lattice constant가
두배가 된다.

Neel temperature 아래에서는
spin ordering이 생겨난다.

Anti ferromagnetic은 AB가 서로 같은 saturation을 갖는다.

$$\chi = \frac{(C_A + C_B)T - 2\mu C_A C_B}{T^2 - T_C^2}$$

$$= \frac{CT - 2\mu C^2}{T^2 - (\mu C)^2} = \frac{2C(T - 2\mu C)}{(T - \mu C)(T + \mu C)} = \frac{2C}{T + \mu C} = \frac{2C}{T + T_n}$$

실험결과 $T > T_N$(Neel temperature)

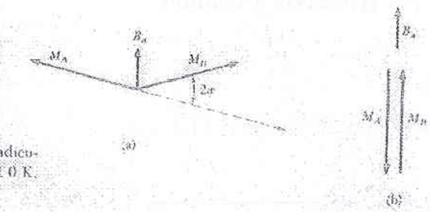
답$T > T_N$ $\chi = \dfrac{2C}{T + T_n}$

$$\chi = \frac{2C}{T + \theta}$$

아마 next-nearest neighborhood 고려해야 한다.

Susceptibility below the Neel temperature
두 가지 경우

Figure 24 Calculation of (a) perpendicular and (b) parallel susceptibilities at 0 K, in the mean field approximation.



perpendicular parallel

$|M_A| = |M_B| = M$ 거의 변화 없다.

$$u = \mu \vec{M}_A \cdot \vec{M}_b - B_a \cdot (\vec{M}_A + \vec{M}_b)$$

$$\simeq \mu M^2(-\cos 2\phi) - 2B_a M\cos(90^\circ - \phi)$$

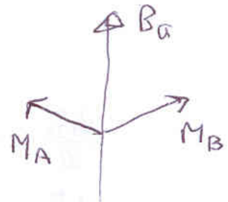
$$= \mu M^2\left(-1 + \frac{1}{2} \cdot 4\phi^2\right) - 2B_a M\phi$$

$$= -\mu M^2(1 - 2\phi^2) - 2B_a M\phi$$

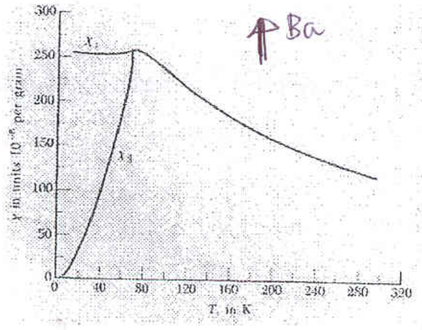
$$0 = \frac{du}{d\phi} = 4\mu M^2\phi - 2B_a M$$

$$\therefore \phi = \frac{B_a}{2\mu M}$$

$$\chi_\perp = \frac{2M\phi}{B_a} = \frac{2M}{B_a} \cdot \frac{B_a}{2\mu M} = \frac{1}{\mu}$$



Parallel case ↑↓↑↓



↑B_a

T=0 에서는 $\chi=0$

↑↓↑↓↑↓

Antiferromagnetic Magnon

ferromagnetic 경우와 비교

2p $s^z = s$

2p+1 $s^z = -s$

nearest nbd J is negtive

$$\frac{ds_p^x}{dt} = \frac{2Js}{\hbar}(2s_p^y - 2s_{p-1}^y - 2s_{p+1}^y)$$

$$\frac{ds_p^y}{dt} = -\frac{2Js}{\hbar}(2s_p^x - 2s_{p-1}^x - 2s_{p+1}^x)$$

$$\frac{ds_p^z}{dt} = 0$$

같은 방법으로

$$\frac{ds_{2p}^x}{dt} = \frac{2Js}{\hbar}(-2s_{2p}^y - 2s_{2p-1}^y - 2s_{2p+1}^y) \quad \text{---①}$$

$$\frac{ds_{2p}^y}{dt} = -\frac{2Js}{\hbar}(-2s_{2p}^x - 2s_{2p-1}^x - 2s_{2p+1}^x) \quad \text{---②}$$

$$\frac{ds_{2p+1}^x}{dt} = \frac{2Js}{\hbar}(2s_2\, 2p+1^y + 2s_{2p}^y + 2s_{2p+2}^y) \quad \text{---③}$$

$$\frac{ds_{2p+1}^y}{dt} = -\frac{2Js}{\hbar}(2s_{2p+1}^x + 2s_{2p}^x + 2s_{2p+2}^x) \quad \text{---④}$$

①과 ②에서 ① +② i.

$$\frac{ds^{+}_{2p}}{dt} = \frac{2iJs}{\hbar}\left(2s^{+}_{2p} + 2s^{+}_{2p-1} + 2s^{+}_{2p+1}\right)$$

$$\frac{ds^{+}_{2p+1}}{dt} = -\frac{2iJs}{\hbar}\left(2s^{+}_{2p+1} + 2s^{+}_{2p} + 2s^{+}_{2p+2}\right)$$

$$s^{1}_{2p} = u\exp(ipka - i\omega t), \qquad s_2\ p+1^{\,1} = v\exp(ipka - i\omega t)$$

$$\omega_{ex} = -4Js/\hbar = 4\,|J|\,s/\hbar$$

$$\omega u = \frac{1}{2}\,\omega_{er}(2u + ve^{-ika} + ve^{ika})$$

$$-\omega v = \frac{1}{2}\,\omega_{er}(2v + ue^{-ika} + ue^{ika})$$

$$\therefore \begin{vmatrix} \omega_{ex} - \omega & \omega_{ex}\cos ka \\ \omega_{ex}\cos ka & \omega_{ex} + \omega \end{vmatrix} = 0$$

$$\omega^2 = \omega_{ex}^2(1 - \cos^2 ka)$$

$\omega = \omega_{ex}\,|\sin ka\,|$ large region in which magnon freq in linear in wave vector

ferromagnet 인 경우와 매우 다르다. $\hbar\omega = 4Js(1 - \cos ka)$

다음의 topic은 too advanced