Chapter 10.

Plasmons , Polaritons , Polarons

Dielectric function of the electron gas

$\varepsilon(\omega, k)$ frequency와 wave vector 의 strong function 이다.

$\varepsilon(\omega, 0)$: collective excitations of the Fermi Sea

$\varepsilon(0, \mathrm{k})$: electrostatic screening of the electron − electron , electron − lattice , electron − impurity

Definitions of the Dielectric function

$$D = E + 4\pi P = \varepsilon E$$

$$\nabla \cdot D = 4\pi\rho_{\text{ext}} = \varepsilon E$$

$$\nabla \cdot E = 4\pi(\rho_{\text{ext}} + \rho_{\text{ind}})$$

k - space 에서

$D(\vec{k}) = \varepsilon(\vec{k})E(\vec{k})$ $\varepsilon(k)$ 를 define 한다.

$$\mathrm{div}\vec{E} = \mathrm{div}\sum E(\vec{k})\exp[i\vec{k}\cdot\vec{r}] = 4\pi\sum\rho(\vec{k})\exp[i\vec{k}\cdot\vec{r}]$$

$$\mathrm{div}\vec{D} = \mathrm{div}\sum\varepsilon(k)E(k)\exp[i\vec{k}\cdot\vec{r}] = 4\pi\sum\rho_{\text{ext}}(\vec{k})\exp[i\vec{k}\cdot\vec{r}]$$

$$\varepsilon(\vec{k}) = \frac{\rho_{\text{ext}}(\vec{k})}{\rho(\vec{k})} = \frac{\rho - \rho_{\text{ind}}(k)}{\rho(k)} = 1 - \frac{\rho_{\text{ind}}(k)}{\rho(k)}$$

$\rho = \rho_{\text{ext}} + \rho_{\text{ind}}$

electrostatic potential

φ_{ext} : $-\nabla\varphi_{\text{ext}} = \vec{D}$ satisfies Poisson equation $\nabla^2\varphi_{\text{ext}} = -4\pi\rho_{\text{ext}}$

$$\nabla^2\varphi = -4\pi\rho$$

$\nabla\varphi = -E$

$\nabla\varphi_{\text{ext}} = -D$ 라하면

$$\frac{\varphi_{\text{ext}}(\vec{k})}{\varphi(\vec{k})} = \frac{\rho_{\text{ext}}(\vec{k})}{\rho(\vec{k})} = \rho(\vec{k})$$

Plasma Optics

$\varepsilon(\omega, 0)$ *or* $\varepsilon(\omega)$ *long wave length*

Plasma Optics $k=0$

$$m\frac{d^2x}{dt^2}=-e\vec{E}$$

$$-\omega^2 mx-e\vec{E}$$

$$\therefore x=\frac{eE}{m\omega^2}$$

dipole moment of one electron :

$$-ex=-\frac{e^2E}{m\omega^2}$$

$$P=-nex=-ne\frac{eE}{m\omega^2}=-\frac{ne^2}{m\omega^2}E$$

The dielectric function at frequency ω is

$$\varepsilon(\omega)\equiv\frac{D(\omega)}{E(\omega)}=1+4\pi\frac{P(\omega)}{E(\omega)}$$

$$=1-4\pi\frac{ne^2}{m\omega^2}$$

$$Let\ \frac{4\pi ne^2}{m}\equiv\omega_P$$

$$=1-\frac{\omega_P{}^2}{\omega^2}$$

$$\therefore\quad \varepsilon(\omega)=1-\frac{\omega_P{}^2}{\omega^2}$$

만약 background 의 dielectric constant 가 $\varepsilon(\omega)$ 이면

$$\varepsilon(\omega)=\varepsilon(\infty)-\frac{4\pi me^2}{m\omega^2}=\varepsilon(\infty)[1-\frac{\widetilde{\omega}_P{}^2}{\omega^2}]\qquad \text{where}\quad \widetilde{\omega}_P{}^2=\frac{4\pi me^2}{\varepsilon(\infty)m}$$

Dispersion relation for Electromagnetic Waves

Nonmagnetic isotropic medium

$$\frac{\partial^2 D}{\partial t^2}=c^2\nabla^2E^2\qquad E\propto\exp[-i\omega t]\exp[i\vec{k}\cdot\vec{r}]$$

$$D=\varepsilon(\omega,k)E$$

$$\varepsilon(\omega,k)\omega^2=c^2k^2$$

- ε real and positive

  k is real, transverse electromagnetic wave가 $\frac{c}{\sqrt{\varepsilon}}$ 로 propagate

- ε real and negative

  w real , k imaginary

  wave 가 $1/|k|$ 로 damping 한다.

- ε complex w real, k는 complex, wave is damping in space.

· ε = ∞ system이 applied force가 없어도 finite response를 한다.

· ε = 0 longitudinally polarized wave가 가능

Transverse Optical modes in a Plasma

$$\varepsilon(\omega)\omega^2 = c^2k^2$$

$$\varepsilon(\infty)[1-\frac{\widetilde{\omega_P}^2}{\omega^2}]\omega^2$$

$$\therefore \varepsilon(\infty)[\omega^2 - \widetilde{\omega_P}^2] = c^2k^2$$

$\omega < \widetilde{\omega_P}$ 이면 $k^2 < 0$ $\therefore$ k는 *imaginary*

wave form 이 $e^{(-i|k|x)}$, $0 < \omega \le \widetilde{\omega_P}$

$\omega > \widetilde{\omega_P}$ 이면 *dielectric fct.*이 *real and positive*하다.

$$\omega^2 = \omega_P{}^2 + \frac{c^2k^2}{\varepsilon(\infty)}$$

propagate in free space

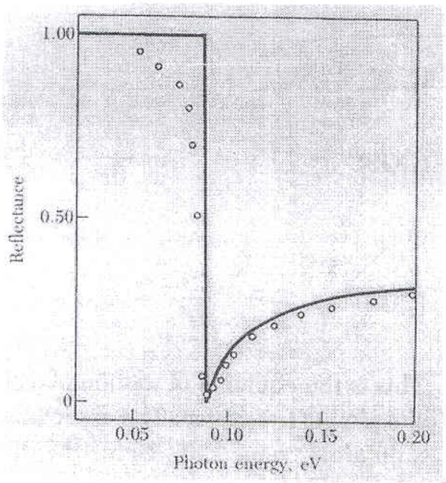
otherwise reflected.

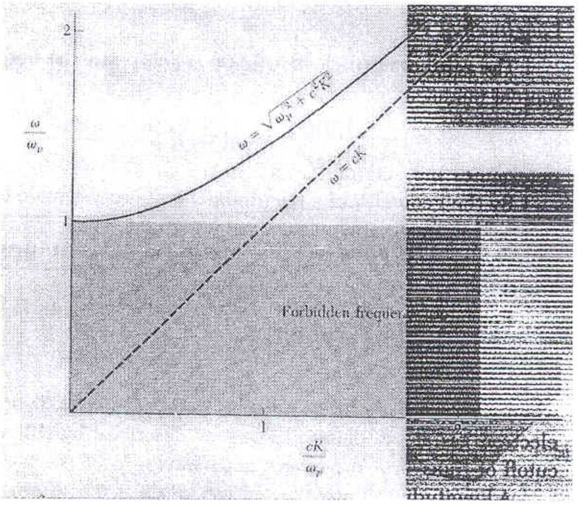
Transparancy of Alkali Metals in the UV

Wood 와 Zener

(발전) (증명)

InSb $n = 4*10^8 cm^{-3}$





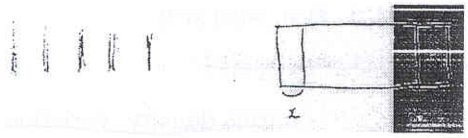
:plasma freq.가 0.09eV.

Longitudinal Plasma Oscillations

$\varepsilon(\omega_L)=0 \quad \Rightarrow$ *longitudinal modes of Oscillation* 결정 $\;(\because\; k=0)$

$$E=-4\pi P(\because D\equiv E+4\pi P=0)$$

Electric field 와 *polarization* 이 같은 방향



$$E = 4\pi\sigma = 4\pi neu$$

electron gas 의 concentration

$$nm\frac{d^2u}{dt^2}=-neE=-ne(4\pi neu)$$
$$=-4\pi n^2e^2u$$

$$\frac{d^2u}{dt^2}+\omega_P{}^2u=0 \qquad \omega_P^2=\frac{4\pi ne^2}{m}\quad (CGS)$$

$$\text{where}\quad \omega_P=\sqrt{\frac{ne^2}{\varepsilon_0 m}}\quad \text{in SI unit}$$

Plasma oscillation of small wave vector

$$\varepsilon(\omega)\omega^2=\varepsilon(\infty)(\omega^2-\omega_P{}^2)=c^2k^2$$

$\omega \sim \omega_P \quad \varepsilon(\omega)\sim 0$ 에서 *expansion*

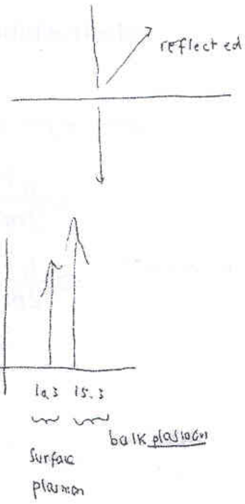
$$\omega^2=\omega_P{}^2+c^2k^2$$

$$\omega^2=\omega_P{}^2\left(1+\frac{c^2k^2}{\omega_P{}^2}\right)$$

$$\omega=\omega_P\left(1+\frac{c^2k^2}{2\,\omega_P{}^2}+\circ\circ\circ\right)$$

$$=\omega_P+\frac{c^2k^2}{2\omega_P\varepsilon(\infty)}=\omega_P+\frac{c^2k^2}{2\omega_P}=\omega_P\left(1+\frac{c^2k^2}{2\,\omega_P{}^2}\right)$$

$$\omega_P=\frac{4\pi ne^2}{m}$$

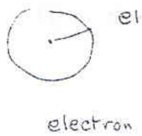


Plasmon : plasma oscillation 의 Quantum

thin film을 놓고 plasmon를 excite 시킬 수 있다.

thin film: 에너지 loss가 plasmon energy의 정수배이다.

Electrostatic Screening



electron $-n_0 e$ 와 ion $n_0 e$ 가 uniform하게 있다고 하고 시작하자.

Poisson equation에 의해 $$\nabla^2 \varphi = -4\pi\rho$$

$$k^2 \varphi(k) = 4\pi\rho(k)$$

Thomas - Fermi approximation

total chemical potential of the electron gas is constant in equilibrium , independent of position

$$\mu_0 = \varepsilon_F{}^0(x) = \frac{\hbar^2}{2m}(3\pi^2 n_0)^{2/3}$$

electrostatic potential $\varphi(x)$ 가 있으면

$$\begin{aligned} \mu = \ & \varepsilon_F - e\varphi(x) \\ & \cong \frac{\hbar^2}{2m}[3\pi^2 n(x)]^{2/3} - e\varphi(x) \\ & \cong \frac{\hbar^2}{2m}[3\pi^2 n_0]^{2/3} \end{aligned}$$

$\varepsilon_F(x)$ is the local value of the Fermi energy.

$$e\varphi(x) = \frac{\hbar^2}{2m}\{(3\pi^2 n(x))^{\frac{2}{3}} - (3\pi^2 n_0)^{\frac{2}{3}}] \quad ㉠$$

$$= \frac{\hbar^2}{2m}(3\pi^2)^{\frac{2}{3}} * [n(x)^{\frac{2}{3}} - n_0^{\frac{2}{3}}] \quad ㉡$$

㉠: $n(x)^{2/3} = (n - n_0 + n_0)^{2/3}$

$$= n_0^{2/3}(1 + \frac{n - n_0}{n_0})^{2/3}$$

$$\cong n_0^{2/3}[1 + \frac{2}{3}\frac{n - n_0}{n_0}]$$

㉡: $e\varphi(x) = \frac{\hbar^2}{2m}(3\pi^2)^{2/3}\frac{2}{3}n_0^{2/3}\frac{n - n_0}{n_0}$

$$= \frac{2}{3}\frac{\hbar^2}{2m}(3\pi^2 n_0)^{2/3}\frac{n - n_0}{n_0}$$

$$\because \varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n_0)^{\frac{2}{3}}$$

$$e\varphi(x) = \frac{2}{3}\varepsilon_F\frac{n - n_0}{n_0}$$

$$\therefore n - n_0 \cong \frac{3}{2}n_0\frac{e\varphi(x)}{\varepsilon_F}$$

induced part of electron concentration

$$\rho_{\mathrm{ind}}(\mathrm{k}) = -\frac{3\mathrm{n}_0\mathrm{e}^2}{2\varepsilon_\mathrm{F}}\varphi(\mathrm{k})$$

$$= -\frac{3\mathrm{n}_0\mathrm{e}^2}{2\varepsilon_\mathrm{F}}\frac{4\pi}{\mathrm{k}^2}\rho(\mathrm{k})$$

$$= -\frac{6\mathrm{n}_0\mathrm{e}^2\pi}{\varepsilon_\mathrm{F}\mathrm{k}^2}\rho(\mathrm{k})$$

$$\therefore \varepsilon(0,\mathrm{k}) = 1 - \frac{\rho_{\mathrm{ind}}(\mathrm{k})}{\rho(\mathrm{k})} = 1 + \frac{6\mathrm{n}_0\mathrm{e}^2\pi}{\varepsilon_F\mathrm{k}^2}$$

$$= 1 + \frac{k_S^2}{k^2} \qquad where \qquad k_S^2 = \frac{6n_0 e^2\pi}{\varepsilon_F}$$

$\varepsilon(0,k)$ 를 Thomas - Fermi dielectric function 이라 하고

$1/k_S$ 를 Thomas Fermi screening length 라 한다.

Copper : $n_0 = 8.5\times10^{22}cm^{-3}$ 일 때 screening length가 0.55Å 이다.

$$\varepsilon(0,k) = 1 + \frac{k_S^2}{k^2} \quad : \quad \varepsilon(\omega,0) = 1 - \frac{\omega_P^2}{\omega^2}$$

Full theory $\varepsilon(\omega,k)$를 Lindbard 가 derive

Screened Coulomb Potential

$\nabla^2\varphi_0 = -4\pi q\delta(\mathrm{r})$ 일 때 $\varphi_0 = \frac{\mathrm{q}}{\mathrm{r}}$ 임을 알고 있다.

구하는 법

$$\varphi_0 = (2\pi)^{-3}\int d\vec{k}\,\varphi_0(\vec{k})\exp[i\vec{k}\cdot\vec{r}]$$

$$\delta(r) = (2\pi)^3\int d\vec{k}\,\exp[i\vec{k}\cdot\vec{r}]$$

$$\therefore \nabla^2\varphi_0 = \int -k^2\varphi_0(\vec{k})$$

$$= -\int 4\pi q$$

$$k^2\varphi_0(\vec{k}) = 4\pi q$$

$$\nabla^2\varphi_0 = -4\pi\frac{p_{ext}}{\varepsilon}$$

$$= -4\pi\frac{q\delta(r)}{\varepsilon}$$

$$\varphi_{ext} = \frac{4\pi q}{k^2}$$

$$\frac{\varphi_{ext}(k)}{\varphi(k)} = \varepsilon(k) \qquad \varphi_{ext}(\vec{k}) = \frac{4\pi q}{k^2} \qquad \varphi(k) = \frac{4\pi q}{k^2\left(1+\frac{k_s^2}{k^2}\right)}$$

따라서 $\varphi(k) = \frac{\varphi_{ext}(k)}{\varepsilon(k)}$

$$= \frac{\varphi_{ext}(k)}{1+\frac{k_s^2}{k^2}}$$

$$= \frac{k^2\varphi_{ext}(k)}{k^2+k_s^2}$$

$$= \frac{4\pi q}{k^2+k_s^2}$$

$$\varphi(r) = (2\pi)^{-3}\int d\vec{k}\,\varphi(k)\exp[i\vec{k}\cdot\vec{r}]$$

$$= \frac{1}{(2\pi)^3}\int\frac{4\pi q}{k^2+k_s^2}k^2dk\exp[ikr\cos\theta]\sin\theta d\theta d\phi$$

$$= \frac{4\pi q}{(2\pi)^3}\int\frac{k^2 2\pi}{k^2+k_s^2}dk\int_{-1}^{1}d(\cos\theta)\exp[ikr\cos\theta]$$

$$= \frac{4\pi q}{(2\pi)^2}\int\frac{k^2dk}{k^2+k_s^2}\,\frac{\exp[ikr]-\exp[-ikr]}{2ikr}\,2$$

$$= \frac{8\pi q}{(2\pi)^2 r}\int_0^\infty\frac{k}{k^2+k_s^2}\sin krdk$$

$$= \frac{2q}{\pi r}\int_0^\infty dk\frac{k\sin kr}{k^2+k_s^2} = \frac{2q}{\pi r}\int_{-\infty}^{\infty}dkk\frac{\exp[ikr]\cancel{-\exp[-ikr]}}{2i}\,\frac{1}{(k+k_si)(k-k_si)}$$

$$= \frac{2q}{\pi r} 2\pi i(Residue)$$

$$= \frac{2q}{r} \exp[-k_s r]$$

Pseudopotential component

K가 작으면 potential 은 $-\frac{2}{3}E_F$

39식 $\varphi(\ ) = \frac{4\pi q}{k_s^2}$

전자 charge e

metal valency z

n_0 i ons per unit volume

$$U(0) = -ezn_0\varphi(0)$$

$$= -ezn_0\frac{4\pi e}{k_s^2}$$

$$k_s^2 = \frac{6\pi n_0 ze^2}{\varepsilon_F}$$

$$= -\frac{4\pi e^2 zn_0}{k_s^2} = -\frac{4\pi e^2 zn_0}{6\pi n_0 e^2 z}\varepsilon_F$$

$$= -\frac{2}{3}\varepsilon_F$$

Mott의 Metal -Insulator Transition

수소원자 하나가 primitive cell 속에 있다면

half filled energy band가 되어 metal이 될 것이다.

그러나 수소 분자 하나가 primitive cell 속에 있다면

두 개 filled ∴ Insulator

매우 pressure 가 높다면 수소는 metal이 될 것이다.

수소가 온도가 zero일 때 Metal인가 Insulator인가? lattice constant의 함수이다.

Mott의 criteria

a가 작으면 metal

a가 크면 Insulator

from conducting side $a_c = 4.5a_0$

Screened potential 에서 출발을 하자.

$$U(r) = -\frac{e^2}{r}\exp[-k_s r]$$

k_s가 커지면 U --> 0

metal이 된다.

bound state가 있을 수 있는 조건

$$k_s < \frac{1.19}{a_0} \qquad k_s = 4(\frac{3}{\pi})^{\frac{1}{3}} n_0^{\frac{1}{3}}/a_0$$

$$\sqrt{4(\frac{3}{\pi})^{\frac{1}{3}} n_0^{\frac{1}{3}}/a_0} < \frac{1.19}{a_0}$$

초전도연구단
단장 이 성 익

다시쓰면

$$3.939\frac{n_0^{\frac{1}{3}}}{a_0} < \frac{1.42}{a_0^2}$$

$n_0 = \frac{1}{a^3}$ *simple cubic* $\qquad a_c > 2.78a_0$ *simple cubic*

Mott의 결과 $4.5a_0$와 차이가 거의 없다.

Metal - insulator transition 은

composition , pressure, strain, magnetic field 에 따라 다르다.

metallic phase는 independent electron model로 이해

insulaor phase : electron-electron interaction으로 해석한다.

randomly occupation : percolation theory

impurity (or composition)

dopping에 의해 metallic phase가 된다.

Observed value of the critical concentration in Si : p

$$n_c = 3.74\times 10^{18} cm^{-3}, \quad n_c = \frac{1}{l^3}$$

$r = 4\times 10$ (7) *Mott의 criteria* $\quad 4.5a_0$

$$r^3 = \frac{3}{4\pi n_c} = \frac{1}{15}\times 10^{-8} = 6\times 10^{-7} cm$$

Mott의 criteria

$$n_c = \frac{1}{a_c^3} = 0.33\times 10^{18} cm^{-3}$$ degenerate semiconductor

실험값보다 매우 작다 Semiconductor는

degenerate semiconductor

heavily doped.

Screening and Phonon in metal

For longitudinal mode , dielectric fct. is zero.

$$\varepsilon_{el}(\omega, k) = 1 + \frac{k_s^2}{k^2}$$

Ion이 moving indep. sound velocity

$$\varepsilon(\omega, k) = 1 - \frac{4\pi ne^2}{m\omega^2} + \frac{k_s^2}{k^2}$$

plasma $\varepsilon(\omega, 0)$ 사용

k과 w 가 작을 때는 1은 제외 $\varepsilon(\omega, k)=0$ $\quad \omega^2 = \dfrac{4\pi n e^2 k^2}{m\, k_s^{\,2}}$

$$\varepsilon_F = \frac{1}{2} m\, v_F^{\,2}$$

$$\omega^2 = \frac{4\pi n e^2 k^2}{M\, k_s^{\,2}} = \frac{4\pi n e^2}{M}\,\frac{\varepsilon_F}{6\pi n e^2}\,k^2 = \frac{m}{3M}\, v_F^{\,2} k^2$$

or $\omega = vk$ $\quad$ *where* $v = (\frac{m}{3M})^{\frac{1}{3}} v_F$

long wavelength of acoustic phonon

실험 : Akakli metal 에서는 longitudinal wave velocity와 일치

High frequency 에서는 $\quad \varepsilon \sim 1 - \dfrac{\omega_p^{\,2}}{\omega^2}$

$$\therefore \varepsilon(\omega, 0) = 1 - \frac{4\pi n e^2}{M\omega^2} - \frac{4\pi n e^2}{m\omega^2}$$

$$\varepsilon = 0 \qquad 0 = 1 - \frac{4\pi n e^2}{1}\left(\frac{1}{M} + \frac{1}{m}\right)\frac{1}{\omega^2}$$

$$\therefore \omega^2 = \frac{4\pi n e^2}{\mu} \quad where \quad \frac{1}{\mu} = \frac{1}{M} + \frac{1}{m}$$

Polariton

transverse optical phonon이 transverse electromagnetic wave와 intercaction 한다. coupling 하여 resonance 근처에서는 propagation characteristics 가 totally 변한다. forbidden band까지 나타난다.

w, k 가 비슷한 곳에서 두개가 합쳐진다.

이렇게 phonon과 photon이 합쳐지는 것을 polariton이라 한다.

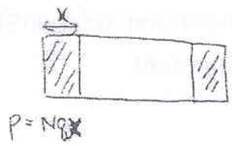
$$\omega(photon) = ck(photon) = \omega(phonon) \approx 10^{13} s^{-1} 1 : \quad k \approx 300\, cm^{-1}$$

electric field E of photon, dielectric polarization Pp of TO phonon

$$c^2k^2E = \omega^2(E + 4\pi P) \qquad \frac{\partial^2 D}{\partial t^2} = c^2\nabla^2 E \qquad \therefore (c^2k^2 - \omega^2)E - 4\pi\omega^2 P = 0$$

P = Nqu

Harmonic oscillator 처럼 작용하니까 P = Nqu



$E = 4\pi\ Nqx$

$$M\frac{d^2x}{dt^2} = 4\pi Nq^2x + qE$$

$$\omega_p^2 = \frac{4\pi Ne^2}{M} \qquad\qquad \frac{d^2x}{dt^2} = \frac{4\pi Ne^2}{M}x + \frac{qE}{M}$$

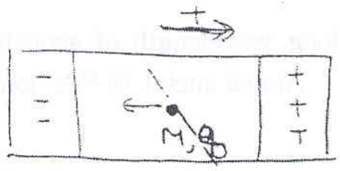
$$-\omega^2 x = \omega_{\cancel{p}\,T}^2 x + \frac{eE}{M}$$

$$-\omega^2 P = \omega_{\cancel{p}}^2 P + \frac{Nq^2}{M}E \qquad\qquad (x = \frac{P}{Nq})$$

Positive ion이 Negative ion에 대해 얼마만큼 움직이나?

Spring에 붙어있다. q : positive charge

$$M\frac{d^2x}{dt^2} + kx = qE$$



$$-M\omega^2 x + kx = qE$$

$$P = Nqx$$

$$-M\omega^2\frac{P}{Nq} + k\frac{P}{Nq} = qE \quad \Rightarrow \quad -\omega^2 P + \frac{k}{\cancel{Nq}}\cdot\frac{\cancel{Nq}}{M}P = \frac{Nq^2}{M}E$$

$$\frac{Nq}{M}$$

$$\therefore \omega^2 P + (-\frac{k}{M}P) = \frac{Nq^2E}{M}$$

$$(\omega^2 - \omega_T^2)P = \frac{Nq^2E}{M} \quad \Rightarrow \quad (\omega^2 - \omega_T^2)P - \frac{Nq^2}{M}E = 0$$

$$\therefore \quad \begin{vmatrix} c^2k^2 - \omega^2 & -4\pi\omega^2 \\ \frac{Nq^2}{M} & \omega_T^2 - \omega^2 \end{vmatrix} = 0$$

Polariton의 dispersion relation이 나온다.

dielectric constant

$$\varepsilon(\omega) = 1 + \frac{4\pi P}{E} = 1 - \frac{4\pi \frac{Nq^2}{M}}{\omega^2 - \omega_T^{\ 2}}$$

$$= 1 + \frac{4\pi Nq^2/M}{\omega_T^{\ 2} - \omega^2}$$

frequency zero 에서 *IR* 까지는

$$\varepsilon(\omega) = \varepsilon(\infty) + \frac{4\pi Nq^2/M}{\omega_T^{\ 2} - \omega^2}$$

$$\varepsilon(0) = \varepsilon(\infty) + \frac{4\pi \frac{Nq^2}{M}}{\omega_T^{\ 2}} \quad = \varepsilon(\infty) + \frac{\omega_L^{\ 2}}{\omega_T^{\ 2}}$$

$$\therefore \varepsilon(\omega) = \varepsilon(\infty) + \frac{\omega_T^{\ 2}}{\omega_T^{\ 2} - \omega^2}[\varepsilon(0) - \varepsilon(\infty)]$$

$$= \frac{\omega_T^{\ 2}\varepsilon(0) - \omega^2\varepsilon(\infty)}{\omega_T^{\ 2} - \omega^2}$$ 직접 계산

OR

$\varepsilon = 0$ 일때의 ω를 ω_L 이라 하므로

$$\varepsilon(\infty) + \frac{4\pi Nq^2/M}{\omega_T^{\ 2} - \omega^2} = 0$$

$$4\pi \frac{Nq^2}{M} = \varepsilon(\infty)(\ \omega_L^{\ 2} - \ \omega_T^{\ 2})$$

Lyddane - Sachs - Teller relation

$$\therefore \quad \varepsilon(\omega) = \frac{\omega_T^{\ 2}\varepsilon(0) - \omega^2\varepsilon(\infty)}{\omega_T^{\ 2} - \omega^2} = \varepsilon(\infty)\frac{\omega_L^{\ 2} - \omega^2}{\omega_T^{\ 2} - \omega^2}$$

ω_T와 ω_L사이에서는 ε이
negative number 가 된다.
k 가 imaginary for real w
exp[ikx] --
--> exp[- |k|x]

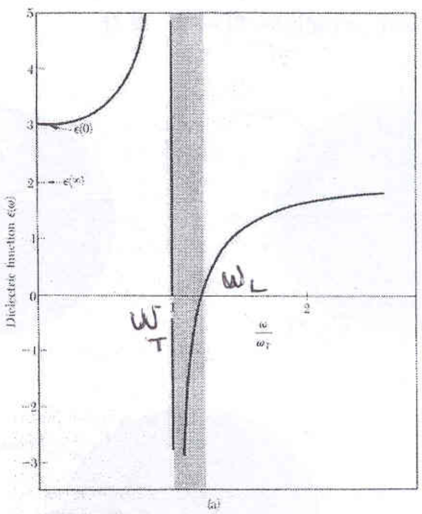


Figure 13a Plot of $\epsilon(\omega)$ from (60) for $\epsilon(\infty) = 2$ and $\epsilon(0) = 3$. The dielectric constar between $\omega = \omega_T$ and $\omega_L = (3/2)^{1/2}\omega_T$; that is, between the pole(infinity) of $\epsilon(\omega)$ and th Incident electromagnetic waves of frequencies $\omega_T < \omega < \omega_L$ will not propagate in the will be reflected at the boundary.

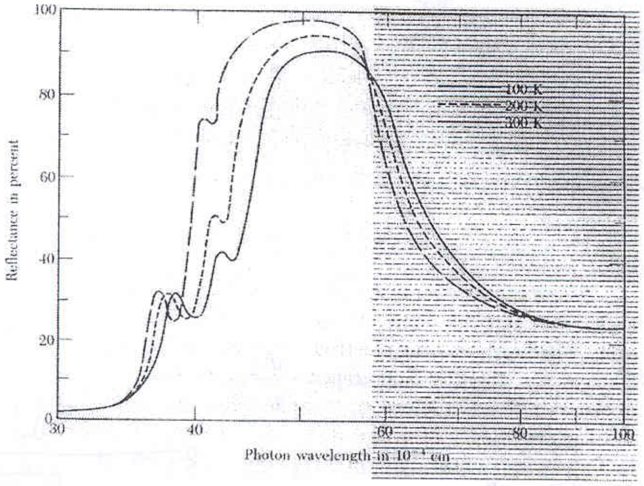
LST relation

$$\frac{\omega_L^{\ 2}}{\omega_T^{\ 2}} = \frac{\varepsilon(0)}{\varepsilon(\infty)}$$

⟹ Lyddane-Sachs-Teller relation

$\omega_T \to 0$ *as* $\varepsilon \to \infty$

:characteristics of ferroelectricity



$\omega_T < \omega < \omega_L$ 에서는 reflectivity가 크다. (∵ 통과 못하므로)

Electron - electron interaction

Fermi liquid : interaction 이 있을 때를 Fermi liquid라 한다.

electron이 주위에 있는 electrostatic force에 의해 움직일 때 mass가 달라진다.

Landau theory of Fermi liquid

Single electron이 excited 되어 있을 때 quasi particle이라 한다.

quasi particle 은 electron gas 속에서 cloud를 끌고 다닌다.

alkali metal은 order 1 증가

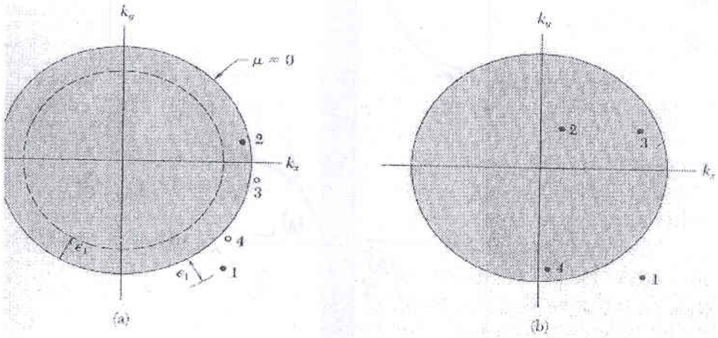
Electron

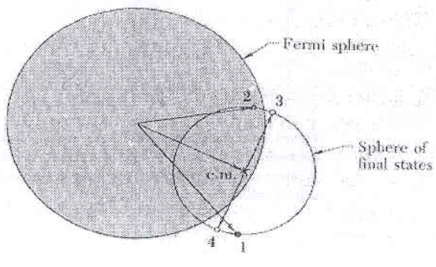
전자가 서로 2Å떨어져 있어도 mean free path는 10^4Å보다 더 크다. (room temp.) 10 cm at 1k

가장 powerfull한 factor는 exclusion principle이다.

두 번째 factor는 screening of coulomb interaction

exclusion principle 하나만 생각





$\varepsilon_1, \varepsilon_2$: Fermi energy를 중심으로 생각한다.

ε_1 *positive*

ε_2 *negative*

$\varepsilon_1 > |\varepsilon_2|$ 이어야 밖에서 두개가 생겨난다.

ε_1 입장 $\frac{\varepsilon_1}{\varepsilon_F}$ 정도의 *fraction* 이 밖에 있다.

ε_2 에서 *conservation of energy* $\frac{\varepsilon_1}{\varepsilon_F}$

두 개의 fraction

$$(\frac{\varepsilon_1}{\varepsilon_F})^2 \sim (2\times10^{-5})^2 \sim 4\times10^{-10}$$

thermal distribution $\varepsilon_1 \sim K_B T$

electron - electron collision

$$\sigma \approx (K_B \frac{T}{\varepsilon_F})^2 \sigma_0$$ Scattering Cross Section

σ_0 : *electron — electron interaction*에 의한 *cross section* 이다.

Room temp.

$$(\frac{K_B T}{\varepsilon_F}) \sim 10^{-2}$$

$$\sigma \sim 10^{-4}\sigma_0 \sim 10^{-19} cm^{-2}$$

$$l \simeq \frac{1}{ns} \sim 10^{-4} cm \quad \textit{at room temp.}$$

electron phonon collision 보다 10 배 정도 크다.

liquid temp. 온도에서 $\sigma \sim T^2$ 임이 발견되다.

Electron - Phonon Interaction : Polarons

0 ℃ $\rho \sim$ 1.55 microhm - cm

100℃ 2.28 microhm - cm

electrons are scattered by the phonons

온도가 높으면 phonon이 더 scattering 된다.

$T > \theta_D$ *phonon* 의 수 $\propto T$

저항이 T에 *linear* 하게 변화

electron - phonon interaction은 electron이 heavy ion core를 끌어 들인다.

electron과 heavy ion core가 서로 잡아 다닌다.

ionic crystal 일 때 이 effect가 더 크다.

electron - lattice interaction 을 dimensionless coupling constant α로 표시

$$\alpha = \textit{deformation energy}/\hbar\omega_L$$

α는 ionic crystal에서는 크고

covalent crystal에서는 작다.

band effective mass m^* 를 m^*_{pol} 에서 계산

Peierls Instability ; G= $2k_f$의 lattic deformation에 instable한다.

one - D 일 때 $\frac{d}{d\triangle}(E_{elastic} + E_{electronic}) = 0$ $E_{elastic} = \frac{1}{2} c\triangle^2 \langle \cos^2 2k_f x \rangle$

$= \frac{1}{4} c\triangle^2$

$U(x) = 2A\triangle\cos(2k_F x)$

$$\varepsilon_k = \frac{\hbar^2}{2m}(k_f^2 + k^2) \pm 4\left(\frac{\hbar^2 k_f^2}{2m}\frac{\hbar^2 k^2}{2m} + A^2\triangle^2\right)^{\frac{1}{2}}$$

$$\frac{d\varepsilon_k}{d\triangle} = \frac{-A^2\triangle}{(x_f x_k + A^2\triangle^2)^{\frac{1}{2}}} \Rightarrow \frac{dE_{electronic}}{d\triangle} = \frac{2}{\pi}\int_0^{k_F} \frac{d\varepsilon_k}{d\triangle} d$$

$$= -2A^2\triangle\pi \int_0^{k_F} \frac{dk}{\sqrt{x_F x_k + \triangle^2 A^2}}$$

$$|A|\triangle \simeq 4\omega \exp\left[-\frac{1}{N(0)V}\right]$$

BCS 결과와 같다