

Chapter 8

Semiconductor Crystals

semiconductor crystals

$$\rho : 10^{-7} \sim 10^{10} \text{ ohm} \cdot \text{cm}$$

온도의 함수 (strongly depend on temperature)

[insulator $\rho > 10^{14} \text{ ohm} \cdot \text{cm}$ 라 하자.]

Detector : transistor, switches, diode,
photovoltaic cell, detector, thermistors

III ~ V indium antimonide

galium arsenide

II ~ VI zinc sulfide

cadmium sulfide

IV silicon germanium

Diamond는 insulator,

Silicon과 Germanium은 semiconductor but: Diamond structure

Highly purified semiconductor

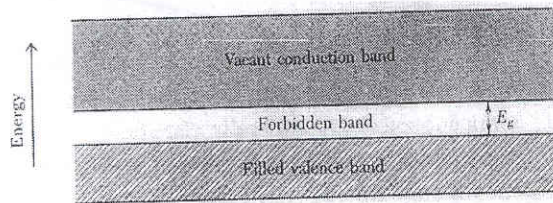
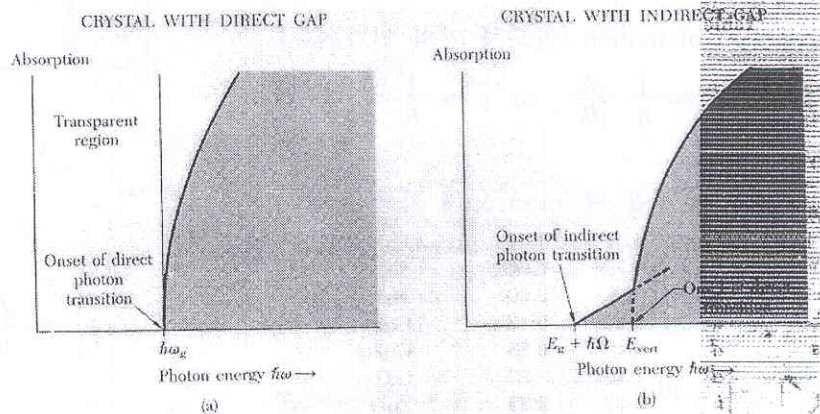


Figure 2 Band scheme for intrinsic conductivity in a semiconductor. At 0 K the conductivity is zero because all states in the valence band are filled and all states in the conduction band are vacant. As the temperature is increased, electrons are thermally excited from the valence band to the conduction band, where they become mobile.

intrinsic
temperature
range에서 pure

Band gap

E_g/kT : intrinsic
conductivity, intrinsic
carrier concentration

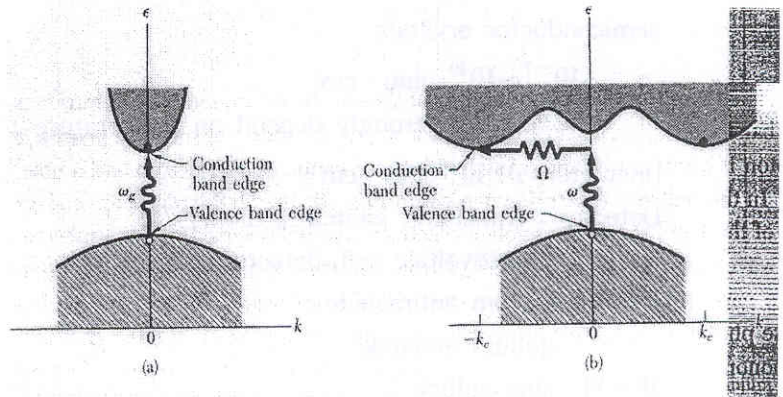


① 0K에서는 phonon이 주변에 없어서 생성에너지가 소모된다.

[phonon 생성에너지: $\hbar \Omega$]

threshold energy:

$$\hbar \omega = E_g + \hbar \Omega$$



Band gap energy E_g

① 온도가 높은 경우에는 phonon이 이미 존재한다. threshold 에너지는 $\hbar \omega = E_g - \hbar \Omega$

InSb를 사용한 optical absorption [그림]

- ① Conductivity - 온도 의존도를 측정
- ② Carrier concentration은 Hall voltage로 잰다.
- ③ Optical measurement : directly or indirectly

Equation of motion [반도체 안의 전자운동]

$$v_g = \frac{d\omega}{dk} = \frac{1}{\hbar} \frac{d\epsilon}{dk} \quad \text{or} \quad \vec{v} = \frac{1}{\hbar} \nabla_k \epsilon$$

e-field가 걸렸을 때, electron에 가해진 일

$$\delta\epsilon = -eE v_g \delta t \quad -eE v_g \delta t = \hbar v_g \delta k$$

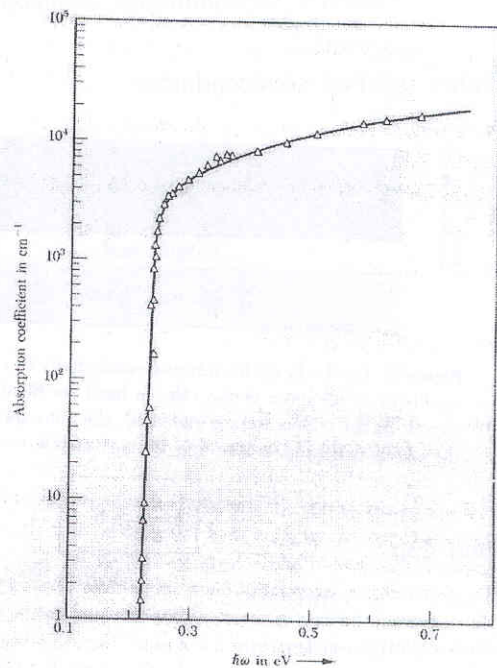
$$= \frac{d\epsilon}{dk} \delta k$$

$$\therefore \delta k = -\frac{eE}{\hbar} \delta t$$

$$\hbar \frac{d\vec{k}}{dt} = \vec{F}$$

$$= \hbar v_g \delta k$$

$$\hbar \frac{\delta k}{\delta t} = -eE$$



Magnetic field가 있으면

$$\hbar \frac{d\vec{k}}{dt} = -\frac{e}{c} \vec{v} \times \vec{B}$$

$$\therefore \frac{d\vec{k}}{dt} = -\frac{e}{\hbar^2 c} \nabla_{\vec{k}} \epsilon \times \vec{B}$$

$$= -\frac{e}{c} \frac{1}{\hbar} \nabla_{\vec{k}} \epsilon \times \vec{B}$$

\therefore Electron moves on a surface of constant energy [$\vec{k} \perp \nabla_{\vec{k}} \epsilon$]
& on a plane normal to \vec{B} [$\vec{k} \perp \vec{B}$]

Physical derivation of $\hbar \frac{d\vec{k}}{dt} = \vec{F}$

eigen value: $\epsilon_{\vec{k}}$, eigen vector: \vec{k}

\vec{J} : change of momentum

electron 하나만 있다.

conduction electron이 lattice와 부딪힌다.

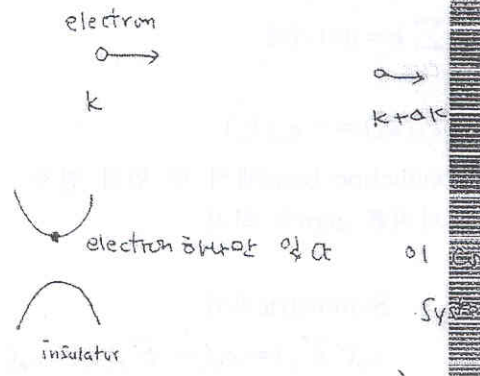
Bloch eigen function $\Psi_{\vec{k}}$

$$\Psi_{\vec{k}} = \sum_{\vec{G}} C(\vec{k} + \vec{G}) \cdot \exp[i(\vec{k} + \vec{G}) \cdot \vec{r}]$$

$$\vec{P}_{el} = \langle \vec{k} | -i\hbar \nabla | \vec{k} \rangle$$

$$= \sum_{\vec{G}} \hbar (\vec{k} + \vec{G}) |C(\vec{k} + \vec{G})|^2$$

$$= \hbar (\vec{k} + \sum_{\vec{G}} \vec{G} |C(\vec{k} + \vec{G})|^2)$$



초전도연구단 단장 이성익

impulse에 의해 system (crystal의 momentum change는 electron에 의해)

$$\vec{J} = \Delta \vec{P}_{tot} = \Delta \vec{p}_{el} = \hbar \Delta \vec{k}$$

Conduction electron은 crystal lattice의 periodic potential에 의해

$$\vec{J} = \Delta \vec{P}_{tot} = \Delta \vec{P}_{latt} + \Delta \vec{P}_{el}$$

$$\nabla \vec{P}_{el} = \hbar \nabla \vec{k} + \sum_{\vec{G}} \hbar \vec{G} [\nabla_{\vec{k}} |C(\vec{k} + \vec{G})|^2 \cdot \Delta \vec{k}]$$

Momentum transfer to the lattice. [by momentum conservation]

$\hbar \vec{k}$ 가 가다 $\hbar (\vec{k} + \vec{G})$ 가 되면 lattice는 momentum $-\hbar \vec{G}$ 얻는다.

$$\nabla \vec{P}_{latt} = -\hbar \sum_{\vec{G}} \vec{G} [\nabla_{\vec{k}} |C(\vec{k} + \vec{G})|^2 \cdot \Delta \vec{k}]$$

$$\nabla_{\vec{k}} |C(\vec{k} + \vec{G})|^2 \cdot \Delta \vec{k}$$

total momentum change는

$$\Delta \vec{P}_{el} + \Delta \vec{P}_{latt} = \vec{J} = \hbar \Delta \vec{k} \quad \text{By the definition of } \vec{J} ; \hbar \frac{d\vec{k}}{dt} = \vec{F}$$

Holes

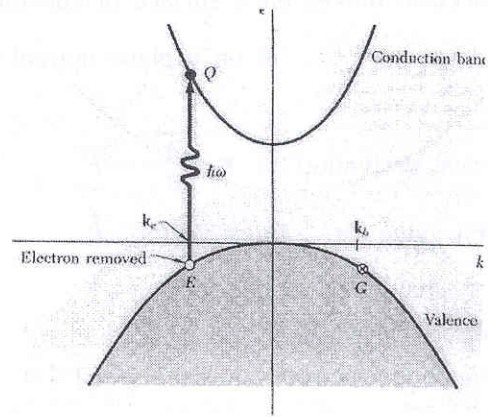
Semiconductor에서 vacant orbital은 매우 중요 \Rightarrow called Holes
Hole은 E field나 B field에 대해 positive charge처럼 행동한다.

$$1. \quad \vec{k}_h = -\vec{k}_e$$

$$\because \sum k = 0 \text{이기에}$$

$$2. \quad \varepsilon_h(k_h) = -\varepsilon_e(k_e)$$

Conduction band에서 맨 위의 전자 에너지를 zero로 하자.



Symmetric하면

$$\varepsilon_e(\vec{k}_e) = \varepsilon_e(-\vec{k}_e) = -\varepsilon_h(-\vec{k}_e) = -\varepsilon_h(\vec{k}_h)$$

$$3. \quad \vec{v}_h = \vec{v}_e$$

$$\nabla_h \varepsilon_h(k_h) = -\nabla_e \varepsilon_e(k_e) = -\nabla_e \varepsilon_e(-k_e)$$

$$= -\nabla_e(-\varepsilon_e(-k_e)) = \nabla_e \varepsilon_e(-k_e) = \nabla_e \varepsilon_e(k_e)$$

$$\therefore \nabla_h \varepsilon_h(k_h) = \nabla_e \varepsilon_e(k_e)$$

$$\vec{v}_h(\vec{k}_h) = \vec{v}_e(\vec{k}_e)$$

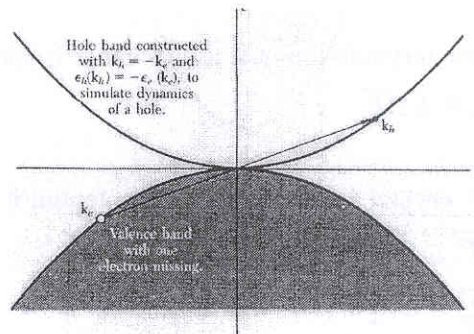


Figure 8 The upper half of the figure shows the hole band that simulates the dynamics of a constructed by inversion of the valence band in the origin. The wavevector and energy of the are equal, but opposite in sign, to the wavevector and energy of the empty electron orbital in the valence band. We do not show the disposition of the electron removed from the valence band.

$$4. \quad m_h = -m_e$$

pf) $\frac{d^2 \varepsilon_k}{dk^2}$ 이 서로 반대이다.

5. 따라서 eq. of motion은 다음과 같다.

$$\hbar \frac{d\vec{k}_h}{dt} = e\left(\vec{E} + \frac{1}{c} \vec{v}_h \times \vec{B}\right)$$

[eq. of motion]

$$\hbar \frac{d\vec{k}_e}{dt} = -e\left(\vec{E} + \frac{1}{c} \vec{v}_e \times \vec{B}\right)$$

pf)

$$\hbar \frac{d\vec{k}_e}{dt} = \hbar \frac{d(-\vec{k}_h)}{dt} = -\hbar \frac{d\vec{k}_h}{dt}$$

$$= -e\left(\vec{E} + \frac{1}{c} \vec{v}_e \times \vec{B}\right)$$

$$= -e\left(\vec{E} + \frac{1}{c} \vec{v}_h \times \vec{B}\right)$$

$$\therefore \hbar \frac{d\vec{k}_h}{dt} = e\left(\vec{E} + \frac{1}{c} \vec{v}_h \times \vec{B}\right)$$



Effective mass

$$\epsilon = \frac{\hbar^2}{2m} k^2$$

ϵ 의 curvature를 결정하는 것은 m 이다.

$$\frac{1}{m} = \frac{1}{\hbar^2} \frac{d^2\epsilon}{dk^2}$$

$$\therefore m = \frac{1}{\frac{1}{\hbar^2} \frac{d^2\epsilon}{dk^2}} = \frac{\hbar^2}{\frac{d^2\epsilon}{dk^2}}$$

$$\epsilon(k) = \epsilon_c + \frac{\hbar^2}{2m_e} k^2$$

$$\frac{m_e}{m} = \frac{1}{\frac{2\lambda}{u} - 1} \quad \text{2nd band}$$

$$\epsilon(k) = \epsilon_v - \frac{\hbar^2}{2m_h} k^2 \quad \text{first band}$$

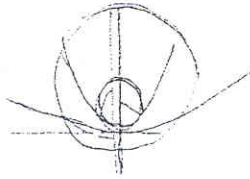
$$\frac{m_h}{m} = \frac{1}{\frac{2\lambda}{u} + 1}$$

$$\epsilon = ak^2$$

a 는 curvature에 비례

다른 말로 표현하면, effective mass는 $\frac{E_g}{\lambda}$ 에 비례한다.

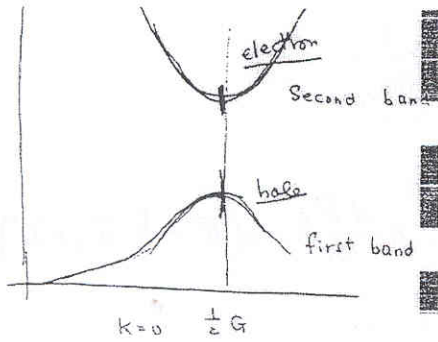
Chapter 7에서



$$\epsilon_K = \frac{\hbar^2}{2m} \left\{ K^2 + \frac{1}{4} G^2 \right\} \pm u \left\{ 1 + 2 \cdot \frac{\lambda}{u^2} \cdot \frac{\hbar^2 K^2}{2m} \right\}$$

$$\frac{\hbar^2 GK}{2m} \ll u, \quad \lambda = \frac{\hbar^2}{2m} \left(\frac{1}{2} G \right)^2 \text{ 이므로}$$

$$= \epsilon(\pm) + \frac{\hbar^2 K^2}{2m} \left(1 \pm \frac{2\lambda}{u} \right)$$



$$\epsilon_K = \epsilon(\pm) + \frac{\hbar^2 K^2}{2m} \left(1 \pm \frac{2\lambda}{u} \right)$$

$$\epsilon_+ = \frac{\hbar^2}{2m} \frac{1}{4} G^2 + u$$

$$\epsilon_- = \frac{\hbar^2}{2m} \frac{1}{4} G^2 - u$$

$$\epsilon_K = \epsilon_+ + \frac{\hbar^2 K^2}{2m_e} \left(1 + \frac{2\lambda}{u} \right) : \text{2nd band}$$

$$\epsilon_K = \epsilon_- + \frac{\hbar^2 K^2}{2m_h} \left(1 - \frac{2\lambda}{u} \right) : \text{first band}$$

$$m = \frac{m_e}{\left(1 + \frac{2\lambda}{u} \right)}$$

at the top band

electron band

$$m = \frac{m_e}{\left(1 - \frac{2\lambda}{u} \right)}$$

at the bottom band

$\frac{2\lambda}{u} > 1$ 이면 hole band

mass negative

$$\vec{v} = \frac{1}{\hbar} \nabla_k \epsilon(\vec{k})$$

$$\frac{dv_g}{dt} = \hbar^{-1} \frac{d^2 \epsilon}{dk dt} = \hbar^{-1} \frac{d^2 \epsilon}{dk^2} \frac{dk}{dt}$$

$$\frac{dk}{dt} = \frac{F}{\hbar} \quad \therefore \frac{dv_g}{dt} = \left(\frac{1}{\hbar^2} \frac{d^2 \epsilon}{dk^2} \right) F$$

$$\therefore F = \left(\frac{\hbar^2}{d^2 \epsilon / dk^2} \right) \frac{dv_g}{dt} = m^* \cdot a$$

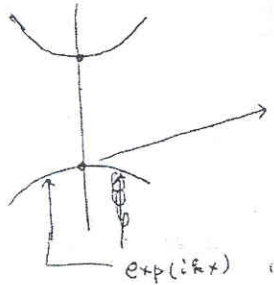
Newton's 2nd law에 의해 effective mass m^* 은

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2 \epsilon}{dk^2} \text{ 이 된다.}$$

$$\left(\frac{1}{m^*} \right)_{\mu\nu} = \frac{1}{\hbar^2} \frac{d^2 \epsilon_k}{dk_\mu dk_\nu} \quad \text{or} \quad \frac{dv_\mu}{dt} = \left(\frac{1}{m^*} \right)_{\mu\nu} F_\nu$$

$$F_\nu = \left(\frac{\hbar^2}{d^2 \epsilon / dk_\mu dk_\nu} \right) \frac{dv_{\mu}}{dt} = m^*_{\mu\nu} \frac{dv_{\mu}}{dt} \quad [m^*_{\mu\nu} : \text{recipsocal effective mass tensor}]$$

Physical Interpretation of the effective mass



k 가 증가하면 $\exp[i(k-G)x]$ 는 증가하고 reflected component가 증가한다.
 eigen vector가 boundary 근방에서 standing wave가 된다.
 >> momentum component $\hbar(-\frac{1}{2}G)$ 는 $\hbar(\frac{1}{2}G)$ component와 cancel된다.

m^* 가 negative가 된다하여 놀랄 것이 못된다.

k 가 가다가 $k+\Delta k$ 가 된다.

momentum transfer to the lattice가 electron에 가해준 momentum보다 크다.

k 가 E field에 대해 Δk 만큼 증가한다.

Bragg reflection에 의해 momentum이 감소한다.

\therefore momentum이 감소한다.

2nd Band : boundary에서 멀어지면 $\exp[i(k-G)x]$ 는 급히 작아진다.

m^* 는 positive이다.

free electron이 받은 impulse보다 larger. $\exp[i(k-G)x]$ 는 감소한다.

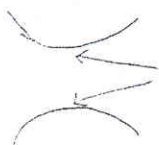


넓으면 $\frac{d^2\varepsilon}{dk^2}$ 이 매우 작아지고 m 이 매우 커진다.

예 : 4f electron인 경우 overlap이 매우 작고

따라서 quantum tunneling에 의해 overlap이 결정되고 effective mass는 매우 크다.

Effective Masses in Semiconductor



이 근처에서 cyclotron resonance로 band edge 모양을 알 수 있다.

즉 effective mass tensor를 결정하자.

Centimeter wave, millimeter wave radiation으로 결정하자.

Angular rotation freq. ω_c 는

$$\omega_c = \frac{eB}{m^*c}$$

B field 가한 후 rf freq. 가하면 확 흡수되는 freq. ($\omega_{rf} = \omega_c$)가 있다.

Hole과 electron은 서로 반대로 돈다.

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line width를 collision time(τ)로 알 수 있다.

실험 결과 $\left(\frac{m^*}{m} \approx 0.1, f_c = 24 \text{ GHz}, \omega_c = 1.5 \times 10^{11} \text{ s}^{-1}\right) \Rightarrow B = 860 \text{ G}$

$$B = \frac{\omega_c m^* c}{e} = \frac{1.5 \times 10^{11} \times 0.9 \times 10^{-27} \times 10^{-1} \times 3 \times 10^{10}}{4.8 \times 10^{-10}}$$

$$= 0.86 \times 10^3 \text{ G}$$

$$= 860 \text{ Gauss}$$

distinctive resonance를 얻으려면 $\omega_c \tau \geq 1$ 은 되어야 한다.

(one radian 가는 동안 부딪히지 않아야 한다.)

higher freq. radiation

higher magnetic field

high purity crystal in liquid helium에서 조건 만족

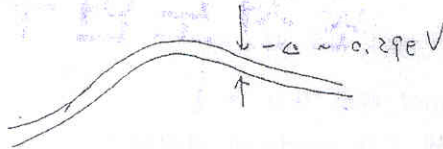
Conduction band edge is spherical with effective mass

$$\epsilon_c = E_g + \frac{1}{2m_e} \hbar^2 k^2$$

Valance band: heavy hole, light hole, spin orbit splitting Δ 가 있다.

$$\epsilon_v(hh) \cong -\frac{\hbar^2 k^2}{2m_{hh}}, \quad \epsilon_v(lh) \cong -\frac{\hbar^2 k^2}{2m_{lh}} : \text{doubly deg enaracy}$$

$$\epsilon_v(soh) \cong -\Delta - \frac{\hbar^2 k^2}{2m_{soh}}$$



electron의 effective mass는 band gap에 비례

$$\frac{m_e}{mE_g} = 0.063, 0.060, 0.051 \text{ (eV)}^{-1}$$

Silicon, Germanium

$k=0$ 근처에서 $p_{3/2}$ tight binding에서 $\pm \frac{3}{2}, \pm \frac{1}{2}$

$p_{1/2}$ $\pm \frac{1}{2}$

Valence band

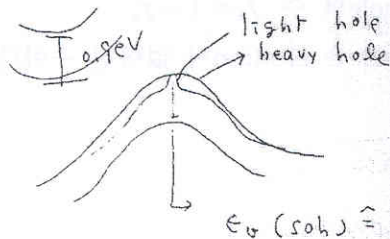
$$\Rightarrow \epsilon(k) = Ak^2 \pm [B^2 k^4 + C^2 (k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2)]^{1/2}$$

$$= -4.29 k^2 \pm [0.68^2 k^4 + 4.87^2 (k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2)]^{1/2}$$

Si : $A = -4.29, |B| = -0.68, |C| = 4.89 : \Delta = 0.044 \text{ eV}$

$$\Rightarrow \frac{\partial^2 \epsilon(k)}{\partial k_x \partial k_x} = -4.29 \cdot 2 \pm \frac{1}{2} [\dots]^{-1/2}$$

Heavy hole, light hole Germanium



0.9 eV의 indirect gap이 있다.

Germanium 0.043 m, 0.3 m

Si 0.15 m, 0.52 m

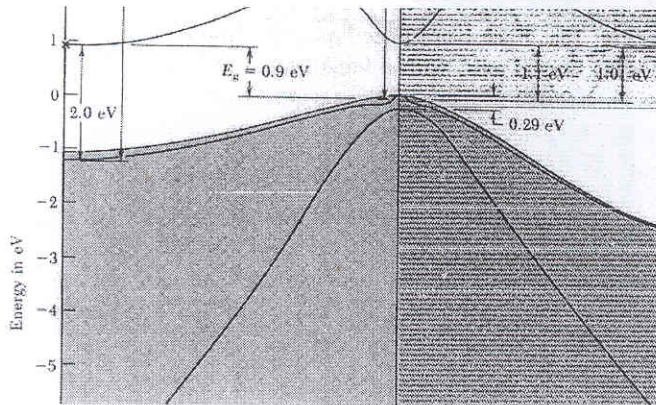
$$\epsilon_v(\text{soh}) \cong -\Delta - \frac{\hbar^2 k^2}{2m_{\text{soh}}}$$

(111)방향

$$m_e = 1.59m$$

$$m_t = 0.082m \text{ 이다.}$$

$$\frac{1}{m_c^2} = \frac{\cos^2 \theta}{m_t^2} + \frac{\sin^2 \theta}{m_l m_l}$$



Intrinsic carrier concentration

chemical potential μ : the number of electrons excited to the conduction band at temp. T

Semiconductor에서 Conduction band인 경우

$$\epsilon - \mu \gg k_B T$$

Fermi-Dirac distribution ft. : $f \cong \exp[(\mu - \epsilon)/k_B T]$

Conduction band에서의 전자 에너지

$$\epsilon_k = E_c + \frac{\hbar^2 k^2}{2m_e}$$

$$D_e(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \cdot (\epsilon - E_c)^{1/2}$$

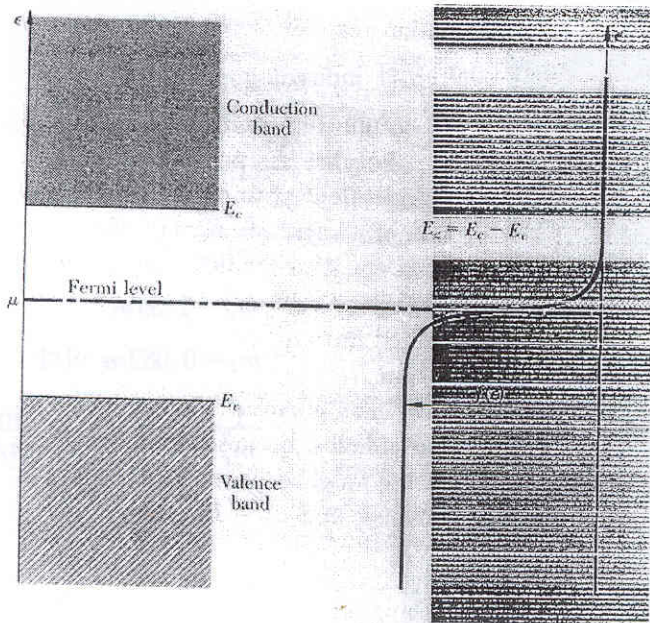
Concentration of electrons in the conduction band

$$n = \int_{E_c}^{\infty} D_e(\epsilon) f_e(\epsilon) d\epsilon$$

$$= \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \exp(\mu/k_B T) \int_{E_c}^{\infty} (\epsilon - E_c)^{1/2} \cdot \exp(-\epsilon/k_B T) d\epsilon ; \int_{-\infty}^{\infty} t^2 e^{-t} dt = \frac{\sqrt{\pi}}{2}$$

$$= 2 \cdot \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} \exp[(\mu - E_c)/k_B T]$$

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hole의 수 $f_h = 1 - f_e$
hole은 electron이 없어진 수이다.

$$f_h = 1 - \frac{1}{\exp[(\epsilon - \mu)/k_B T] + 1} = \frac{1}{\exp[(\mu - \epsilon)/k_B T] + 1} \approx \exp[(\epsilon - \mu)/k_B T]$$

$$D_h(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} \cdot (E_v - \epsilon)^{1/2}$$

$$p = \int_{-\infty}^{E_v} D_h(\epsilon) f_h(\epsilon) d\epsilon = 2 \cdot \left(\frac{m_h k_B T}{2\pi \hbar^2} \right)^{3/2} \cdot \exp[(E_v - \mu)/k_B T]$$

$$\begin{aligned} np &= 2 \cdot \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} \exp[(\mu - E_c)/k_B T] \cdot 2 \cdot \left(\frac{m_h k_B T}{2\pi \hbar^2} \right)^{3/2} \cdot \exp[(E_v - \mu)/k_B T] \\ &= 4 \cdot \left(\frac{k_B T}{2\pi \hbar^2} \right)^3 (m_e m_h)^{3/2} \cdot \exp(-E_g/k_B T) \end{aligned}$$

300K에서는 np :	$2.10 \times 10^{19} \text{ cm}^{-6}$	Si
	2.89×10^{26}	Ge
	6.55×10^{12}	GeAs

equilibrium population of electrons & holes: 빛을 쬐으로써

$A(T)$: generation rate / $B(T)np$: recombination rate

$$\frac{dn}{dt} = A(T) - B(T)np = \frac{dp}{dt} = 0 \Rightarrow \therefore np = \frac{A(T)}{B(T)}$$

impurity를 넣으면 total carrier concentration $n + p$ 는 매우 적다.

이 때의 reduction은 "compensation"이라 한다.

intrinsic 경우(n=p)

$$n_i = p_i = 2 \cdot \left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} (m_e m_h)^{3/4} \cdot \exp(-E_g/2k_B T)$$

또한

$n = p$

$$2 \cdot \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} \exp[(\mu - E_c)/k_B T] = 2 \cdot \left(\frac{m_h k_B T}{2\pi\hbar^2} \right)^{3/2} \exp[(E_v - \mu)/k_B T]$$

$$\left(\frac{m_e}{m_h} \right)^{3/2} = \exp[(E_v + E_c - 2\mu)/k_B T]$$

$$\frac{(E_v + E_c - 2\mu)}{k_B T} = \frac{3}{2} \log \left(\frac{m_e}{m_h} \right)$$

$$\mu = \frac{1}{2}(E_v + E_c) - \frac{3}{4} k_B T \log \left(\frac{m_e}{m_h} \right)$$

$$= \frac{1}{2} E_g + \frac{3}{4} k_B T \cdot \ln \left(\frac{m_h}{m_e} \right)$$

Intrinsic Mobility

$$\mu = |v|/E$$

mobility는 (전자, hole)이 각각 다르다.: (μ_e , μ_h)

$$\sigma = (ne\mu_e + pe\mu_h)$$

$$\mu_e = e\tau_e/m_e, \mu_h = \eta\mu_h/m_h$$

σ 의 온도 의존성(Intrinsic case)

$$\exp(-E_g/2k_B T)$$

hole의 mobility는 electron mobility에 비해 매우 작다.

band degeneracy 때문

특별히 ionic crystal에서는 hole이 immobile

only by thermally activated hopping from ion to ion

small band gap at direct band gap \Rightarrow electron의 mobility가 큰 경향이 있다.

Impurity conductivity

impurity, imperfection에 의해 electrical properties 많이 바뀐다.

boron을 silicon에다 넣으면 conductivity 증가

[$\frac{1}{10^5}$ boron이 10^3 증가시킨다.]

stoichiometric deficiency of one constituent : "deficit semiconductors"

[doping: 고의로 불순물을 첨가]

5가가 들어가면 valence는 five
 impurity valence는 5개
 phosphorus, arsenic, antimony

Donor state :

impurity가 전자 하나를 포기. crystal은 still neutral

electron이 느끼는 potential $\frac{e}{\epsilon r}$ ϵ : 주위의 polarization 때문이다.

1. r 이 큰 경우
2. electron이 slow한 경우 orbital freq. $\omega < \omega_g$ (gap)

P, As, Sb, in Ge, Si

ionization energy of atomic hydrogen

$$-\frac{e^4 m}{2 \hbar^2}$$

Semiconductor $e^2 \rightarrow \frac{e^2}{\epsilon}$, $m \rightarrow m_h$

$$E_d = \frac{e^4 m_e}{2 \epsilon^2 \hbar^2} = \frac{13.6}{\epsilon^2} \cdot \frac{m_e}{m} \text{ eV}$$

$$\text{radius } a_d = \frac{\epsilon \hbar^2}{m_e e^2} = \left(\frac{0.53 \epsilon}{m_e/m} \right) \text{ \AA}$$

impurity level : $m_e \approx 0.1m$ germanium 속에서 $\epsilon = 15.8$
 $m_e \approx 0.2m$ silicon에서 $\epsilon = 11.7$

Germanium 속에서 $E_d = 5 \text{ meV}$

$$\frac{m_e}{m} \frac{1}{\epsilon^2} = 4 \times 10^{-4}$$

Silicon 속에서 $E_d = 20 \text{ meV}$

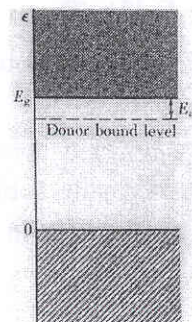
좀 더 정확하게 계산 (anisotropic mass tensor)

Ge : 9.05 meV Si : 29.8 meV

	P	As	Sb	prediction
Si	45	49	39	29.8 meV
Ge	12	12.7	9.6	9.05 meV

Table 5 Donor ionization energies E_d of pentavalent impurities in germanium and silicon, in meV

Impurity	Germanium (meV)	Silicon (meV)
P	45	39
As	49	39
Sb	12.7	9.6



Corresponding radius $\left(\frac{\hbar^2}{m_e^2}\right) \times \left(\frac{\epsilon m}{m_e}\right) = \frac{\epsilon \hbar^2}{m_e e^2}$

Si : $60 \times 0.53 \cong 30 \text{ \AA}$ donor orbit은 overlap이 된다.

Ge : $160 \times 0.53 \cong 80 \text{ \AA}$ => donor band 형성!

Acceptor state

3가가 4가 속에 있다. B, Al, Ga, In

leaving holes in the band

hole is free

Hall voltage n, p type 결정

또는 thermoelectric potential로 sign을 결정한다.

intrinsic electron concentration n_i (300K)

$1.7 \times 10^{13} \text{ cm}^{-3}$ in Ge $\rho : 43 \text{ ohm} \cdot \text{cm}$

$4.6 \times 10^9 \text{ cm}^{-3}$ in Silicon $\rho : 2.6 \times 10^5 \text{ ohm} \cdot \text{cm}$

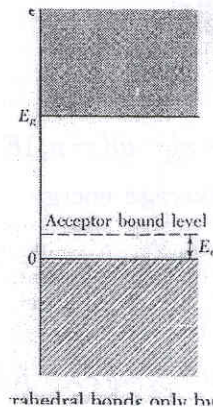


Table 6 Acceptor ionization energies E_a of trivalent impurities in germanium and silicon, in meV

	B	Al	Ga	In
Si	45	57	65	157
Ge	10.4	10.2	10.8	11.2

acceptor, donor의 수를 얼마나 줄일 수 있나?

1 impurity in 10^{11} Ge atom

(실험상 10^7 까지 켤다. photothermal ionization spectroscopy)

P in Ge : $4 \times 10^{10} \text{ cm}^{-5}$

H, O, Si, C는 Ge 속에서 $10^{12} \sim 10^{14} \text{ cm}^{-3}$

electrical measurement에 관계없다.

Thermal Ionization of Donor and Acceptor

low temp limit $k_B T \ll E_d$, donor만 있다.

$n \cong (n_0 N_d)^{1/2} \cdot \exp(-E_d/2k_B T)$

$n_0 \equiv 2(m_e k_B T / 2\pi \hbar^2)^{3/2}$

N_d : donor의 concentration

초전도연구단
단장 이성익

만약 donor와 acceptor concentration이 비슷하면 매우 풀기 어렵다.

그러나 mass action law는 같다.

donor가 증가하면 전자가 증가. hole은 감소

$n + p$ 는 증가한다.

Thermoelectric effects

$$\text{current} : j_q = n(-e)(-\mu_e)E = ne\mu_e E$$

전자가 갖는 average energy

$$(E_c - \mu) + \frac{3}{2} k_B T \quad E_c : \text{conduction band edge}$$

에너지 flux는

$$j_U = n(E_c - \mu + \frac{3}{2} k_B T)(-e)E$$

$$\text{Peltier coefficient } j_U = \pi j_q \quad \pi_e = - \frac{(E_c - \mu + \frac{3}{2} k_B T)}{e}$$

hole에 관해서는

$$j_g = p e \mu_h E \quad , \quad j_u = p(\mu - E_v + \frac{3}{2} K_B T)(u_h)E$$

$$\therefore \text{Peltier coefficient} \quad \pi_h = (\mu - E_v + \frac{3}{2} K_B T)/e$$

Absolute thermoelectric Power $E = Q \nabla T$

$$(j_u = \pi j_g)$$

$$\pi = QT$$

Peltier coefficient 와

thermoelectric power 사이의 관계

Semi - Metal : small overlap of valance and conduction band

valence에 small hole

conduction에 small electron

예: arsenic(As) , antimony(Sb) , bismuth