Cotunneling Transport and Quantum Phase Transitions in Coupled Josephson-Junction Chains with Charge Frustration

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We investigate the quantum phase transitions in two capacitively coupled chains of ultrasmall Josephson junctions, where the particle-hole symmetry is broken by the gate voltage applied to each superconducting island. Near the maximal-frustration line, cotunneling of the particles along the two chains is shown to play a major role in the transport and to drive a quantum phase transition out of the charge-density-wave insulator, as the Josephson-coupling energy is increased. We also argue briefly that slightly off the symmetry line the universality class of the transition remains the same as that right on the line, being driven by the particle-hole pairs. [S0031-9007(98)07601-7]

Systems of ultrasmall tunnel junctions composed of metallic or superconducting electrodes have been the source of a great number of experimental and theoretical works [1]. Single charge (electron or Cooper-pair) tunneling observed in those systems demonstrates the remarkable effects of Coulomb blockade. Especially, in Josephson-junction arrays, the charging energy in competition with the Josephson-coupling energy further brings about the noble effects of quantum fluctuations, which induce quantum phase transitions at zero temperature [2]. Very recently, another fascinating manifestation of Coulomb blockade has been revealed in capacitively coupled one-dimensional (1D) arrays of metallic tunnel junctions [3,4]: In such coupled chains, the major transport along both chains occurs via *cotunneling* of the electron-hole pairs, which is a quantum mechanical process through an intermediate virtual state. Such a cotunneling transport leads to the interesting phenomenon of the current mirror.

In capacitively coupled Josephson-junction chains, the counterpart of the electron-hole pair is the particle-hole pair, i.e., the pair of an excess and a deficit in Cooper pairs across the two chains. Such particle-hole pairs, combined with the quantum fluctuations, have been proposed to drive the insulator-to-superconductor transition [5]. Here it should be noticed that the particle-hole pair is stable only near the particle-hole symmetry line; far away from the symmetry line, it no longer makes the lowest charging-energy configuration. Moreover, in a single chain of Josephson junctions, breaking the particle-hole symmetry (by applying a gate voltage) is known to change immediately the universality class of the transition [6,7]. Therefore, it is necessary to find another relevant cotunneling process, if any, off the particle-hole symmetry line and examine how the transitions change in coupled Josephson-junction chains.

As an attempt toward that goal, we investigate in this paper the quantum phase transitions in two chains of ultrasmall Josephson junctions, coupled capacitively with each other. The particle-hole symmetry is broken by the gate voltage applied to each superconducting island; the resulting induced charge introduces *frustration* to the system. Near the maximal-frustration line, cotunneling of the particles along the two chains is found to play a major role in the transport and to drive a quantum phase transition out of the charge-density-wave (CDW) insulator, as the Josephson-coupling energy is increased. We also argue that slightly off the symmetry line the universality class of the transition remains the same as that right on the line, i.e., a Berezinskii-Kosterlitz-Thouless (BKT) transition [8], driven by the particle-hole pairs.

We consider two 1D arrays, i.e., chains of Josephson junctions, each of which is characterized by the Josephson-coupling energy E_J and the charging energies $E_0 \equiv e^2/2C_0$ and $E_1 \equiv e^2/2C_1$, associated with the self-capacitance C_0 and the junction capacitance C_1, respectively (see Fig. 1). The two chains are coupled with each other via the capacitance C_I, with which the electrostatic energy $E_I \equiv e^2/2C_I$ is associated, while no Cooper-pair tunneling is allowed between the two chains [9]. The intrachain capacitances are assumed to be so small ($E_J \ll E_0, E_1$) that, without the coupling, each chain would be in the insulating phase [10]. We are interested in the limit where the coupling capacitance is sufficiently large compared with the intrachain capacitances, $C_I \gg C_0, C_1$, i.e., $E_I \ll E_0, E_1$ [see Eq. (4) below]. On each superconducting island, external gate voltage V_g is applied, and, accordingly, the external charge $n_g \equiv C_0 V_g/2e$ is induced, with e being the electric charge. The external charge n_g breaks the particle-hole symmetry of the system, introducing charge frustration. We restrict our discussion to two regions: near the particle-hole symmetry line ($|n_g - \mathcal{N}| \ll 1/4$ with $\mathcal{N}$ integer) and near the maximal-frustration line ($|n_g - \mathcal{N} - 1/2| \ll 1/4$), where the properties of the system are severely different. We further note the invariance with respect to the substitution $n_g \rightarrow n_g + 1$, and take $\mathcal{N} = 0$ without loss of generality.
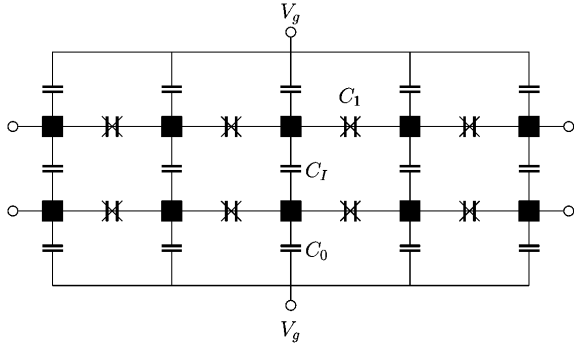
FIG. 1. Schematic diagram of the system.

The Hamiltonian describing the system is given by

$$H = 2e^2 \sum_{\ell,\ell';x,x'} [n_\ell(x) - n_g]\mathbb{C}_{\ell\ell'}^{-1}(x,x')[n_{\ell'}(x') - n_g]$$
$$- E_J \sum_{\ell,x} \cos[\phi_\ell(x) - \phi_\ell(x+1)], \qquad (1)$$

where the number $n_\ell(x)$ of the Cooper pairs and the phase $\phi_\ell(x)$ of the superconducting order parameter at site x on the ℓth chain ($\ell = 1,2$) are quantum-mechanically conjugate variables: $[n_\ell(x), \phi_{\ell'}(x')] = i\delta_{\ell\ell'}\delta_{xx'}$. The capacitance matrix $\mathbb{C}$ in Eq. (1) can be written in the block form:

$$\mathbb{C}_{\ell\ell'}(x,x') \equiv C(x,x')\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta_{x,x'}C_I\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
$$(2)$$

with the intrachain capacitance matrix

$$C(x,x') \equiv C_0\delta_{xx'} + C_1[2\delta_{xx'} - \delta_{x,x'+1} - \delta_{x,x'-1}].$$

For simplicity, we keep only the on-site and the nearest-neighbor interactions between the charges (i.e., $C_1/C_0 \lesssim 1$) although this is not essential in the subsequent discussion (as long as the interaction range is finite). With the block form of the capacitance matrix in Eq. (2), the Hamiltonian can be conveniently expressed as the sum

$$H = H_C^0 + H_C^1 + H_J \qquad (3)$$

with the components

$$H_C^0 \equiv U_0 \sum_x [n_+(x) - 2n_g]^2 + V_0 \sum_x [n_-(x)]^2,$$

$$H_C^1 \equiv U_1 \sum_x [n_+(x) - 2n_g][n_+(x+1) - 2n_g]$$
$$+ V_1 \sum_x n_-(x)n_-(x+1), \qquad (4)$$

$$H_J \equiv -E_J \sum_{\ell,x} \cos[\phi_\ell(x) - \phi_\ell(x+1)],$$

where $n_\pm(x) \equiv n_1(x) \pm n_2(x)$ and the coupling strengths are given by $U_0 \simeq 2E_0$, $U_1 \simeq 4(C_1/C_0)E_0$, $V_0 \simeq E_I$, and $V_1 \simeq (C_1/C_I)E_I$.

The on-site charging-energy term of the Hamiltonian H_C^0 in Eq. (4) reveals clearly the crucial difference between the charge configurations in the system near the maximal-frustration line $n_g = 1/2$ and those near the

particle-hole symmetry line $n_g = 0$. In the former region ($|n_g - 1/2| \ll 1$), the charge configurations which do not satisfy the condition $n_+(x) = 1$ (for all x) have a huge excitation gap of the order of E_0. (Note that we are interested in the parameter regime E_I, $E_J \ll E_0$, E_1.) Furthermore, the ground states of H_C^0, separated from the excited states by the gap of the order of E_I, have twofold degeneracy for each x, corresponding to $n_-(x) = \pm 1$. This degeneracy is lifted as the Josephson-coupling energy E_J is turned on. As a result, it is convenient in this case to work within the reduced Hilbert space $\mathcal{E}_d$, where $n_+(x) = 0$ and $n_-(x) = \pm 1$ for each x. In the latter region ($|n_g| \ll 1/4$), on the other hand, the low-energy charge configuration should satisfy the condition $n_+(x) = 0$ for all x. Unlike the former case, the ground state of H_C^0 is nondegenerate and forms a Mott insulator characterized by $n_1(x) = n_2(x) = 0$ for all x. As E_J is turned on, the ground state of H_C^0 is mixed with the states with $n_-(x) = \pm 2$. Accordingly, the relevant reduced Hilbert space is given by $\mathcal{E}_s$, where $n_+(x) = 0$ and $n_-(x) = 0, \pm 2$ for all x (see also Ref. [5]).

We first consider the region near the maximal-frustration lines, where we project the Hamiltonian Eq. (1) onto $\mathcal{E}_d$, and analyze the properties of the system near the maximal-frustration line ($|n_g - 1/2| \ll 1/4$), based on the resulting effective Hamiltonian. Given the projection operator P onto $\mathcal{E}_d$, the effective Hamiltonian up to the second order in E_J/E_0,

$$H_{\rm eff} \equiv P\left[H + H_J \frac{1 - P}{E - H_C^0} H_J\right]P, \qquad (5)$$

can be obtained via the standard procedure [6,11]. Implementing explicitly the projection procedure, we obtain the effective Hamiltonian describing a *single* spin-1/2 antiferromagnetic Heisenberg chain [12]

$$H_{\rm eff} = \gamma J \sum_x S^z(x)S^z(x+1)$$
$$- \frac{1}{2}J \sum_x [S^+(x)S^-(x+1) + S^-(x)S^+(x+1)],$$
$$(6)$$

where the exchange interaction and the uniaxial anisotropy factor are given by $J \equiv E_J^2/4E_0$ and $\gamma \equiv 16\lambda^2 E_I^2/E_J^2$, respectively. The pseudospin operators have been defined according to

$$S^z(x) \equiv P\frac{n_1(x) - n_2(x)}{2}P,$$
$$S^+(x) \equiv Pe^{-i\phi_1(x)}(1 - P)e^{+i\phi_2(x)}P, \qquad (7)$$
$$S^-(x) \equiv Pe^{-i\phi_2(x)}(1 - P)e^{+i\phi_1(x)}P.$$

(Note the difference from the standard definition.)

The effective Hamiltonian in Eq. (6) includes contributions from several complex processes, back in the charge picture: The first term in Eq. (6), which comes from the projection PH_C^1P, simply describes the nearest-neighbor interaction of the charges in the form $n_-(x)$. On the other
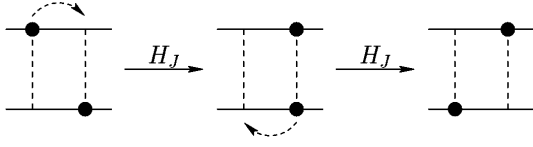
FIG. 2. A typical second-order process via intermediate virtual states with energies of the order of E_0 near the maximal-frustration line.

hand, the second term in Eq. (6), arising from the second order expansion $PH_J \frac{1-P}{E-H_C^0} H_J P$, describes the *cotunneling* process of two particles on different chains by way of a quantum mechanical virtual state with energy of the order of E_0. Figure 2 shows schematically a particle at x on one chain and another particle at $x + 1$ (or $x - 1$) on the other chain hopping at the same time in the opposite direction. This cotunneling process plays a major role in the charge transport along the two chains, and drives the quantum phase transition in the system, as discussed below.

The antiferromagnetic Heisenberg chain described by Eq. (6) has been extensively studied [13], and is known, from the Bethe ansatz solution or the Sine-Gordon theory [14], to exhibit a quantum phase transition at $\gamma = 1$: For $\gamma > 1$, it belongs to the universality class of the Ising chain in the renormalization group (RG) sense, and its ground state displays genuine long-range order in the staggered magnetization, i.e., $\langle(-1)^x S^z(x)S^z(0)\rangle$ approaches a nonzero constant as $x \to \infty$. This long-range order in the staggered magnetization corresponds to the charge-density wave in the charge picture of the original problem. For $\gamma < 1$, on the other hand, the system described by Eq. (6) is equivalent to the quantum *XY* chain, where the Mermin-Wagner theorem prohibits genuine long-range order. In this case, the system can be mapped to the repulsive Luttinger model, and the transverse component of the magnetization, as well as the z component of the staggered magnetization, exhibits quasi-long-range order. Namely, both $\langle(-1)^x S^z(x)S^z(0)\rangle$ and $\langle S^+(x)S^-(0)\rangle$ decay algebraically with the distance x, and the system in the charge picture displays *both* the diagonal and the off-diagonal quasi-long-range order. This state may be regarded as the counterpart of the supersolid, possessing both the diagonal and off-diagonal (true) long-range order and proposed recently in 2D Josephson-junction arrays [6].

The properties of the repulsive Luttinger liquid phase, with both the diagonal and off-diagonal quasi-long-range order, has been discussed in Ref. [11] for a *single* Josephson-junction chain. It has been suggested that the system is extremely sensitive to impurities [15] and may make another insulator, different in nature from the CDW insulator. In the coupled chains, the repulsive Luttinger liquid phase has another remarkable feature of the current mirror. According to the basic transport mechanism due to the cotunneling process of particles in the two chains, shown in Fig. 2, the current fed through one chain is

accompanied by the secondary current in the other chain, with the same magnitude but in the opposite direction. Similar current mirror effects have also been pointed out for $n_g = 0$, where the mechanism is rather different, and via the particle-hole pair transport (see below and Ref. [5]).

We now turn to the region near the particle-hole symmetry line. In the reduced Hilbert space $\mathcal{E}_s$ with the condition $n_+(x) = 0$ satisfied, $n_-(x)/2$ can be regarded as the number of particle-hole pairs located at x. The role of such particle-hole pairs can be analyzed by means of the imaginary-time path-integral representation of the partition function and its dual transformation [5]. The Euclidean action, in the current-loop representation [7], then reads

$$S = \frac{1}{4K} \sum_{\ell\ell',xx',\tau} [n_\ell(x,\tau) - n_g][2C_I \mathbb{C}_{\ell\ell'}^{-1}(x,x')]$$
$$\times [n_\ell(x',\tau) - n_g] + \frac{1}{4K} \sum_{\ell,x,\tau} [J_\ell(x,\tau)]^2, \quad (8)$$

where the (imaginary) time has been rescaled in units of the inverse Josephson plasma frequency $\omega_p^{-1} \equiv \hbar/\sqrt{4E_I E_J}$, the dimensionless coupling constant defined to be $K \equiv \sqrt{E_J/16E_I}$. Here (n_ℓ, J_ℓ) may be viewed as the current in $(1 + 1)$ dimensions, satisfying the continuity equation

$$\nabla_\tau n_\ell(x,\tau) + \nabla_x J_\ell(x,\tau) = 0. \quad (9)$$

With the capacitance matrix Eq. (2), it is convenient to decompose the action in Eq. (8) into the sum $S = S_+ + S_-$:

$$S_+ \simeq \frac{E_0}{4KE_I} \sum_{x,\tau} [n_+(x,\tau) - 2n_g]^2 + \frac{1}{8K} \sum_{x,\tau} [J_+(x,\tau)]^2,$$

$$S_- \simeq \frac{1}{8K} \sum_{x,\tau} [n_-(x,\tau)]^2 + \frac{1}{8K} \sum_{x,\tau} [J_-(x,\tau)]^2, \quad (10)$$

where $J_\pm(x,\tau) \equiv J_1(x,\tau) \pm J_2(x,\tau)$.

The factor E_0/E_I in the component S_+, which is enormous in the parameter regime of interest, again implies the condition $n_+(x) = 0$ already mentioned. Furthermore, the continuity equation in Eq. (9) requires $J_+(x)$ to be a constant on the average, which should obviously be zero. Consequently, near the transition point, the system is effectively described by the action S_- in Eq. (10), which is equivalent to the 2D *XY* model, and exhibits a BKT transition at $K = K_{\text{BKT}} \approx 2/\pi$ [8]. Here the transition, which is between the Mott insulating phase and the superconducting phase, is driven exclusively by the particle-hole pairs represented by the variable $n_-(x)$, whereas n_+ and J_+ merely renormalize the action S_- and shift slightly the transition point [5]. This shift of the transition point depends on the external charge n_g and may be estimated in the following way: The transition to the superconducting phase occurs when the Josephson-coupling energy E_J also overcomes the Coulomb blockade associated with a particle-hole pair. Since the Coulomb blockade increases with
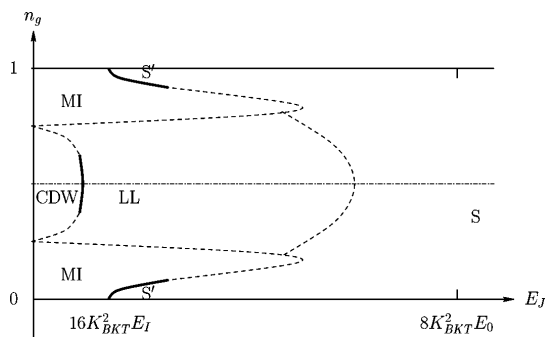
FIG. 3.   Schematic phase diagram of the coupled Josephson-junction chains.

n_g, approximately given by $8E_0 n_g^2 + 4E_I$, the critical value of E_J is concluded to grow from the symmetry-line value $16K_{\mathrm{BKT}}^2 E_I$ as n_g is increased. It is also stressed that the BKT-type transition survives the gate voltage as long as the induced charge is sufficiently small ($|n_g| \ll 1/4$); this is in sharp contrast to the single-chain case, where breaking the particle-hole symmetry by nonzero n_g immediately alters the universality class of the transition [6].

In the parameter regimes other than those considered above, the behavior of the system may be inferred by the following argument: It is obvious that, for $E_J \gg E_0$, the system should be a superconductor with each chain superconducting separately. Note that this superconducting phase, denoted by S, comes from the *particle* (Cooper-pair) transport as usual, thus it is different in character from the superconducting phase in the region $E_J \ll E_0$. In the latter, denoted by S', only the coupled chains as a whole are superconducting, with superconductivity arising from the *particle-hole pair* transport. Far away from both the particle-hole symmetry line and the maximal-frustration lines, the Hamiltonian may be projected onto the subspace, where $n_1(x) = 0, 1$ and $n_2(x) = 0, 1$ for all x, and single-particle processes dominate the transport in the system.

The observations so far are summarized by the phase diagram displayed schematically in Fig. 3. The phase transitions of our main concern are represented by the thick solid lines, separating the CDW from the repulsive Luttinger liquid (LL) and the Mott insulator (MI) from the superconductor (S'); the somewhat speculative boundaries discussed above are depicted by dashed lines. Here it is not clear within our approach whether the boundary between the repulsive Luttinger liquid region and the superconducting region in the phase diagram describes a phase transition or merely a crossover. Furthermore, even in the single-chain case, the properties of the repulsive Luttinger liquid phase is controversial, and the possibility of an intermediate *normal* phase has recently been raised as well [16].

Coupled chain systems are presumably be realized in experiment by current techniques, which have already made it possible to fabricate submicron metallic junction arrays with large interarray capacitances [4] as well as large arrays of ultrasmall Josephson junctions [1]. We also point out that quasiparticles have been safely disregarded in obtaining the equilibrium properties at zero temperature.

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