양자머신러닝

한국전자통신연구원
방 정 호

2023. 07. 31 / 2023.08.03

Part I

양자컴퓨팅 / 알고리즘 연구

양자BLAS (HHL) / 양자서포트벡터머신

Part II

NISQ와 양자머신러닝

변분법 양자알고리즘(VQA)과 양자인공신경망

컴퓨터와 알고리즘

알고리즘 : 문제해결을 위한 일련의 방법론들의 순차적 집합

문제: 라면을 먹으려면?

#1. 물을 끓인다.

#2. 면과 스프를 넣음.

#3. 기다림

} ➡ 알고리즘

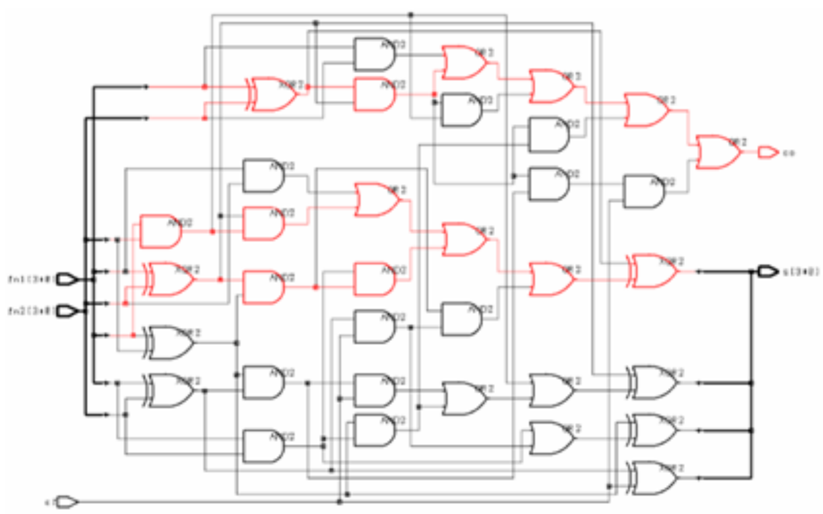
알고리즘의 정의와 표현을 위한 요소

- 문제정의(Problem Definition)
- 알고리즘기술(Description)
- *증명(Solution Proof)
- *성능분석(Performance Analysis)

컴퓨터의 등장 ➡ 알고리즘은 매우 중요해짐

- 비교적 단순한 신호 (Bit: 0 or 1)
- 기본 단위논리 집합으로 연산표현이 가능 (Logic gates; e.g., and, or, etc.)

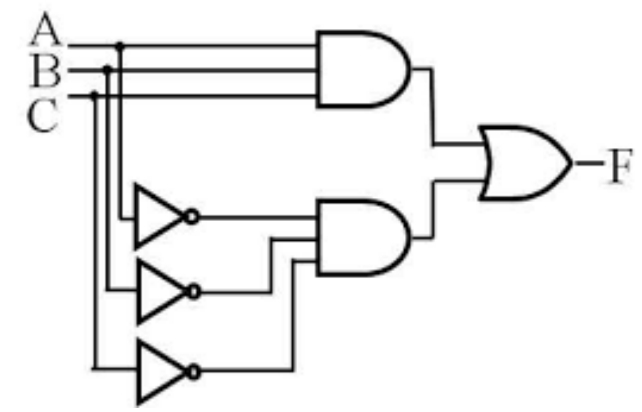
문제난이도와 계산복잡도 이론



복잡한 회로 = 게이트 수: 많음

= 오래걸림 = 알고리즘 성능: Low

VS



간단한 회로 = 게이트 수: 적음

= 금방걸림 = 알고리즘 성능: High

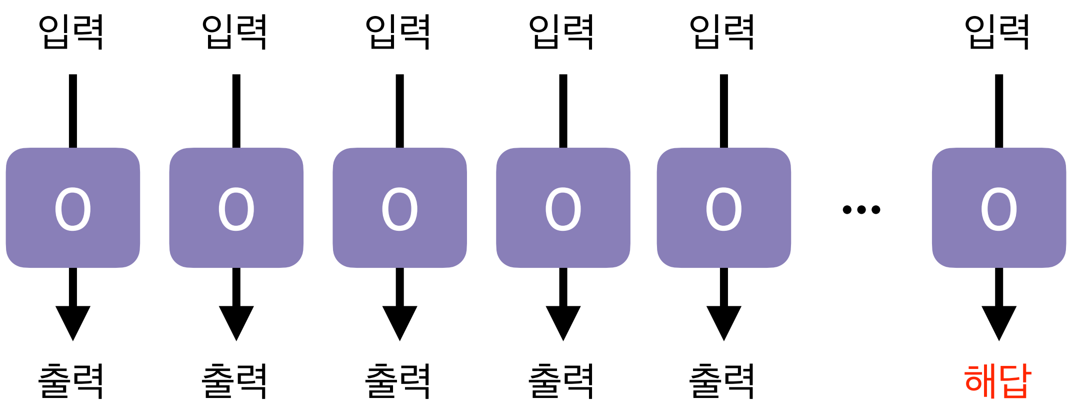
문제 사이즈의 증가 대비 요구되는 *게이트 수의 증가비율

(e.g., 지수함수적 증가 ⇨ 어려움 / 다항함수적 증가 ⇨ 쉬움)

회로복잡도(Circuit complexity) 혹은 게이트복잡도(Gate complexity)

*최근에는 게이트의 레이어 수(회로깊이) 등의 기준을 사용

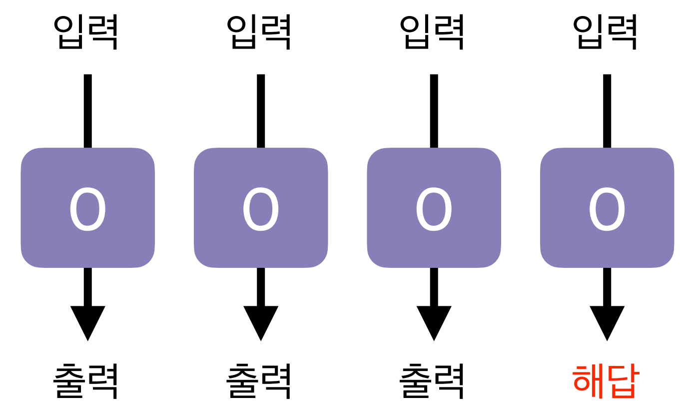
문제난이도와 계산복잡도 이론



문제정보를 자주 확인 = 오라클을 자주 호출

= 오래걸림 = 알고리즘 성능: Low

VS



문제정보를 자주 확인 = 오라클을 자주 호출

= 오래걸림 = 알고리즘 성능: Low

문제 사이즈의 증가 대비 요구되는 *오라클 연산 호출의 증가비율

(e.g., 지수함수적 증가 ⇨ 어려움 / 다항함수적 증가 ⇨ 쉬움)

질의복잡도(Query complexity) 또는 샘플복잡도(Sample complexity)

양자알고리즘 속도향상 및 물리기작 증명

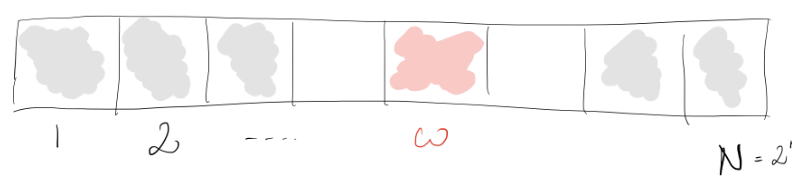
양자 소인수분해 알고리즘

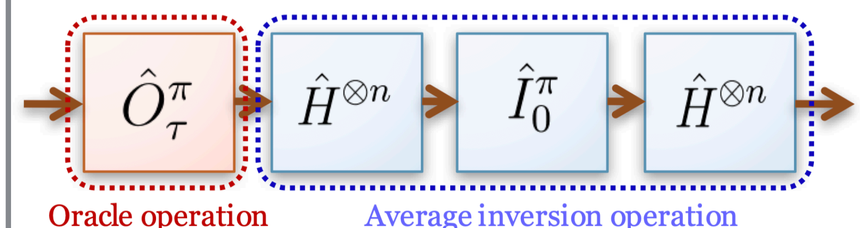
$$p_1 \times p_2 \rightarrow N$$

$$N \rightarrow p_1 \times p_2$$

- 양자 퓨리에 변환
- 양자병렬성 극대화
- $O(e^n) \rightarrow O(n)$

양자 데이터검색 알고리즘





Un-target state, $|\alpha\rangle = \frac{1}{\sqrt{N-M}} \sum_{j \notin \tau} |d_j\rangle$

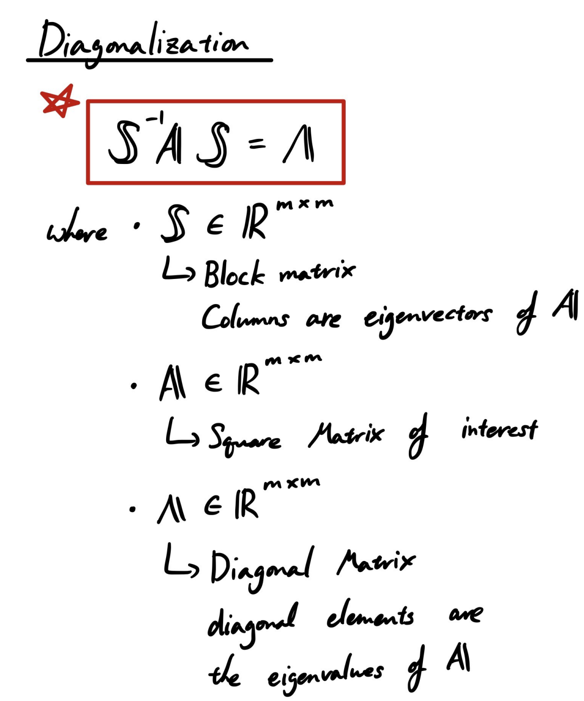
Target state, $|\beta\rangle = \frac{1}{\sqrt{M}} \sum_{j \in \tau} |d_j\rangle$

Initial state : $|\psi\rangle = \hat{H}^{\otimes n}|0\rangle = \cos\frac{\theta}{2}|\alpha\rangle + \sin\frac{\theta}{2}|\beta\rangle \quad \begin{cases} \cos\frac{\theta}{2} = \sqrt{\frac{N-M}{N}} \\ \sin\frac{\theta}{2} = \sqrt{\frac{M}{N}} \end{cases}$

- 양자오라클
- Amplitude Amplification
- $O(N) \rightarrow O(\mathrm{Sqrt}(N))$

*양자선형 알고리즘(HHL)

$Ax=b$ with $A \in \mathbb{R}^{N \times N}$ and $x, b \in \mathbb{R}^N$



- 양자중첩샘플
- 해밀토니안 시뮬레이션
- $O(N) \rightarrow O(\mathrm{Log}(N))$

양자 선형알고리즘 (HHL)

PRL **103,** 150502 (2009) PHYSICAL REVIEW LETTERS week ending 9 OCTOBER 2009

Quantum Algorithm for Linear Systems of Equations

Aram W. Harrow,[1] Avinatan Hassidim,[2] and Seth Lloyd[3]

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[2]*Research Laboratory for Electronics, MIT, Cambridge, Massachusetts 02139, USA*
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(Received 5 July 2009; published 7 October 2009)

Solving linear systems of equations is a common problem that arises both on its own and as a subroutine in more complex problems: given a matrix A and a vector $\vec{b}$, find a vector $\vec{x}$ such that $A\vec{x} = \vec{b}$. We consider the case where one does not need to know the solution $\vec{x}$ itself, but rather an approximation of the expectation value of some operator associated with $\vec{x}$, e.g., $\vec{x}^\dagger M \vec{x}$ for some matrix M. In this case, when A is sparse, $N \times N$ and has condition number κ, the fastest known classical algorithms can find $\vec{x}$ and estimate $\vec{x}^\dagger M \vec{x}$ in time scaling roughly as $N\sqrt{\kappa}$. Here, we exhibit a quantum algorithm for estimating $\vec{x}^\dagger M \vec{x}$ whose runtime is a polynomial of $\log(N)$ and κ. Indeed, for small values of κ [i.e., poly $\log(N)$], we prove (using some common complexity-theoretic assumptions) that any classical algorithm for this problem generically requires exponentially more time than our quantum algorithm.

For a system of linear equations $Ax=b$ with $A \in \mathbb{R}^{N\times N}$ and $x,b \in \mathbb{R}^N$:

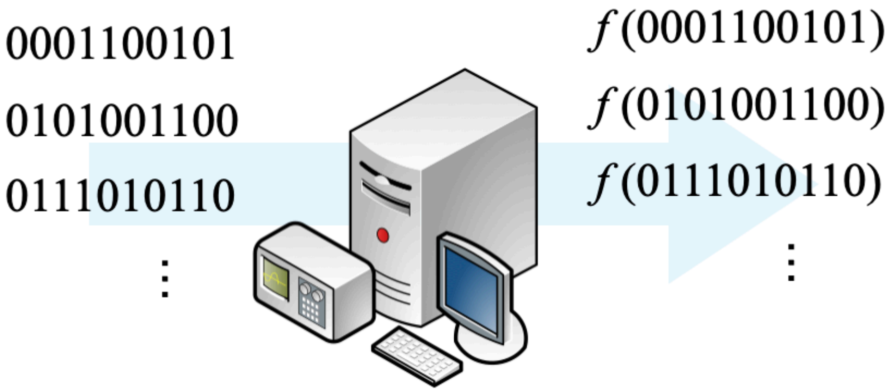
(고전) • Standard classical methods (based QR-factorization) exhibit $O(N^3)$; the best algorithm has a runtime of $\simeq O(N^{2.3})$. [D. Coppersmith and S. Winograd, *Journal of symbolic computation* **9**, 251 (1990)]

(양자) • Quantum linear system algorithm, known as 'HHL' after the three main authors Harrow, Hassidim, and Lloyd, promises to solve the problem in $O(log(N)\kappa^2 s^2/\varepsilon)$, where:

- κ is the ratio of the largest and the smallest eigenvalues;
- s is the sparsity; [A. W. Harrow, A. Hassidim, and S. Lloyd, *Phys. Rev. Lett.* **103**,150502 (2009)]
- ε is the precision (degree of error).

알고리즘과 속도향상: 양자병렬성

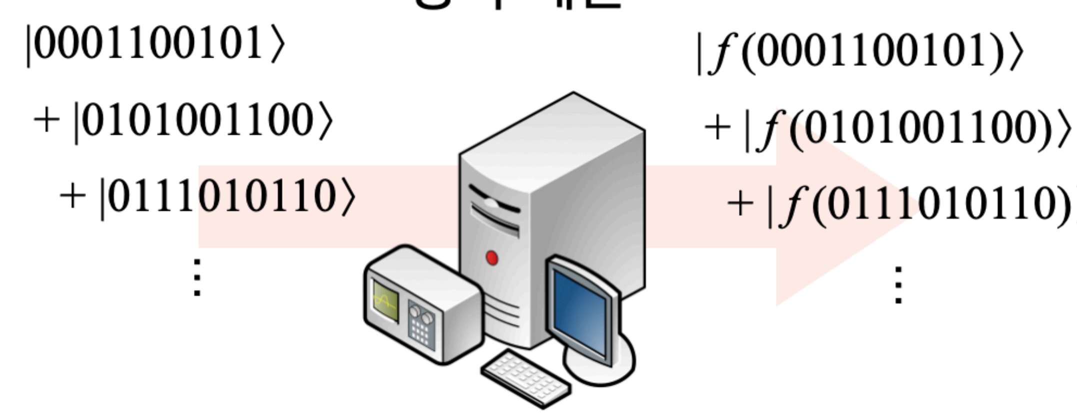
고전 계산



한번에 하나씩…

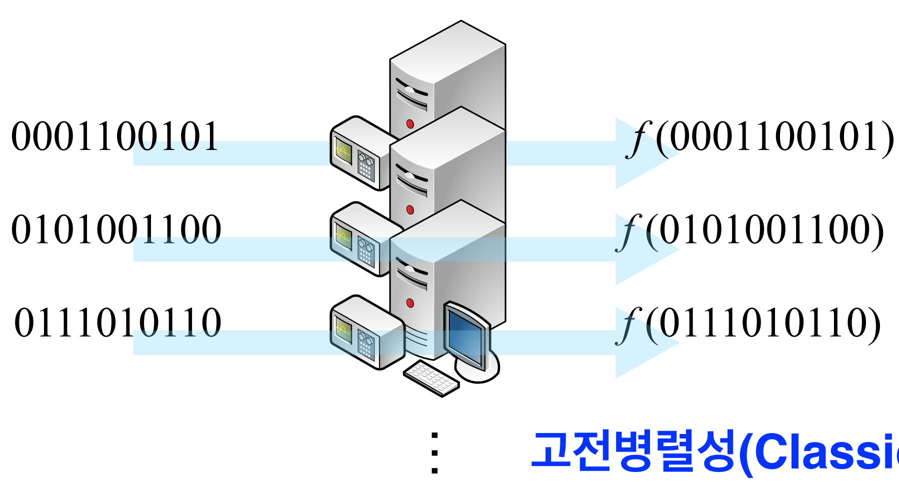
VS

양자 계산



중첩으로 여러개를 한번에…

→ ***양자병렬성(Quantum Parallelism)**



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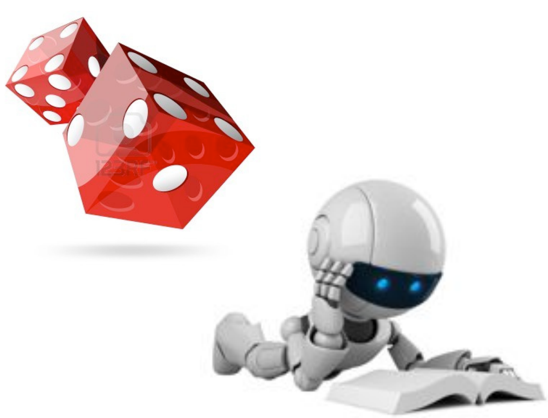
고전병렬성(Classical Parallelism)

양자머신러닝 연구분야 시작

- **양자정보/컴퓨팅**

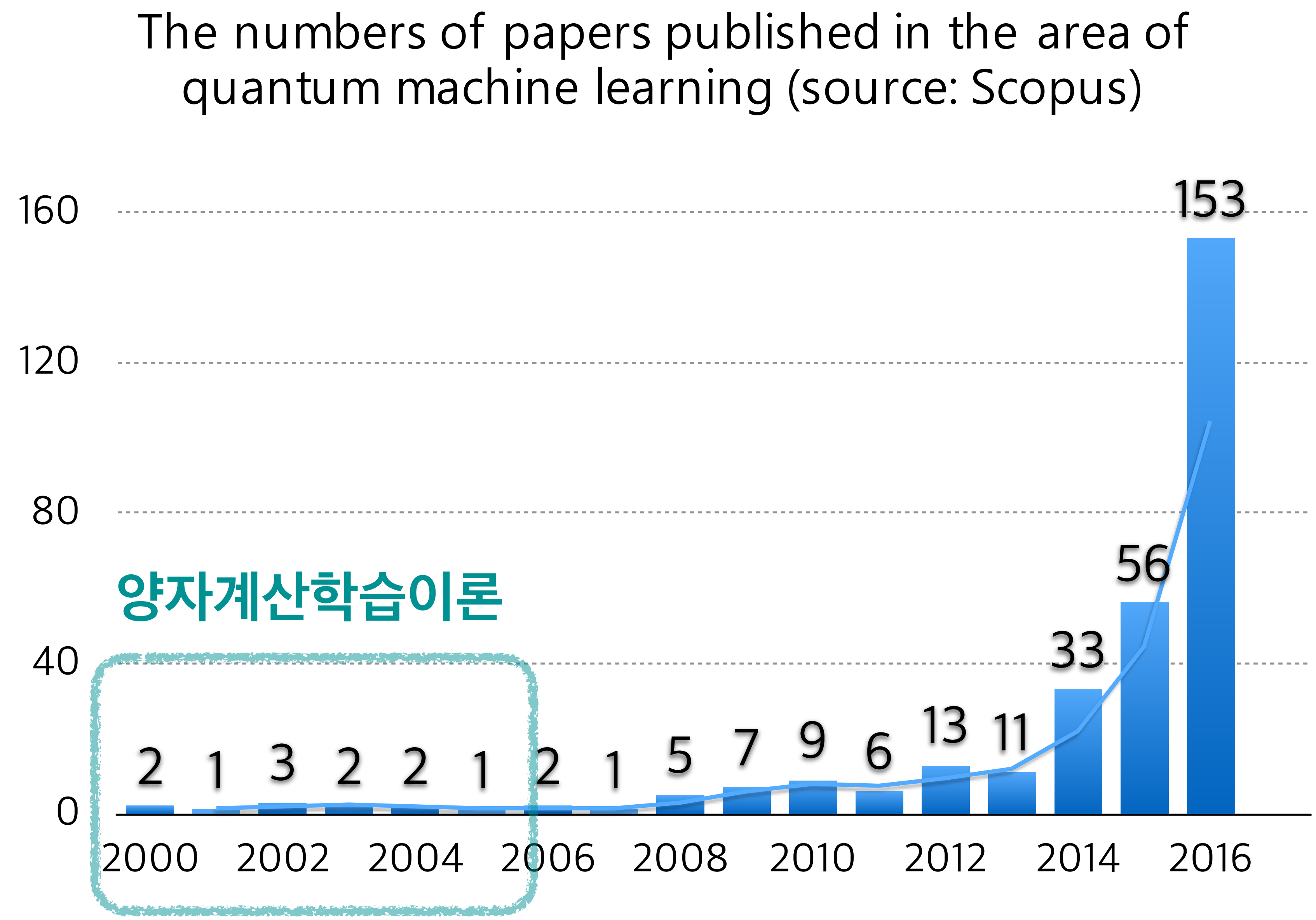
→ 새로운 양자알고리즘 개발에의 난항: 고전 수리/논리적 접근방식에의 한계
→ 하드웨어 플랫폼: 고품질의 큐빗 생성/제어/측정 기술 개발은 여전히 요원
→ 양자정보에의 전반적인 연구 트렌드의 변화



***양자머신러닝(Quantum Machine Learning;QML)**

- 양자컴퓨팅 연구/개발에의 새로운 모멘텀 부여
- 기반이론 확장을 통한 원리적 측면에서의 기술향상 가능성 제시

양자머신러닝 연구의 시작



(고전) 계산학습이론: 정의 및 용어

Definition

Machine learning can be broadly defined as computational methods using experiences to improve the performance or to make the accurate prediction.

* past information available to learner, which typically takes the form of electronic data

Associated problems

* Classification: Assign a category to each data.
* Regression: Predict a real value for each data.
* Ranking: Order data according to some criterion.
* Clustering: Partition data into homogeneous regions.

Learning scenarios & strategies

* Supervised learning: receives the labeled examples as training sample.
* Un-supervised learning: receives the unlabeled examples as training sample.
* Semi-supervised learning: receives both labeled and unlabeled
* Reinforcement learning:

(고전) 계산학습이론: 정의 및 용어

* Examples: Items or instances of data used for learning.
* Features: The set of attributes, associated to an example.
* Labels: Values or categories assigned to examples.

* Input class $\mathcal{X}$: the set of all possible *examples* or *instances*.
* Target class $\mathcal{Y}$: the set of all possible *labels* or *target values*.
* A concept: "$c : \mathcal{X} \mapsto \mathcal{Y}$ " is a mapping from $\mathcal{X}$ to $\mathcal{Y}$. A concept class C is a set of concepts we wish to learn.
* Hypothesis set H:, the set of possible concepts, which may not coincide with C.

Description of the learning problem:

Given a fixed set H, a learner receives a sample $S=(x_1, x_2, \ldots , x_m)$ and labels $(c(x_1), c(x_2), \ldots, c(x_m))$ drawn from a fixed but unknown distribution D. Here, c is a specific target concept in C to learn. The task is to use the labeled sample S to select the best hypothesis $h_S \in H$ that has a small generalization error.

고전 및 양자 계산학습이론: 질의 복잡도

Classical Exact Learning.

For a concept class *C*, a learner *A* is given access to a membership oracle *MQ*(*c*) for the target concept $c \in C$ that A is trying to learn. Given an input $x \in \{0,1\}^n$, *MQ*(*c*) returns the label *c*(*x*). Here, a learning algorithm *A* is an exact learner for *C* if:

- For every $c \in C$, given access to *MQ*(*c*) oracle, *A* outputs hypothesis *h* such that $h(x)=c(x)$ for all *x*, with probability at least 2/3.

The *query complexity of A* is the maximum number of invocations of the *MQ*(*c*), over all concepts $c \in C$ and internal randomness of the learner. The *query complexity of exactly learning C* is the minimum query complexity over all exact learners for *C*.

Each concept $c:\{0,1\}^n \rightarrow \{0,1\}$ specified by its *N*-bit Truth-table ($N=2^n$), we define *(N,M)-query complexity of exact learning* as the maximum query complexity of exactly learning C, maximized over all $C \subseteq \{0,1\}^N$ such that $|C|=M$.

Quantum Exact Learning.

In the quantum setting, instead of having access to an *MQ*(*c*) oracle, a quantum exact learner is given access to a Quantum-*MQ*(*c*) oracle, which maps: $|x, b\rangle \rightarrow |x, b \oplus c(x)\rangle$ for $x \in \{0,1\}^n$, b $\in \{0,1\}$. For a given C, the *quantum query complexity of exactly learning C* can be defined as the quantum analogues to the classical complexity measures.

고전 및 양자 계산학습이론: PAC 학습모델 및 샘플 복잡도

Classical Probably Approximately Correct (PAC) Learning

For a concept class C, a learner A is given access to a random example oracle $PEX(c,D)$, where $c \in C$ is a target concept that A is trying to learn and $D:\{0,1\}^n \rightarrow [0,1]$ is an unknown probability distribution. When invoked $PEX(c,D)$ returns a labeled example (x, $c(x)$) where x is drawn from D. Here, a learning algorithm A is a (ε,δ)-*PAC learner* for C if:

- For every $c \in C$ and distribution D, given access to $PEX(c)$ oracle, A outputs hypothesis h, with probability at least $1-\delta$, such that $\Pr_{x\sim D}[h(x) \neq c(x)] \leq \varepsilon$ for all x.

The *sample complexity of A* is the maximum number of invocations of the $PEX(c,D)$ oracle which the learner makes, over all concepts $c \in C$, distribution D, and the internal randomness of the learner. The *(ε,δ)-PAC sample complexity of a concept class C* is the minimum sample complexity over all (ε,δ)-PAC learners for C.

Quantum Probably Approximately Correct (PAC) Learning

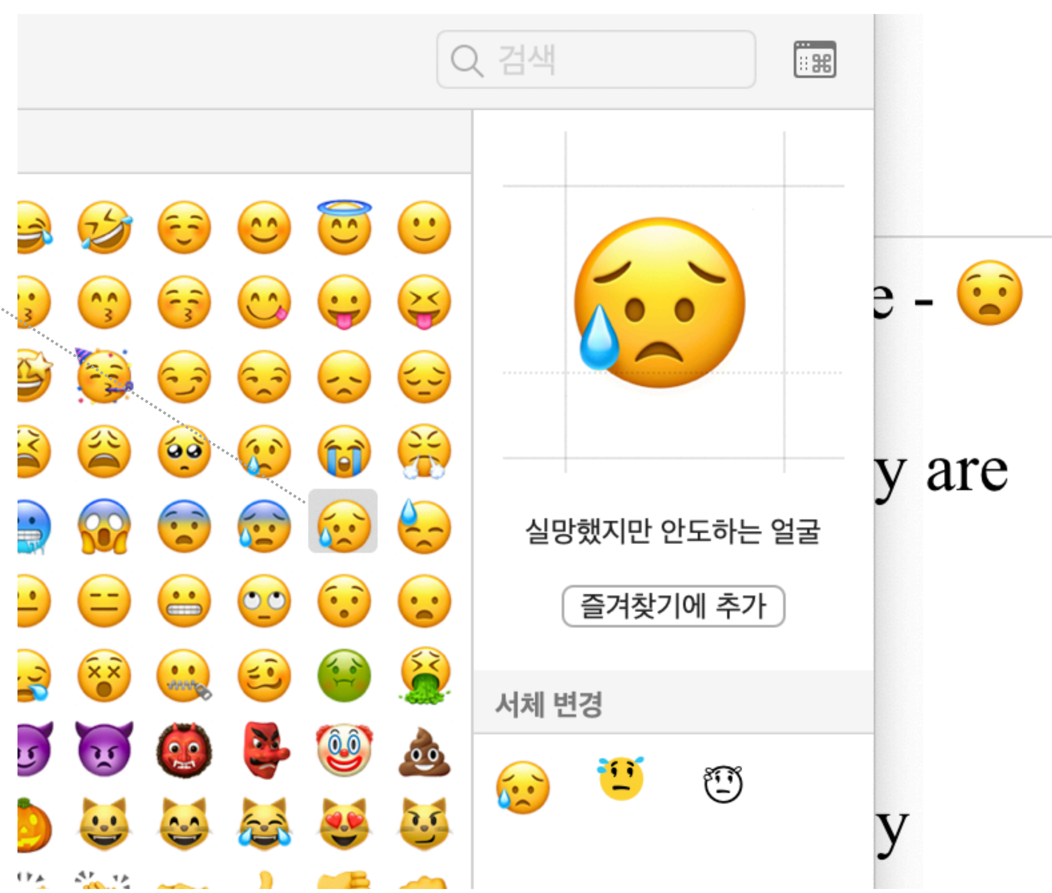
The quantum PAC learner has access to a quantum example oracle QPEX(c, D) that produces a quantum example,

$$\sum_{x\in\{0,1\}^n} \sqrt{D(x)}\,|x, c(x)\rangle$$

고전 및 양자 계산학습이론 연구결과

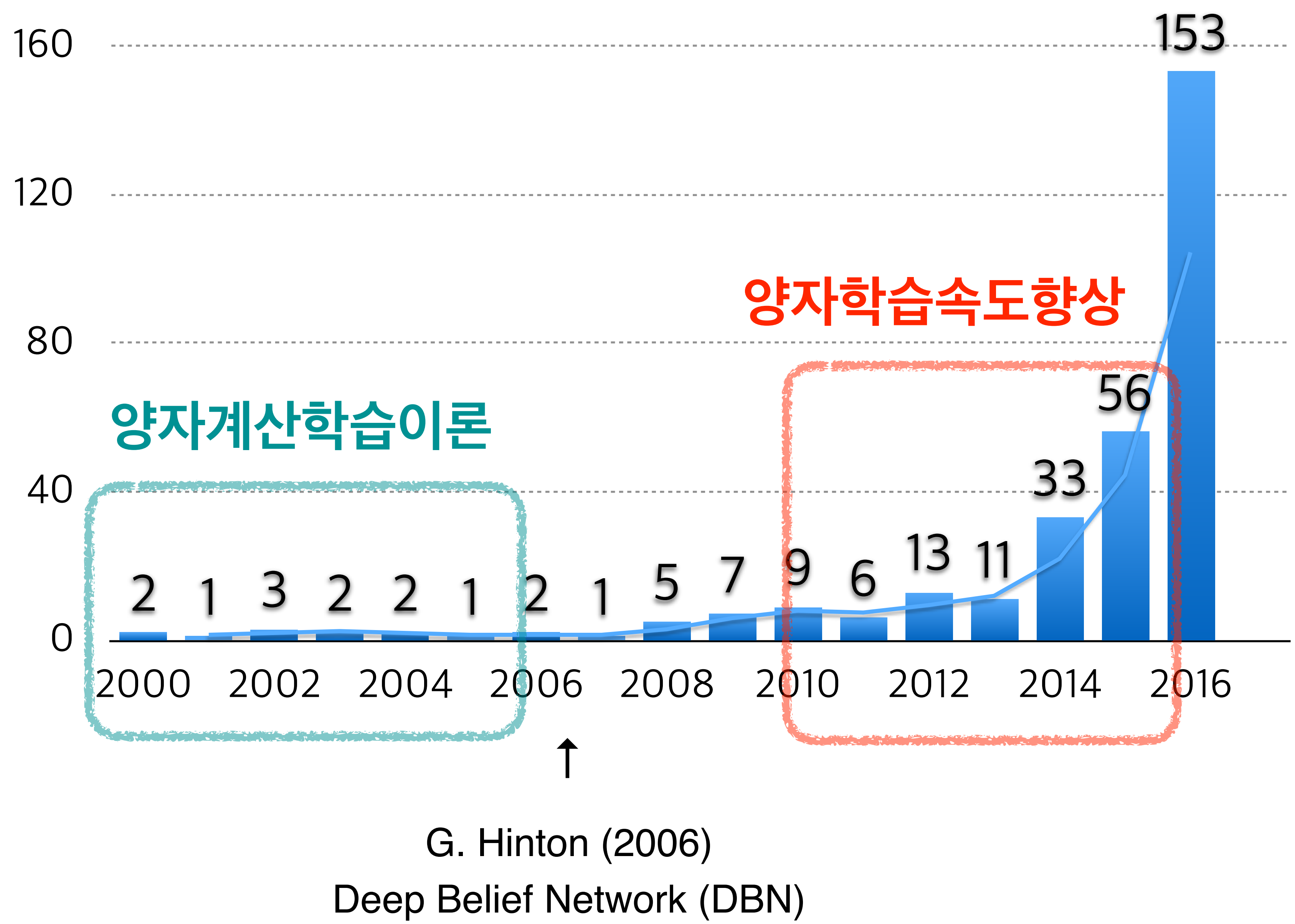
양자 질의/샘플 복잡도 결과

- 양자학습주체(혹은 양자알고리즘)은 다항함수적(polynomial) 속도향상만 가능.
- 확률적(probably-approximately-correct) 학습모델의 경우, 양자학습주체(알고리즘)의 학습은 고전학습주체(알고리즘)와 같은 수준이거나 대략 몇배 정도의 속도향상만 가능함.
- 특정 목적 혹은 비-물리적(?) 조건 하에, 지수함수적(exponential) 속도향상이 가능한 경우가 있음.



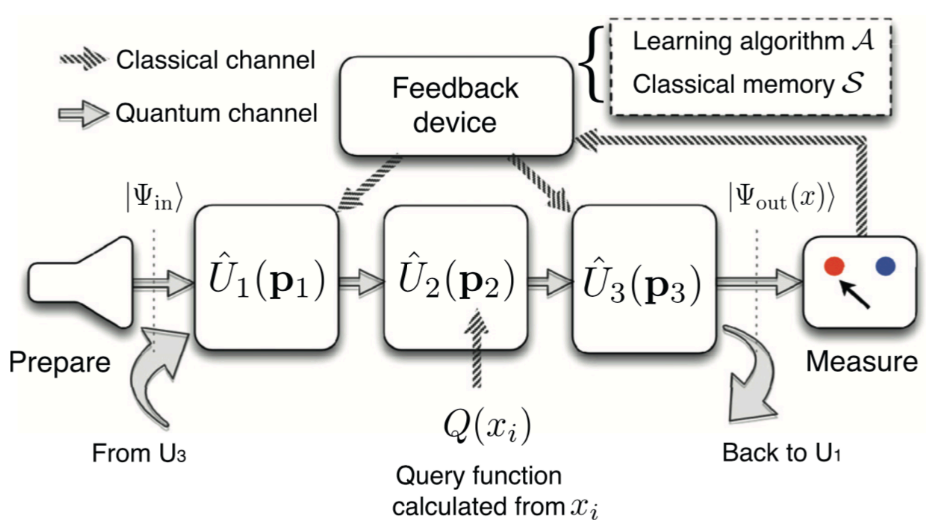
양자머신러닝 연구의 본격화

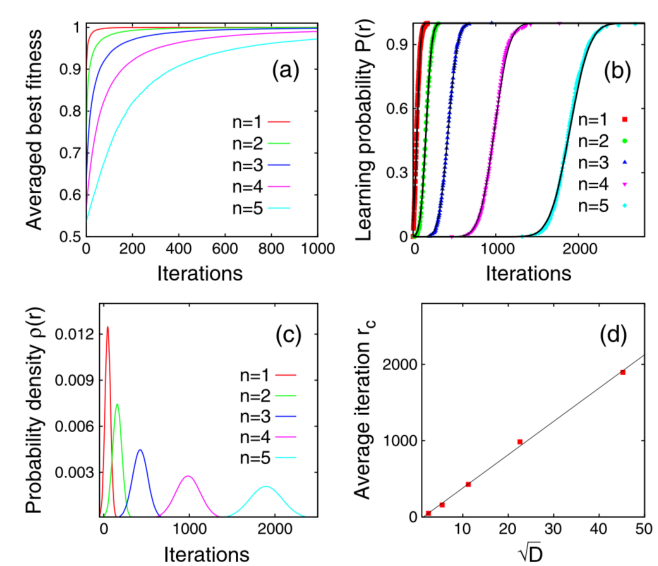
The numbers of papers published in the area of quantum machine learning (source: Scopus)



양자속도향상 가능성 증명

	Speed-Ups	Kernel
J. Bang *et al.* (2014)	*N* → Sqrt(*N*)	Deutsch-Jozsa's & 양자데이터 검색
H. Briegel *et al.* (2016)	*N* → Sqrt(*N*)	양자데이터 검색
C. Lloyd *et al.* (2014)	*N* → Log(*N*)	HHL





▶ 양자머신러닝 용어/연구 방법론 등의 확립 및 양자정보과학 영역 진입

양자 서포트-벡터-머신

PRL **113,** 130503 (2014) PHYSICAL REVIEW LETTERS week ending 26 SEPTEMBER 2014

Quantum Support Vector Machine for Big Data Classification

Patrick Rebentrost,[1,*] Masoud Mohseni,[2] and Seth Lloyd[1,3,†]
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[2]*Google Research, Venice, California 90291, USA*
[3]*Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
(Received 12 February 2014; published 25 September 2014)

Supervised machine learning is the classification of new data based on already classified training examples. In this work, we show that the support vector machine, an optimized binary classifier, can be implemented on a quantum computer, with complexity logarithmic in the size of the vectors and the number of training examples. In cases where classical sampling algorithms require polynomial time, an exponential speedup is obtained. At the core of this quantum big data algorithm is a nonsparse matrix exponentiation technique for efficiently performing a matrix inversion of the training data inner-product (kernel) matrix.

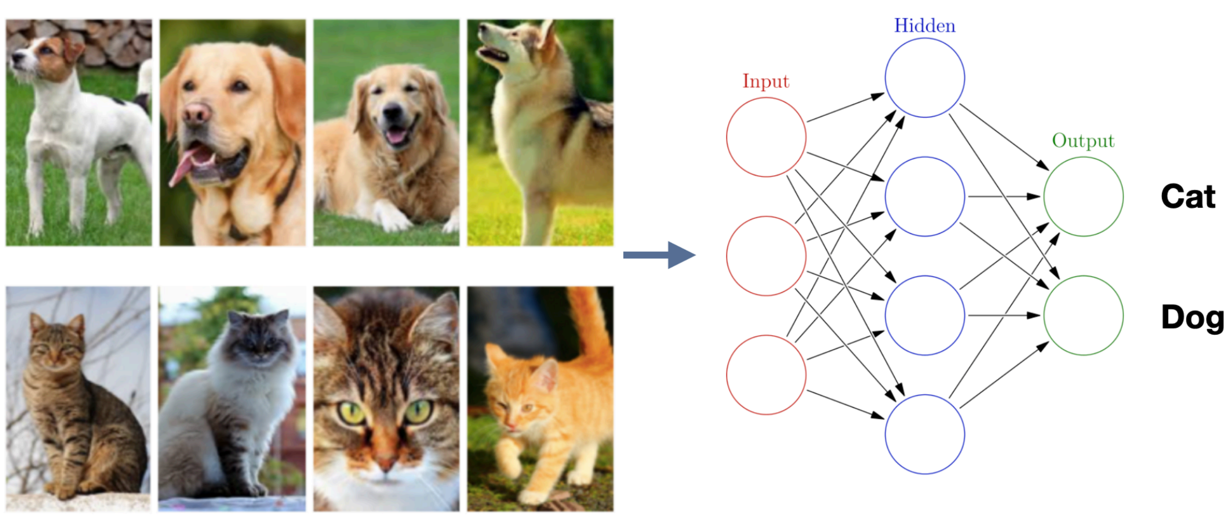
DOI: 10.1103/PhysRevLett.113.130503 PACS numbers: 03.67.Ac, 07.05.Mh

- 데이터의 가공/분류 기작
- 핵심계산 및 최적화 과정에의 유용한 양자 알고리즘/계산모듈 이식
- 양자정보/컴퓨팅 과학이 추구하는 양자속도향상 맥락의 이론분석

양자중첩 데이터-인코딩 & 양자머신러닝 속도향상

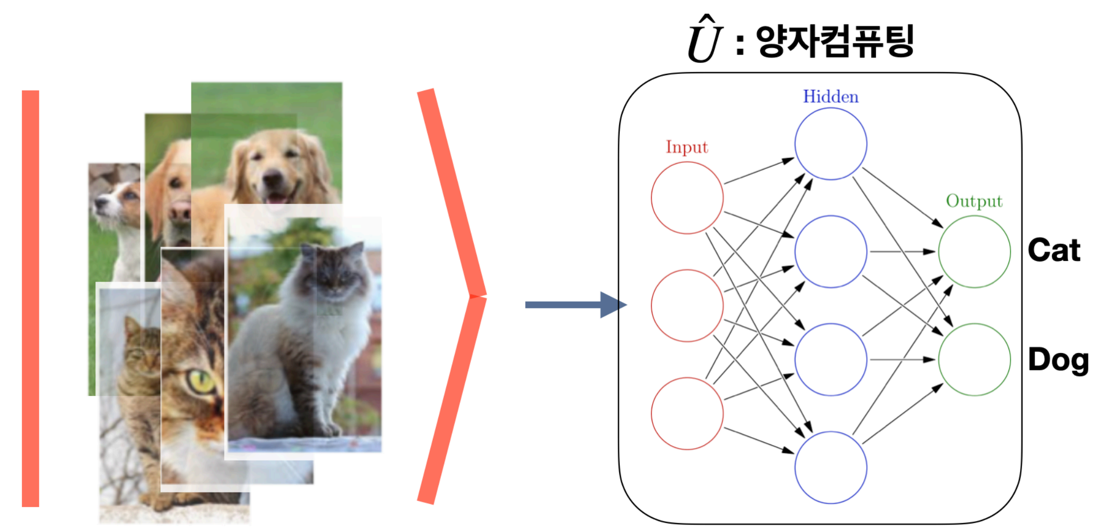
- **예제 : 개/고양이 분류**

고전머신러닝



학습 샘플

양자머신러닝



샘플의 양자 중첩

양자 병렬성을 활용한 빠른 학습

(고전) 서포트-벡터-머신

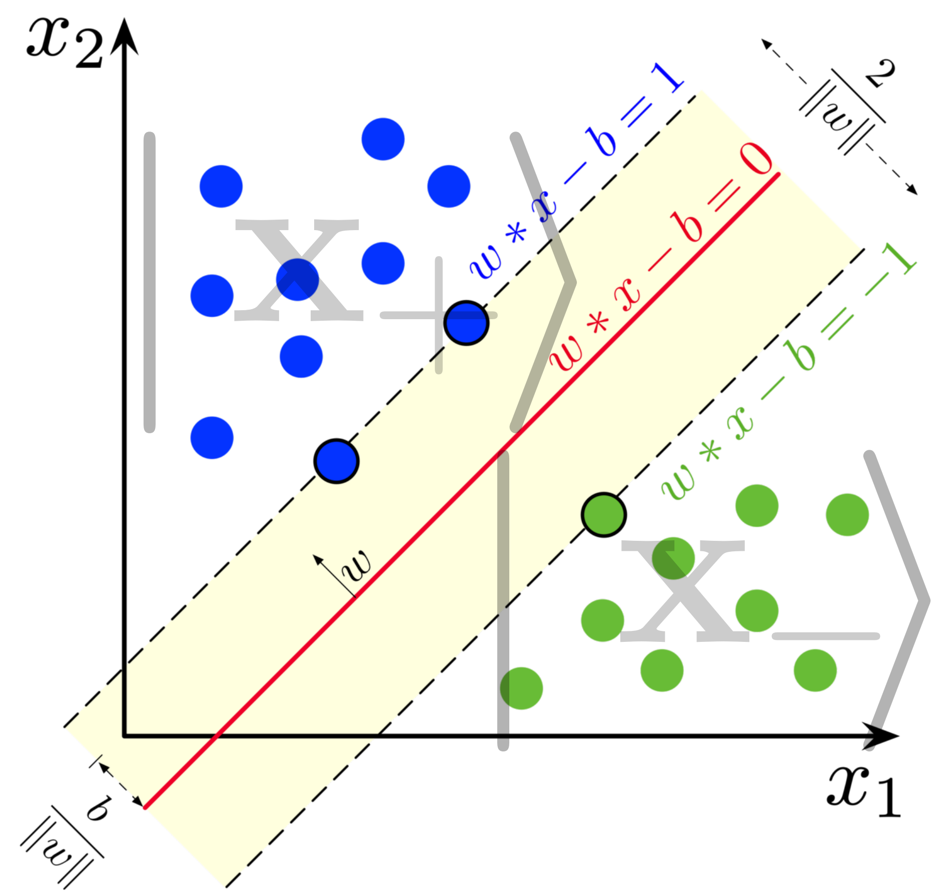
- A method to find an optimal hyper-plain to classify the given data

Decision Rule:

$$\mathbf{w}\cdot\mathbf{x}_{+}+b\geq+1\quad(y=+1\text{ for }\boldsymbol{x}_{+})$$
$$\mathbf{w}\cdot\mathbf{x}_{-}+b\leq-1\quad(y=-1\text{ for }\boldsymbol{x}_{-})$$

➡ $y_k\left(\mathbf{w}\cdot\mathbf{x}_k+b\right)-1\geq 0$

Margin: $\dfrac{2}{|\mathbf{w}|}$



Optimization Problem:

$max(|\mathbf{w}|^{-1})$ or $min(|\mathbf{w}|)$

For the constraints : $y_k\left(\mathbf{w}\ldots\mathbf{x}_k+b\right)-1=0$

➡ $\mathcal{L}=\dfrac{1}{2}|\mathbf{w}|^2-\sum_k\lambda_k y_k\left(\mathbf{w}\cdot\mathbf{x}_k+b\right)-1$ Solving Eq. $\partial\mathcal{L}=0$

(고전) 서포트-벡터-머신

- Linear SVM

$$\mathcal{L} = \frac{1}{2}|\mathbf{w}|^2 - \sum_k \lambda_k y_k \left(\mathbf{w}\cdot\mathbf{x}_k + b\right) - 1 \; + \eta_k \xi_k$$

when there is no solution hyper-plane

$$= \sum_k \lambda_k - \frac{1}{2}\sum_{j,k} \lambda_j K_{jk} \lambda_k \qquad \text{for } \sum_k \lambda_k = 0 \;\text{ and }\; y_k\lambda_k \geq 0$$

Kernel matrix: $K_{jk} = k(\mathbf{x}_j, \mathbf{x}_k) = \mathbf{x}_j \cdot \mathbf{x}_k$

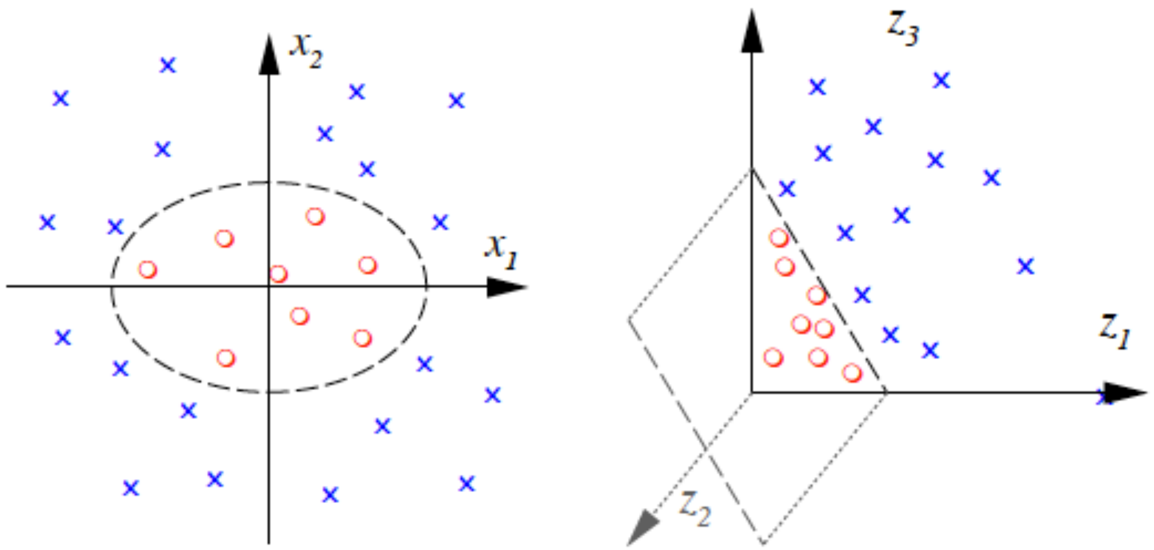
- Nonlinear SVM

<u>Kernel Method:</u> $\mathbf{x} \rightarrow \Phi(\mathbf{x})$

$$K'_{jk} = k(\phi(\mathbf{x}_j), \phi(\mathbf{x}_k)) = \phi(\mathbf{x}_j)\cdot\phi(\mathbf{x}_k)$$

Example: $\Phi : R^2 \rightarrow R^3$

$$(x_1, x_2) \mapsto (z_1, z_2, z_3) := (x_1^2, \sqrt(2) x_1 x_2, x_2^2)$$



양자 서포트-벡터-머신

[*Phys. Rev. Lett.* **113**,130503 (2014)]

- Quantum Support Vector Machine (QSVM)

  - Quantize the training data and labels :

  $$\mathbf{x} \in \mathcal{X} \rightarrow |x\rangle \in \mathcal{H} \;\text{ and }\; y \in \{|+1\rangle, |-1\rangle\}$$

  - The problem:

  $$\left\{(\mathbf{x}, y) : \mathbf{x} \in \mathcal{R}^n, y = \pm 1\right\} \Rightarrow \left\{(|\mathbf{x}\rangle, |y\rangle) : |\mathbf{x}\rangle \in \mathcal{H}^n, |y\rangle = |\pm 1\rangle\right\}$$

  - Adopting the HHL algorithm for solving linear equation

  Classical: $O(s\kappa N \log(1/\varepsilon)) \gg$ Quantum: $O(s^2\kappa^2 \log(N)/\varepsilon)$

- Nonlinear QSVM

  - ex) Polynomial kernel:

  $$k(\mathbf{x}_j, \mathbf{x}_k) = \phi(\mathbf{x}_j) \cdot \phi(\mathbf{x}_k) \Rightarrow \langle\phi(\mathbf{x}_j)|\phi(\mathbf{x}_k)\rangle = \langle\mathbf{x}_j|\mathbf{x}_k\rangle^d$$

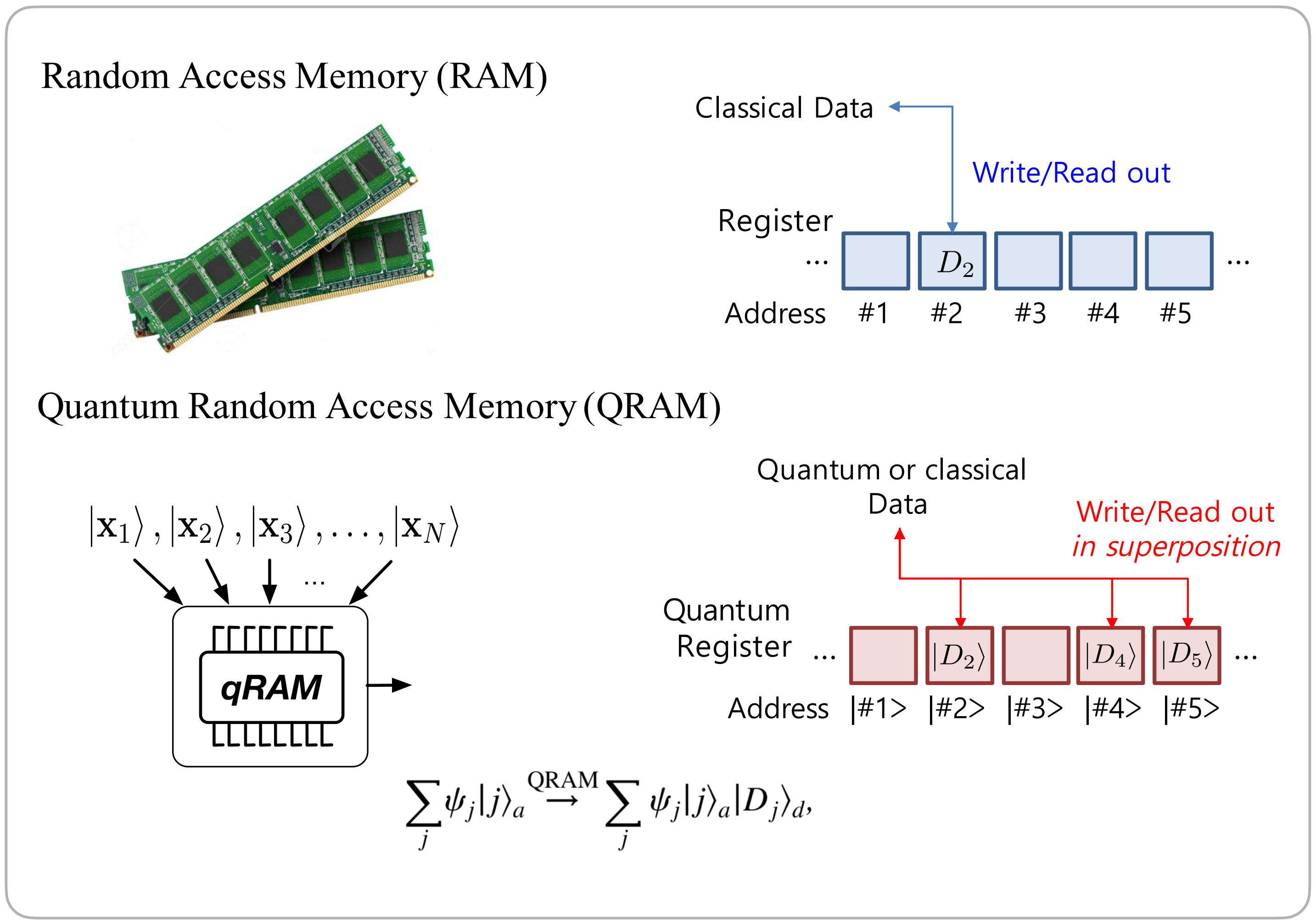
  where $|\phi(\mathbf{x}_j)\rangle = |\mathbf{x}_j\rangle \otimes \cdots \otimes |\mathbf{x}_j\rangle$

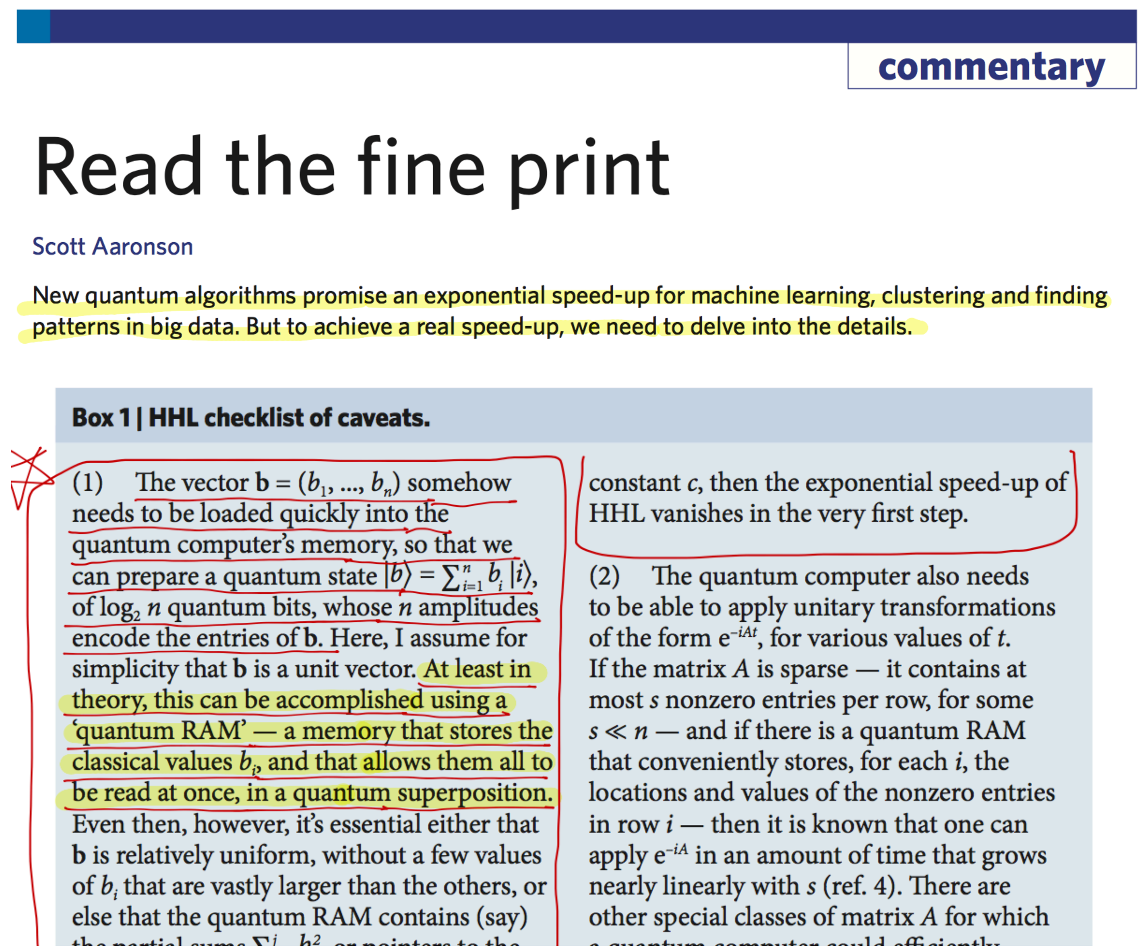
“HHL” (or “QSVE”) 기반 양자머신러닝 속도향상 연구

Problem	Scaling	QML applications
HHL for linear system	Classical: $O(s\kappa\, N\, log(1/\varepsilon))$ Quantum: $O(s^2\kappa^2\, log(N)/\varepsilon)$	Quantum SVM* [*Phys. Rev. Lett.* **113**,130503 (2014)] Quantum Regression [*arXiv:1512.03929* (2015)] Kernel Least Squares [*Phys. Rev. A* **94**, 022342 (2016)]
SVE	Classical: $O(k^2\, N\, log(1/\delta)/\varepsilon)$ Quantum: $O(\log(N)/\varepsilon^3)$	Recommendation System [*arXiv:1603.08675* (2016)] Quantum Linear Regression [*Phys. Rev. A* **94**, 022342 (2016)] Principal Component Analysis [*Nature Physics* **10**, 631 (2014)]

“HHL” 기반 양자머신러닝 속도향상은 가능한가?

- QRAM: 데이터의 (단기)저장 및 접근 등의 기능을 담당하는 양자소자
- QRAM의 정의/활용은 양자알고리즘 연구에 있어서 선택적 요소가 아닌 필수사항임
- QRAM 호출시 소요되는 양자리소스 등은 알고리즘 성능평가 및 양자이득 검증에 반드시 반영되어야 함





commentary

Read the fine print

Scott Aaronson

New quantum algorithms promise an exponential speed-up for machine learning, clustering and finding patterns in big data. But to achieve a real speed-up, we need to delve into the details.

Box 1 | HHL checklist of caveats.

(1) The vector $\mathbf{b} = (b_1, ..., b_n)$ somehow needs to be loaded quickly into the quantum computer's memory, so that we can prepare a quantum state $|b\rangle = \sum_{i=1}^{n} b_i |i\rangle$, of $\log_2 n$ quantum bits, whose n amplitudes encode the entries of $\mathbf{b}$. Here, I assume for simplicity that $\mathbf{b}$ is a unit vector. At least in theory, this can be accomplished using a 'quantum RAM' — a memory that stores the classical values b_i, and that allows them all to be read at once, in a quantum superposition. Even then, however, it's essential either that $\mathbf{b}$ is relatively uniform, without a few values of b_i that are vastly larger than the others, or else that the quantum RAM contains (say) … constant c, then the exponential speed-up of HHL vanishes in the very first step.

(2) The quantum computer also needs to be able to apply unitary transformations of the form e^{-iAt}, for various values of t. If the matrix A is sparse — it contains at most s nonzero entries per row, for some $s \ll n$ — and if there is a quantum RAM that conveniently stores, for each i, the locations and values of the nonzero entries in row i — then it is known that one can apply e^{-iA} in an amount of time that grows nearly linearly with s (ref. 4). There are other special classes of matrix A for which …

양자 랜덤-엑세스-메모리: bucket-brigade scheme

<u>Fast Quantum Random Access Memory (QRAM)</u>: An (imaginary) quantum gadget that is capable to fast encode/decode large data into quantum superposition.

- e.g., proposed a 'bucket-brigade' architecture, it can encode N d-dim. classical vectors into $log(Nd)$ qubits in $O(log(Nd))$ time.

[V. Giovannetti, S. Lloyd, and L. Maccone, *Phys. Rev. A* **78**, 052310 (2008)]
[V. Giovannetti, S. Lloyd, and L. Maccone, *Phys. Rev. Lett.* **100**, 160501 (2008)]

Controversial Issues:

- *First issue is whether all the components require to be error-corrected*.
- The QRAM should have the data distributed in a relatively uniform manner.
- As a last comment, the possibility of fast loading the data (particularly when the data-size is considerably large) is now controversial due to the communication speed limited by light-speed. This would requires very big memory structures.

[S. Arunachalam et al, *New J. Phys.* 17, 123010 (2015)]

결함허용(Fault-tolerant) 관점에서의 QRAM: error robustness

Error correction: exponential resources, again!

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PAPER

On the robustness of bucket brigade quantum RAM

Srinivasan Arunachalam[1,2], Vlad Gheorghiu[2,3,10], Tomas Jochym-O'Connor[2,4], Michele Mosca[2,3,5,6] and Priyaa Varshinee Srinivasan[7,8,9]

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Keywords: quantum memories, quantum error correction, quantum algorithms

Abstract

We study the robustness of the bucket brigade quantum random access memory model introduced by Giovannetti *et al* (2008 *Phys. Rev. Lett.* **100** 160501). Due to a result of Regev and Schiff (ICALP '08 733), we show that for a class of error models the error rate per gate in the bucket brigade quantum memory has to be of order $o(2^{-n/2})$ (where $N = 2^n$ is the size of the memory) whenever the memory is used as an oracle for the quantum searching problem. We conjecture that this is the case for any realistic error model that will be encountered in practice, and that for algorithms with super-polynomially many oracle queries the error rate must be super-polynomially small, which further motivates the need for quantum error correction. By contrast, for algorithms such as matrix inversion Harrow *et al* (2009 *Phys. Rev. Lett.* **103** 150502) or quantum machine learning Rebentrost *et al* (2014 *Phys. Rev. Lett.* **113** 130503) that only require a polynomial number of queries, the error rate only needs to be polynomially small and quantum error correction may not be required. We introduce a circuit model for the quantum bucket brigade architecture and argue that quantum error correction for the circuit causes the quantum bucket brigade architecture to lose its primary advantage of a small number of 'active' gates, since all components have to be actively error corrected.

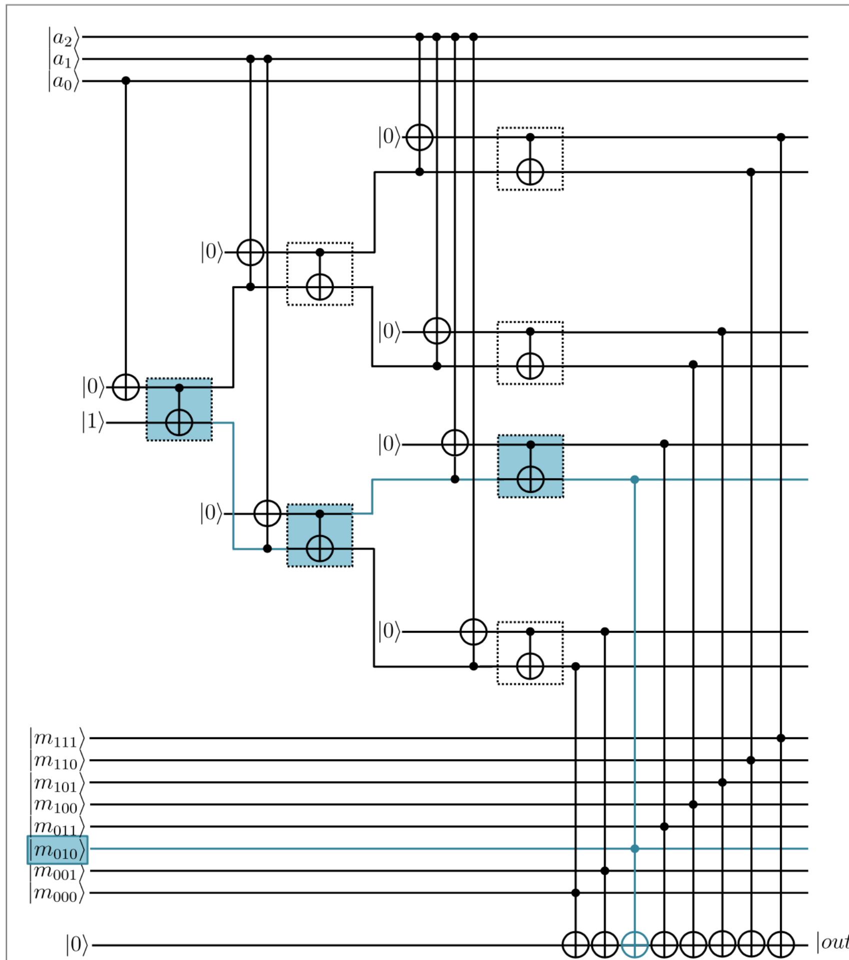


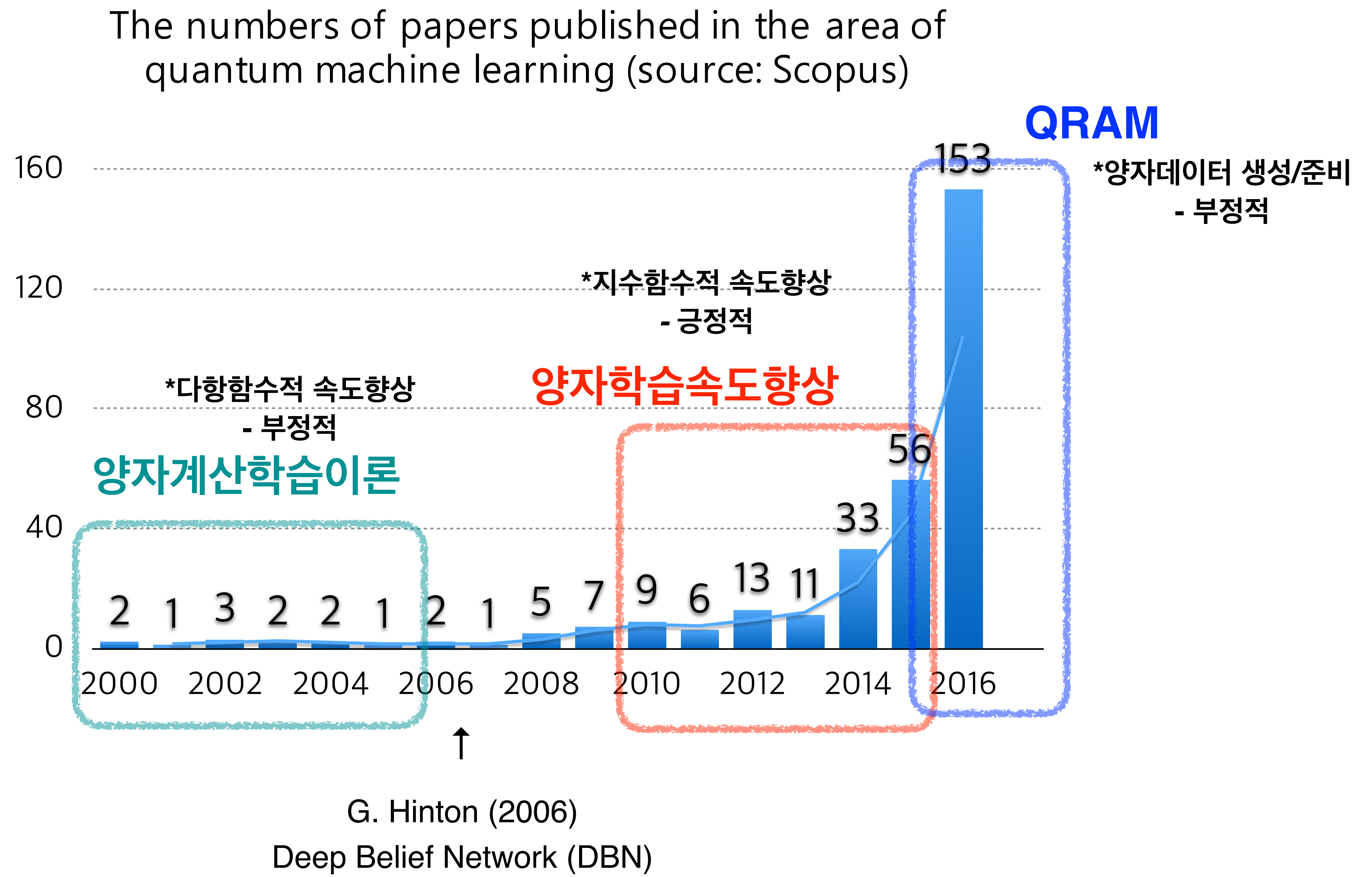
Figure 9. Circuit for bucket brigade qRAM. Nodes to the left of the memory cell are *routing nodes*. The dashed squares represents the memory locations. The first layer of nodes immediately to the right of the memory are the *coupling nodes*. Finally, the nodes on the right are the *read out nodes*. A possible input is e.g. $|a_0 a_1 a_2\rangle = |010\rangle$, for which the circuit reads the memory location m_{010}. The path leading to the location m_{010} is represented in blue color, and the active routing and readout nodes are highlighted. One could more closely mimic the physical flow of information in the bucket brigade qRAM by adding an additional qubit at each node in the binary tree we see in the diagram. Then, for $k \in \{0, \ldots, n-1\}$, we add an initial controlled-NOT (CNOT) gate to copy a_k to the root node, followed by a series of $\mathcal{O}(2^k)$ controlled-SWAPs that will bring the value of a_k to the unique node in level k defined by the bits $a_0, a_1, \ldots, a_{k-1}$. While this adds exponentially many gates, it does not change the overall gate complexity, and these additional gates only add $\mathcal{O}(k)$ to the depth of the circuit. This also illustrates that the exponential depth implicit in the circuit we describe in the diagram can easily be reduced to polynomial depth by further mimicking the ideas presented in the qRAM proposal. We leave the circuit diagram in this simpler form, since it does not affect our arguments in sections 3 and 5.

- QRAM query implementation
  → circuit-model description
- Error rate per gate: $O(e^{-n})$ is required!
  → Computational resources: exponential, again!

양자머신러닝 속도향상 커널 및 QRAM

Method	Speedup	Amplitude amplification	HHL	Adiabatic	qRAM
Bayesian inference[106,107]	$O(\sqrt{N})$	Yes	Yes	No	No
Online perceptron[108]	$O(\sqrt{N})$	Yes	No	No	Optional
Least-squares fitting[9]	$O(\log N)$*	Yes	Yes	No	Yes
Classical Boltzmann machine[20]	$O(\sqrt{N})$	Yes/No	Optional/No	No/Yes	Optional
Quantum Boltzmann machine[22,61]	$O(\log N)$*	Optional/No	No	No/Yes	No
Quantum PCA[11]	$O(\log N)$*	No	Yes	No	Optional
Quantum support vector machine[13]	$O(\log N)$*	No	Yes	No	Yes
Quantum reinforcement learning[30]	$O(\sqrt{N})$	Yes	No	No	No

양자머신러닝 연구의 문제



양자머신러닝 연구의 문제

중첩데이터 생성문제:

- QRAM을 가정하는 양자컴퓨팅/머신러닝의 지수함수적 속도향상 증명은 불완전

[E. Tang, *Phys. Rev. Lett.* **127**, 060503 (2021)]

PHYSICAL REVIEW LETTERS **127,** 060503 (2021)

Quantum Principal Component Analysis Only Achieves an Exponential Speedup Because of Its State Preparation Assumptions

Ewin Tang*
University of Washington, Seattle, Washington 98195, USA

(Received 21 November 2019; revised 3 June 2021; accepted 1 July 2021; published 4 August 2021)

"From this work, we conclude that the exponential speedups of the quantum algorithms that we consider arise from strong input assumptions rather than from the "quantumness" of the algorithms since the speedups vanish when classical algorithms are given analogous assumptions."

- QRAM을 가정하는 양자컴퓨팅/머신러닝의 속도향상은 고전컴퓨터로 흉내(dequantizing)가 가능

[A. Bakshi, E. Tang, arXiv:2303.01492v2 (2023)]

An Improved Classical Singular Value Transformation for Quantum Machine Learning

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Ewin Tang
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University of Washington

March 7, 2023

THANK YOU

Part I

양자컴퓨팅 / 알고리즘 연구

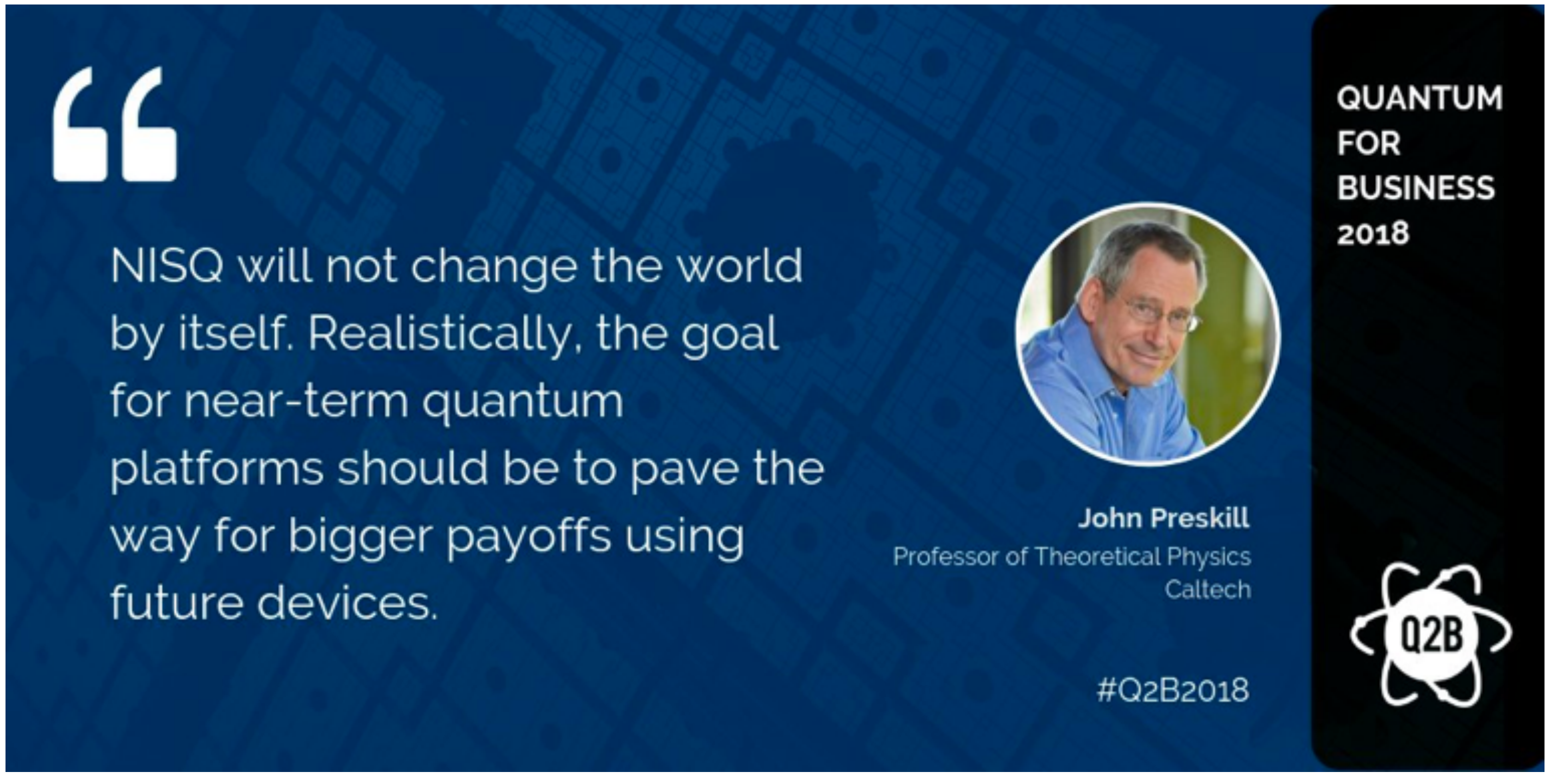
양자BLAS (HHL) / 양자서포트벡터머신

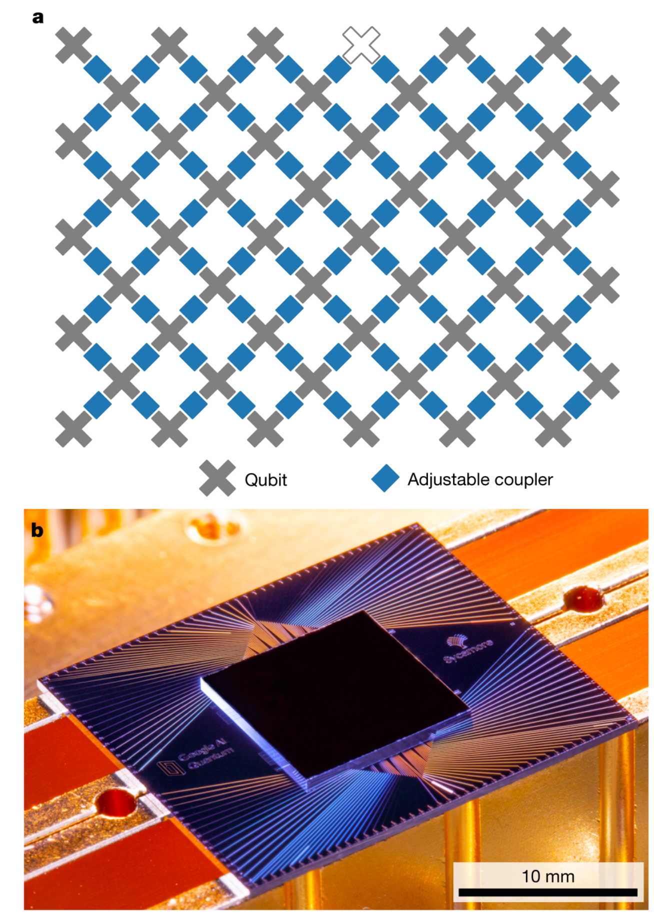
Part II

NISQ와 양자머신러닝

변분법 양자알고리즘(VQA)과 양자인공신경망

Noisy Intermediate-Scale Quantum (NISQ)





Google's Sycamore Processor

- 최근 양자정보 연구동향/비전: 국소적, 작업-편중형(Task-Oriented)
- 양자연산의 기능적 최적화 → 측정데이터/확률분포에의 양자효과 분석/활용 (e.g., 샘플링 문제)
- ***NISQ 시대 도래:**

  - 오류를 허용하고(Noisy)
  - 중규모에서(Intermediate-Scale)에서
  - 실현 가능한 형태의 양자(Quantum) 기술의 추구

Google 양자우월성 증명

Google's quantum supremacy proof

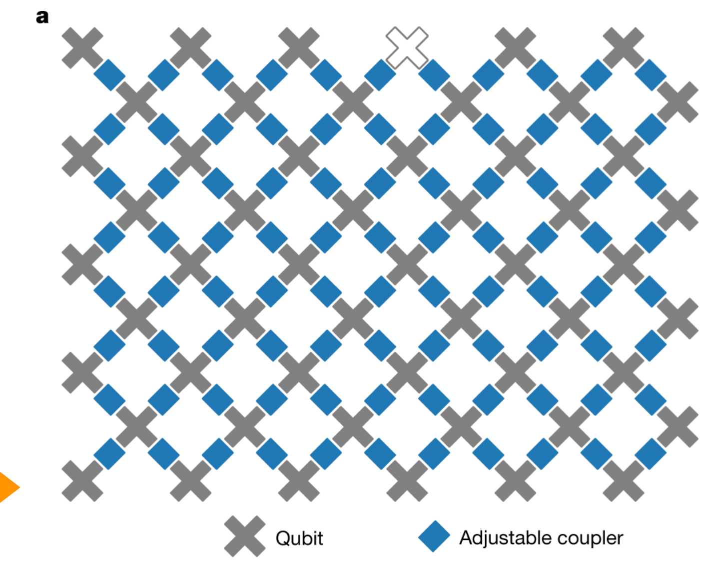
The leading quantum supremacy proposals:

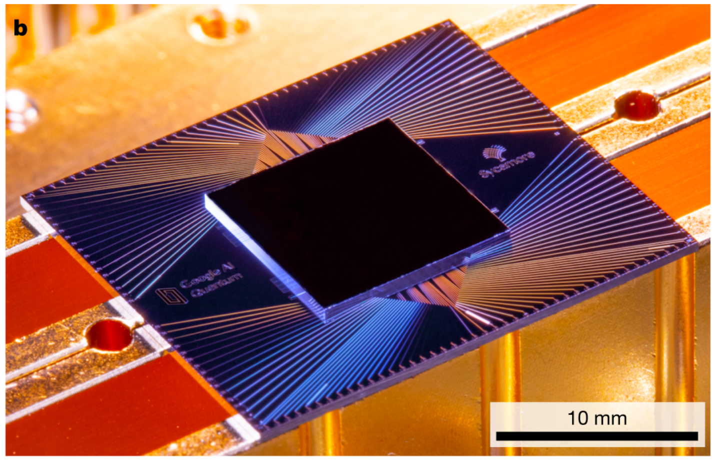
- Boson Sampling
- Fourier Sampling
- Instantaneous Quantum Polynomial-time (IQP)
- Random Circuit Sampling (RCS)

	Worst-case hardness	Average-case hardness	Anti-Concentration	Experimentally Feasible
Boson Sampling	OK	OK		
Fourier Sampling	OK	OK		
IQP	OK		OK	
RCS	OK	OK	OK	OK

[Nature Physics **15**, 159 (2019)] [Nature Physics **14**, 595 (2018)]

Theoretical proof





Google's Sycamore Processor

[Nature **574**, 505 (2019)]

Hardware demonstration

Interplay between software and NISQ hardware

Google 양자우월성 증명

nature physics

ARTICLES

https://doi.org/10.1038/s41567-018-0124-x

Characterizing quantum supremacy in near-term devices

Sergio Boixo[1*], Sergei V. Isakov[2], Vadim N. Smelyanskiy[1], Ryan Babbush[1], Nan Ding[1], Zhang Jiang[3,4], Michael J. Bremner[5], John M. Martinis[6,7] and Hartmut Ne

**A critical question for quantum computing in the near future is whether qu
a well-defined computational task beyond the capabilities of supercompu
quantum supremacy requires a reliable evaluation of the resources requir
propose the task of sampling from the output distribution of random quan
acy. We extend previous results in computational complexity to argue th
a classical computer. We introduce cross-entropy benchmarking to obt
dynamics. This can be estimated and extrapolated to give a success metri
the computational cost of relevant classical algorithms and conclude that
a two-dimensional lattice of 7×7 qubits and around 40 clock cycles. This
gates (0.05% for one-qubit gates), and it would demonstrate the basic bu**

nature physics

ARTICLES

https://doi.org/10.1038/s41567-018-0318-2

On the complexity and verification of quantum random circuit sampling

Adam Bouland[1], Bill Fefferman[1,2*], Chinmay Nirkhe[1] and Umesh Vazirani[1]

A critical milestone on the path to useful quantum computers is the demonstration of a quantum computation that is prohibitively hard for classical computers—a task referred to as quantum supremacy. A leading near-term candidate is sampling from the probability distributions of randomly chosen quantum circuits, which we call random circuit sampling (RCS). RCS was defined with experimental realizations in mind, leaving its computational hardness unproven. Here we give strong complexity-theoretic evidence of classical hardness of RCS, placing it on par with the best theoretical proposals for supremacy. Specifically, we show that RCS satisfies an average-case hardness condition, which is critical to establishing computational hardness in the presence of experimental noise. In addition, it follows from known results that RCS also satisfies an anti-concentration property, namely that errors in estimating output probabilities are small with respect to the probabilities themselves. This makes RCS the first proposal for quantum supremacy with both of these properties. Finally, we also give a natural condition under which an existing statistical measure, cross-entropy, verifies RCS, as well as describe a new verification measure that in some formal sense maximizes the information gained from experimental samples.

Google 양자우월성 증명

Article

Quantum supremacy using a programmable superconducting processor

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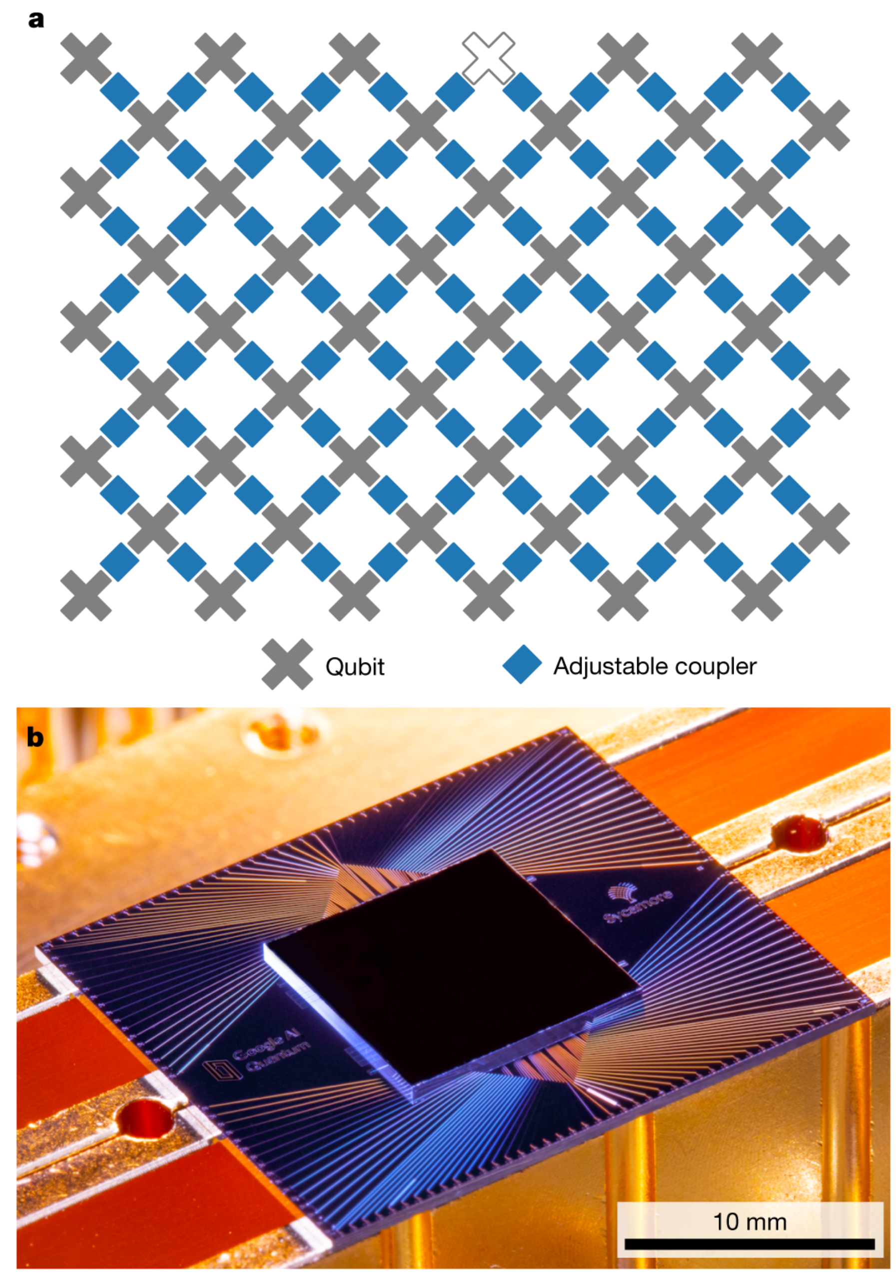
Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1], Rupak Biswas[3], Sergio Boixo[1], Fernando G. S. L. Brandao[1,4], David A. Buell[1], Brian Burkett[1], Yu Chen[1], Zijun Chen[1], Ben Chiaro[5], Roberto Collins[1], William Courtney[1], Andrew Dunsworth[1], Edward Farhi[1], Brooks Foxen[1,5], Austin Fowler[1], Craig Gidney[1], Marissa Giustina[1], Rob Graff[1], Keith Guerin[1], Steve Habegger[1], Matthew P. Harrigan[1], Michael J. Hartmann[1,6], Alan Ho[1], Markus Hoffmann[1], Trent Huang[1], Travis S. Humble[7], Sergei V. Isakov[1], Evan Jeffrey[1], Zhang Jiang[1], Dvir Kafri[1], Kostyantyn Kechedzhi[1], Julian Kelly[1], Paul V. Klimov[1], Sergey Knysh[1], Alexander Korotkov[1,8], Fedor Kostritsa[1], David Landhuis[1], Mike Lindmark[1], Erik Lucero[1], Dmitry Lyakh[9], Salvatore Mandrà[3,10], Jarrod R. McClean[1], Matthew McEwen[5], Anthony Megrant[1], Xiao Mi[1], Kristel Michielsen[11,12], Masoud Mohseni[1], Josh Mutus[1], Ofer Naaman[1], Matthew Neeley[1], Charles Neill[1], Murphy Yuezhen Niu[1], Eric Ostby[1], Andre Petukhov[1], John C. Platt[1], Chris Quintana[1], Eleanor G. Rieffel[3], Pedram Roushan[1], Nicholas C. Rubin[1], Daniel Sank[1], Kevin J. Satzinger[1], Vadim Smelyanskiy[1], Kevin J. Sung[1,13], Matthew D. Trevithick[1], Amit Vainsencher[1], Benjamin Villalonga[1,14], Theodore White[1], Z. Jamie Yao[1], Ping Yeh[1], Adam Zalcman[1], Hartmut Neven[1] & John M. Martinis[1,5]*

The promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor[1]. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here we report the use of a processor with programmable superconducting qubits[2–7] to create quantum states on 53 qubits, corresponding to a computational state-space of dimension 2^{53} (about 10^{16}). Measurements from repeated experiments sample the resulting probability distribution, which we verify using classical simulations. Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years. This dramatic increase in speed compared to all known classical algorithms is an experimental realization of quantum supremacy[8–14] for this specific computational task, heralding a much-anticipated computing paradigm.

In the early 1980s, Richard Feynman proposed that a quantum computer would be an effective tool with which to solve problems in physics and chemistry, given that it is exponentially costly to simulate large quantum systems with classical computers[1]. Realizing Feynman's vision poses substantial experimental and theoretical challenges. First, can a quantum system be engineered to perform a computation in a large enough computational (Hilbert) space and with a low enough error rate to provide a quantum speedup? Second, can we formulate a problem that is hard for a classical computer but easy for a quantum computer? By computing such a benchmark task on our superconducting qubit processor, we tackle both questions. Our experiment achieves quantum supremacy, a milestone on the path to full-scale quantum computing[8–14].

In reaching this milestone, we show that quantum speedup is achievable in a real-world system and is not precluded by any hidden physical laws. Quantum supremacy also heralds the era of noisy intermediate-scale quantum (NISQ) technologies[15]. The benchmark task we demonstrate has an immediate application in generating certifiable random numbers (S. Aaronson, manuscript in preparation); other initial uses for this new computational capability may include optimization[16,17], machine learning[18–21], materials science and chemistry[22–24]. However, realizing the full promise of quantum computing (using Shor's algorithm for factoring, for example) still requires technical leaps to engineer fault-tolerant logical qubits[25–29].

To achieve quantum supremacy, we made a number of technical advances which also pave the way towards error correction. We



Google's Sycamore Processor

NISQ 시대의 양자머신러닝

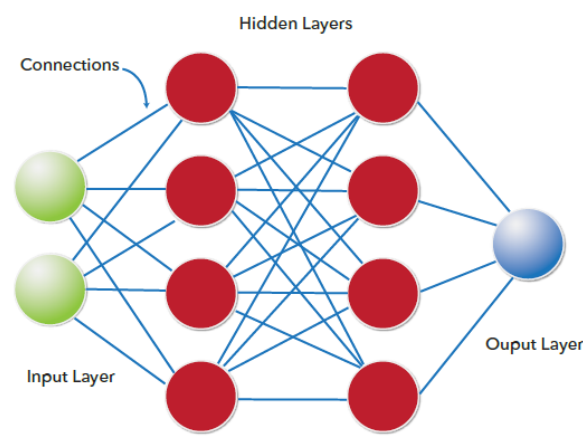
Data

Classical-data

vs

Quantum-data

Model

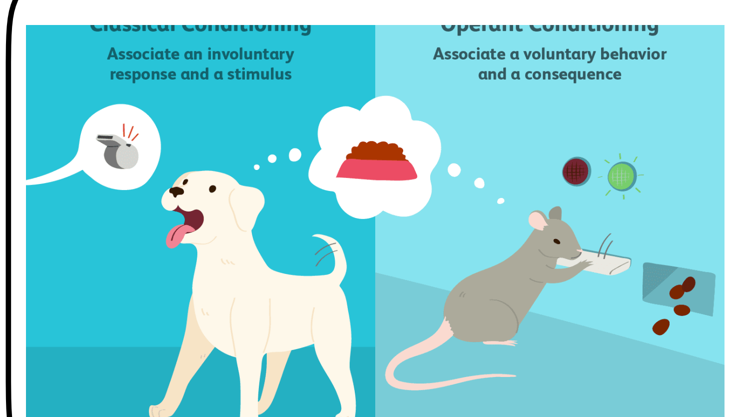


· Kernel Method

· Parametric Quantum Circuits = QNN

(*Variational Quantum Algorithms)

Training & Generalization

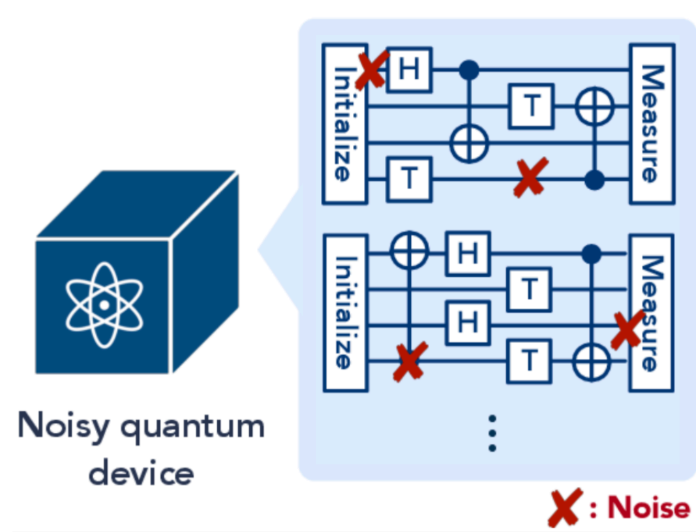


· Loss Function

· Control Parameter Space

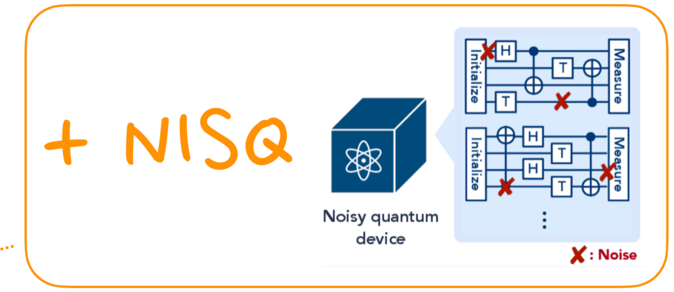
· Quantum Training?

+ NISQ



NISQ & 양자머신러닝: 양자 데이터 생성/활용

■ Quantum Data Embedding



user-recognizable (or classical) data → quantum state

$$\mathbf{x}_j \rightarrow |\psi(\mathbf{x}_j)\rangle \quad \text{or} \quad \hat{U}(\mathbf{x}_j)|00...0\rangle = |\psi(\mathbf{x}_j)\rangle$$

- 원칙적으로 모든 데이터는 큐비트 시스템에 효율적으로 인코딩 가능
- n 고전비트 → n 큐비트 (일반적으로 역은 성립하지 않음)
- 큐비트는(즉, 힐버트 공간은) 어떠한 물리적 과정에서 얻게되는 정보(양자정보 포함)도 인코딩 가능한 궁극적 데이터 표현매체

[*Phys. Rev. Lett.* **122**, 040504 (2019)] [arXiv:2001.03622 (2020)]

1. 얼마나 잘(즉, 효율적으로) 고전 데이터를 양자 상태에 인코딩할 수 있을것인가?
   - 임베딩 프로토콜의 출력(양자)상태간의 내적이 고전적으로 시뮬레이션하기 어려운 형태여야 함 ⇨ Kernel Method 양자이득
   - 임베딩 프로토콜의 출력(양자)상태가 힐버트 공간 내 구별 가능한 영역에 있어야 함 ⇨ Classification 양자이득

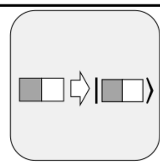
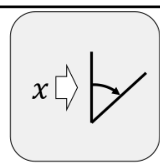
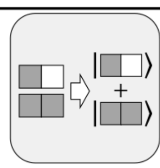
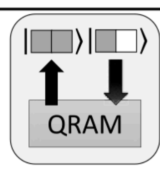
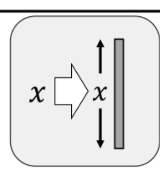
2. Quantum Dataset? (MNIST, dogs vs cats, Iris, etc)

NISQ & 양자머신러닝: 양자 데이터 생성/활용

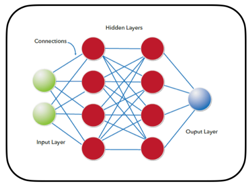
■ 양자 데이터 임베딩

user-recognizable (or classical) data → quantum state

$$\mathbf{x}_j \to |\psi(\mathbf{x}_j)\rangle \quad \text{or} \quad \hat{U}(\mathbf{x}_j)\,|00...0\rangle = |\psi(\mathbf{x}_j)\rangle$$

Encoding pattern		Encoding	Req. qubits
	BASIS ENCODING	$x_i \approx \sum_{i=-k}^{m} b_i 2^i \mapsto \|b_m \dots b_{-k}\rangle$	$l = k+m$ per data-point
	**ANGLE ENCODING**	$x_i \mapsto cos(x_i)\|0\rangle + sin(x_i)\|1\rangle$	1 per data-point
	QUAM ENCODING	$X \mapsto \sum_{i=0}^{n-1} \frac{1}{\sqrt{n}} \|x_i\rangle$	l
	**QRAM ENCODING**	$X \mapsto \sum_{n=0}^{n-1} \frac{1}{\sqrt{n}} \|i\rangle \|x_i\rangle$	$\lceil \log n \rceil + l$
	AMPLITUDE ENCODING	$X \mapsto \sum_{i=0}^{n-1} x_i \|i\rangle$	$\lceil \log n \rceil$

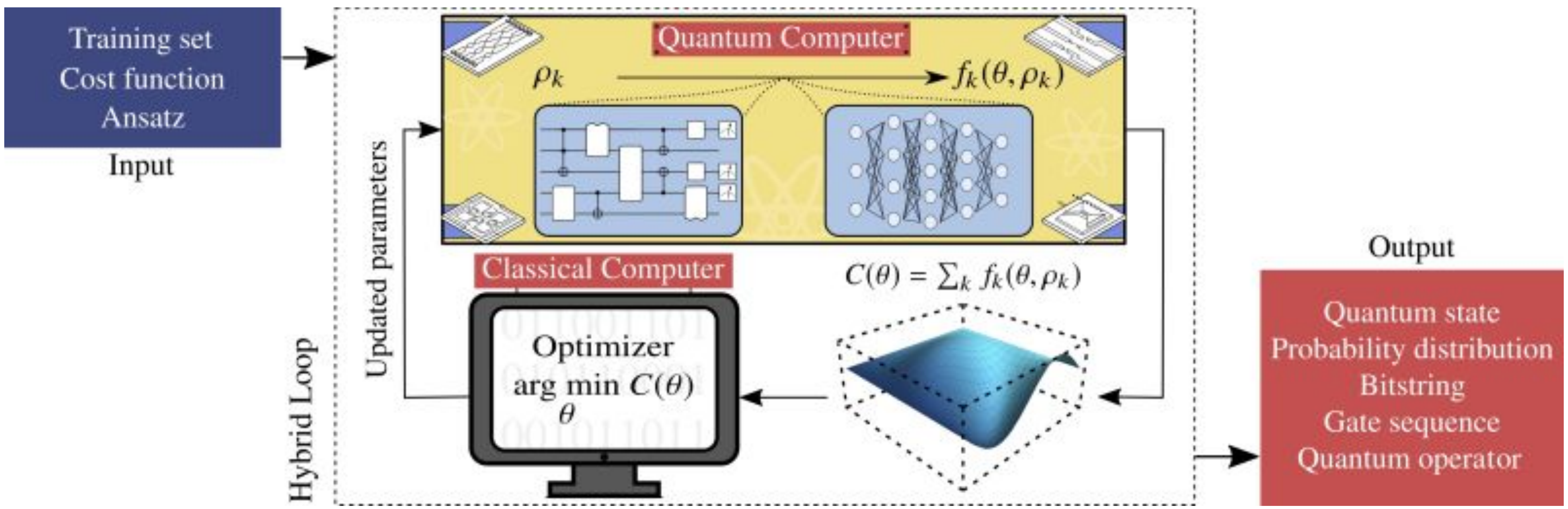
IET Quantum Communications **2**, 141 (2021)



NISQ & 양자머신러닝: 양자모델

■ Variational Quantum Algorithms (VQAs)

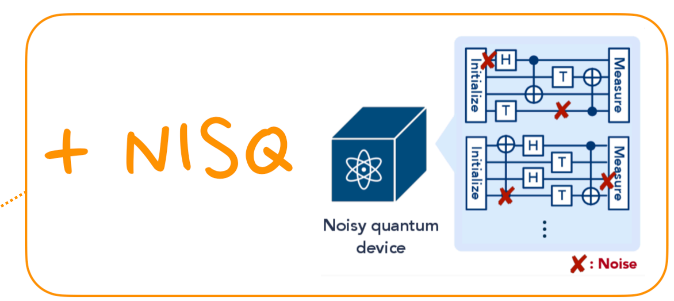
- 양자 + 고전 하이브리드 알고리즘
- 양자인공신경망 기본모형



[Cerezo *et al.*, Nature Review Physics (2021)]

■ Quantum Neural Network (QNN)

- QML 모델의 가장 기본적이고 핵심적인 요소 ⇨ 매개변수화된 양자 회로(PQCs)



e.g.)

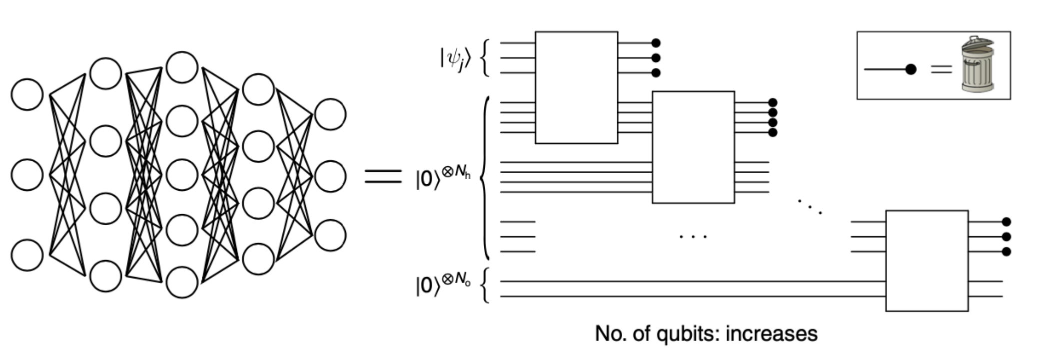
서로다른 클래스의 양자상태들을 힐버트 공간의 구별 가능한 영역으로 매핑 ⇨ Classification (Supervised learning)

MAXCUT 문제 매핑 ⇨ Clustering (Unsupervised learning)

[*Phys. Rev. Lett.* **122**, 040504 (2019)] [arXiv:1712.05771 (2017)]

NISQ & 양자머신러닝: 양자모델

■ Quantum Neural Network (QNN)

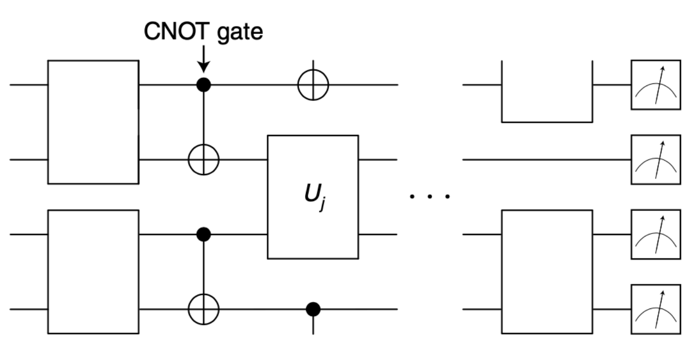


Dissipative model of QNN

- 전통적인 피드포워드 네트워크를 일반화한 유니타리 연산
- 레이어 깊이 증가 → 큐비트 수 증가

* 확산형(dissipative): 레이어 내의 큐비트들이 정보를 다음 레이어의 (새로운) 큐비트들로 전달된 후 버려진다는 것을 의미

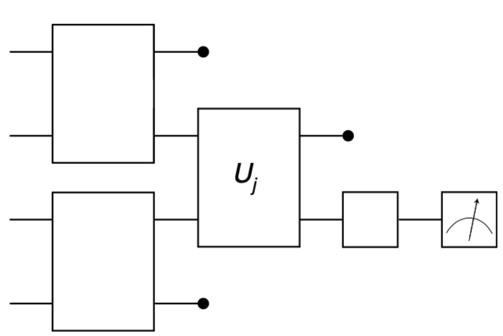
[*Nat. Commun.* **11**, 808 (2020)]



Iterative model of QNN

- 표준적인 네트워크 모델
- 레이어 깊이 증가 → 큐비트 수 고정

[*Quant. Inf. Proc.* **13**, 267 (2014)] [arXiv:1802.06002 (2018)]



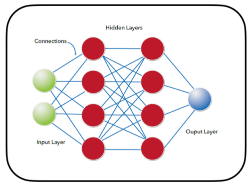
QCNN

- 각 레이어에서 큐비트들이 측정되어 데이터의 차원을 줄임

  ⇨ 관련 특성들의 보존
- 레이어 깊이 증가 → 큐비트 수 고정

[*Nat. Phys.* **15**, 1273 (2019)]

[Cerezo *et al.*, Nature Computational Science (2022)]

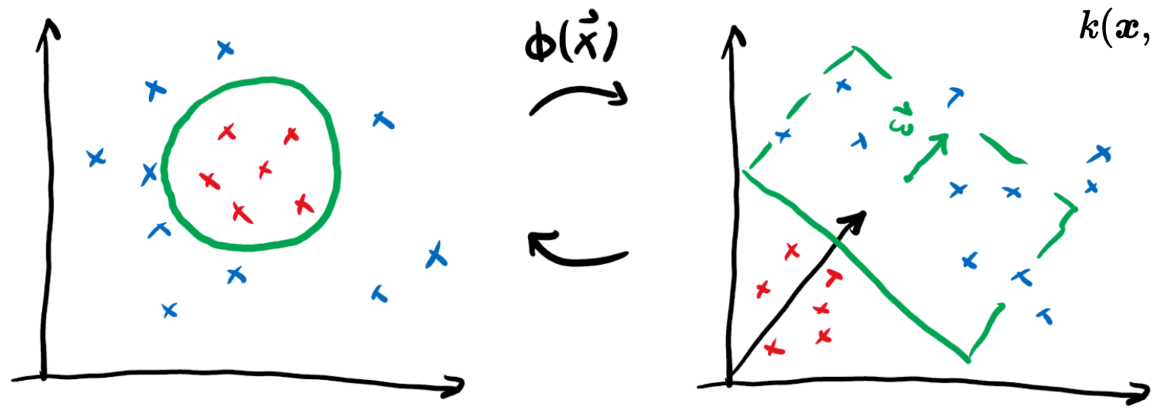


NISQ & 양자머신러닝: 양자모델

■ Kernel Method

Ex) $\phi((x_1, x_2)) = (x_1^2, \sqrt{2}x_1x_2, x_2^2)$

$k(\boldsymbol{x}, \boldsymbol{y}) = x_1^2y_1^2 + 2x_1x_2y_1y_2 + x_2^2y_2^2 = \langle \boldsymbol{x}, \boldsymbol{y} \rangle^2$



Kernel: $k(\boldsymbol{x}_i, \boldsymbol{x}_j) = \langle \phi(\boldsymbol{x}_i), \phi(\boldsymbol{x}_j) \rangle$

$y(\boldsymbol{x}) = \mathrm{sgn}(\langle \boldsymbol{w}, \boldsymbol{x} \rangle + b)$

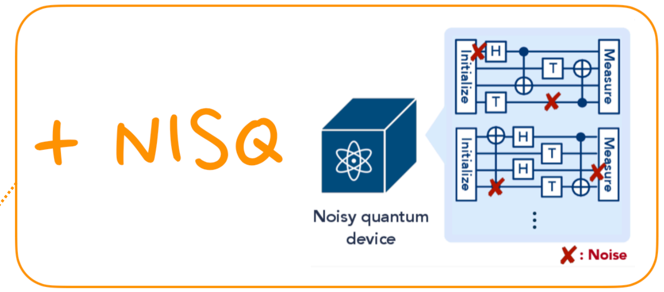
$y(\boldsymbol{x}) = \mathrm{sgn}(\langle \boldsymbol{w}, \phi(\boldsymbol{x}) \rangle + b)$

$\boldsymbol{w} = \sum_i \alpha_i \phi(\boldsymbol{x}_i).$

$$y(\boldsymbol{x}) = \mathrm{sgn}\left(\sum_i \alpha_i \langle \phi(\boldsymbol{x}_i), \phi(\boldsymbol{x}) \rangle + b \right)$$

<https://pennylane.ai/qml/demos/tutorial_kernels_module>

■ Quantum Kernel Method



- QML 모델로서 기존 커널 방법의 양자버전이 제안됨
- 각 입력 데이터를 고차원 벡터 공간인 (재생 커널) 힐버트 공간으로 매핑 ⇨ 재생 커널 힐버트 공간에서 선형 함수를 학습

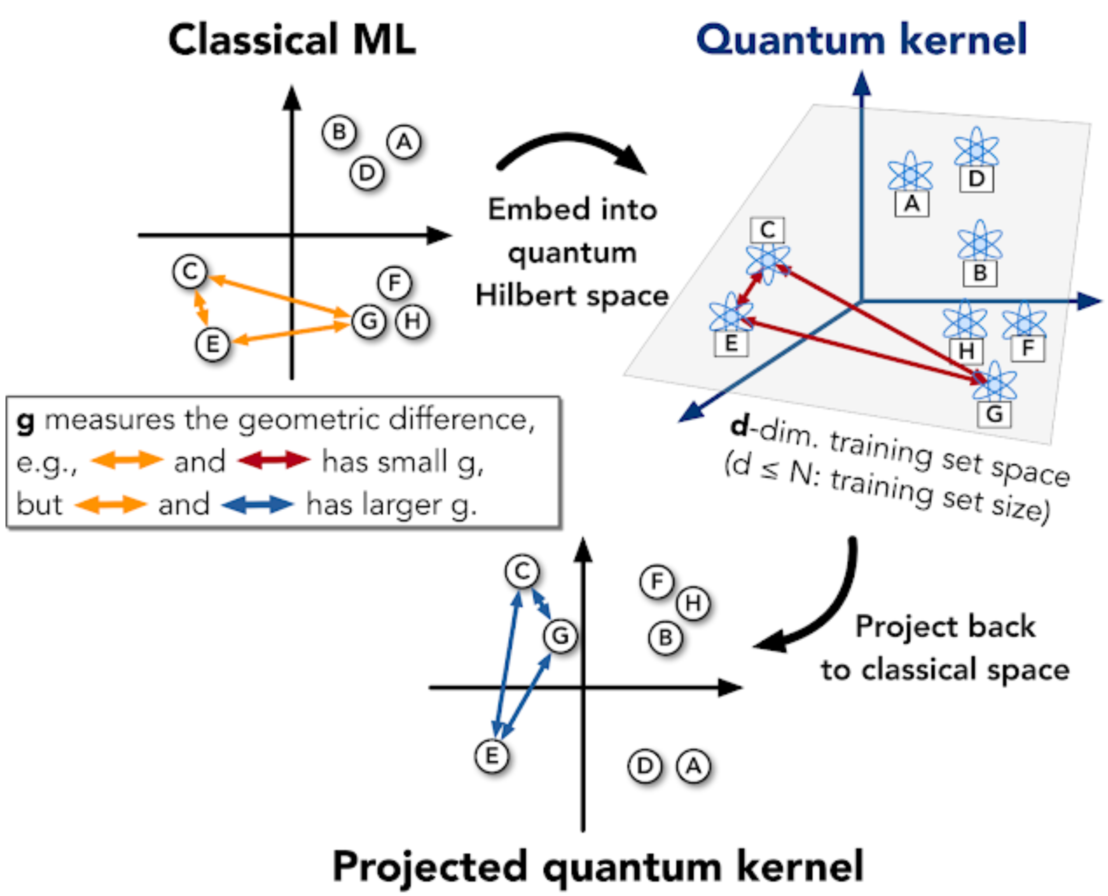
$$|\psi(\boldsymbol{x})\rangle = U(\boldsymbol{x})|0\rangle$$

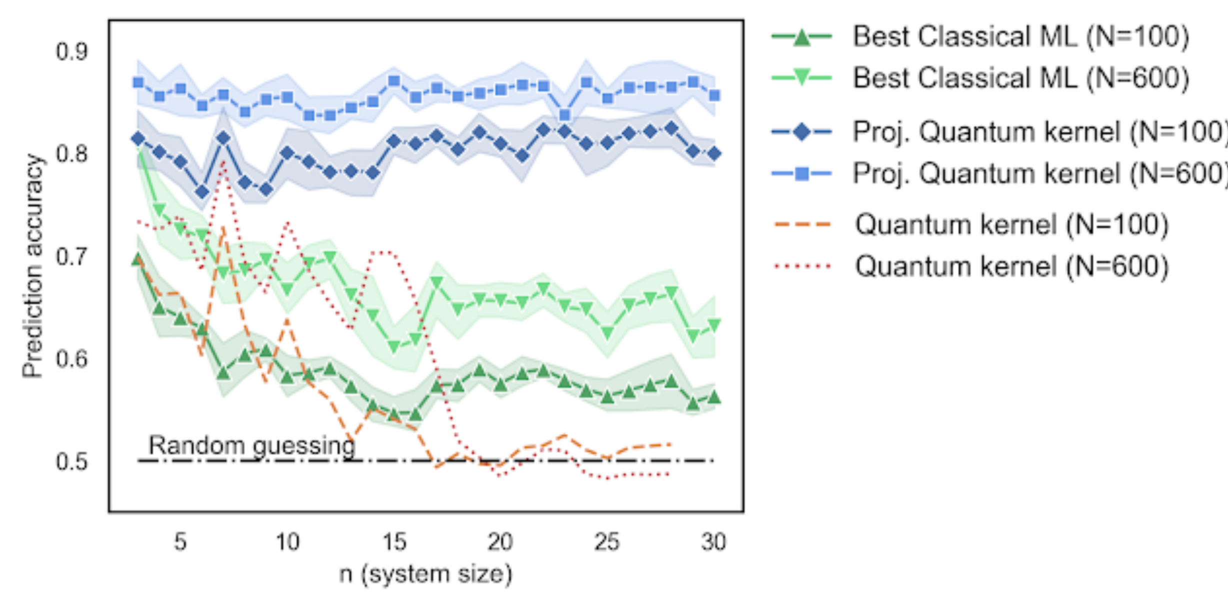
$$k(\boldsymbol{x}_i, \boldsymbol{x}_j) = \langle \phi(\boldsymbol{x}_i), \phi(\boldsymbol{x}_j) \rangle \longrightarrow k(\boldsymbol{x}_i, \boldsymbol{x}_j) = |\langle \psi(\boldsymbol{x}_i) | \psi(\boldsymbol{x}_j) \rangle|^2.$$

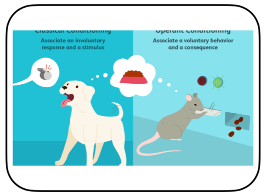
※ 양자 컴퓨터를 사용하여 커널 함수를 계산

NISQ & 양자머신러닝: 양자모델

■ Example: Projected Quantum Kernel Method [Nat. Comm. **12**, 2631 (2021)]







NISQ & 양자머신러닝: 훈련 및 일반화

■ Training (or optimization)

결국 머신러닝(either classical or quantum)의 목표는 주어진 작업을 해결하는 모델을 훈련하는 것

⇨ 최적의 매개변수 집합 {θ} + 손실 함수 L(θ)의 최소화

중요한 점: 양자 상태로부터 정보를 추출하는 것은 관측 가능의 기대값을 계산하는 것을 필요로 함

$$\underbrace{\left|\langle\psi_0|\,\hat{U}(\{\theta\})\,|\psi_0\rangle\right|^2}_{\text{기대값}} \xrightarrow[\text{양자컴퓨터}]{} \underbrace{\mathcal{L}(\{\theta\})}_{\text{손실함수}}$$

■ Challenges: quantum landscape or Barren plateaus problem

- QML 모델의 매개변수를 훈련하는 것 ⇨ 손실 함수 L을 최소화하고 그로부터 (일반적으로 비볼록한)랜드스케이프를 탐색하는 것

(Local minima in quantum landscapes)

- Loss function 이 주는 local minimum 들 때문에 생김 ⇨ 가변 구조의 PQCs 으로 일부 개선

(Barren plateaus from ignorance or insufficient inductive bias)

- PQCs (혹은 Ansatz) 의 구조 자체가 주는 탐색공간에의 편향성 때문에 생기는 문제

(Barren plateaus from global observables)

- 탐색공간 전역의(즉, 모든 큐비트들의) 측정값들에 기반한 Loss function 을 가정하여 BP 발생

(Barren plateaus from entanglement)

- 너무 많은 얽힘이 생성되는 QNN 역시 BP 초래

요 약

- “양자이론 + 정보이론 = 양자정보과학”

  → 보다 빠르고, 보다 안전하고, 보다 정확한 정보 가공/처리에의 기반이론

- 양자 컴퓨팅에의 지수함수적 양자속도향상 증명: 양자정보과학 연구 가속화

  → 범용 양자컴퓨터/양자머신 개발의 목표한계 인식

- “양자이론 + 머신러닝 = **양자머신러닝**”

  → 양자정보/컴퓨팅 연구에의 새로운 모멘텀 부여 및 새로운 서브 연구분야 확립

- 머신러닝에의 양자속도향상 및 물리기작 증명: 원리규명 및 이론확립 단계

  (최근) 중단기 구현 가능성에 대한 비전 제시

  Q1. 머신러닝에의 *양자속도향상 가능성 및 물리기작의 규명

  e.g., HHL/QSVD, 양자샘플복잡도, 등

  Q2. 고전학습모델 → 양자모델개발/구성 → 양자정보처리 등에의 응용

  e.g., 양자신경망이론, 양자강화학습모델, 등

+ (최근) **Q3**. 양자시뮬레이션 및 단기구현 가능성에 대한 비전 제시

  e.g., 고전-양자 융합 접근, 가변회로를 이용한 양자인공신경망 모델 등

THANK YOU