Lab Tutorial (25 March 20 2019)

Tutorial: Superconducting Qubits

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OVERVIEW

1. Introduction
2. Superconductivity
3. Macroscopic Quantization
4. Superconducting Qubits
5. Summary

THIS TUTORIAL IS …

- Obviously, about superconducting qubits;
- But not only about superconducting qubits;
- Introduce general paradigm to understand solid-state, or even most general, qubits.

THE PRIMARY SPIRIT IS …

… UTMOST SIMPLIFICATION

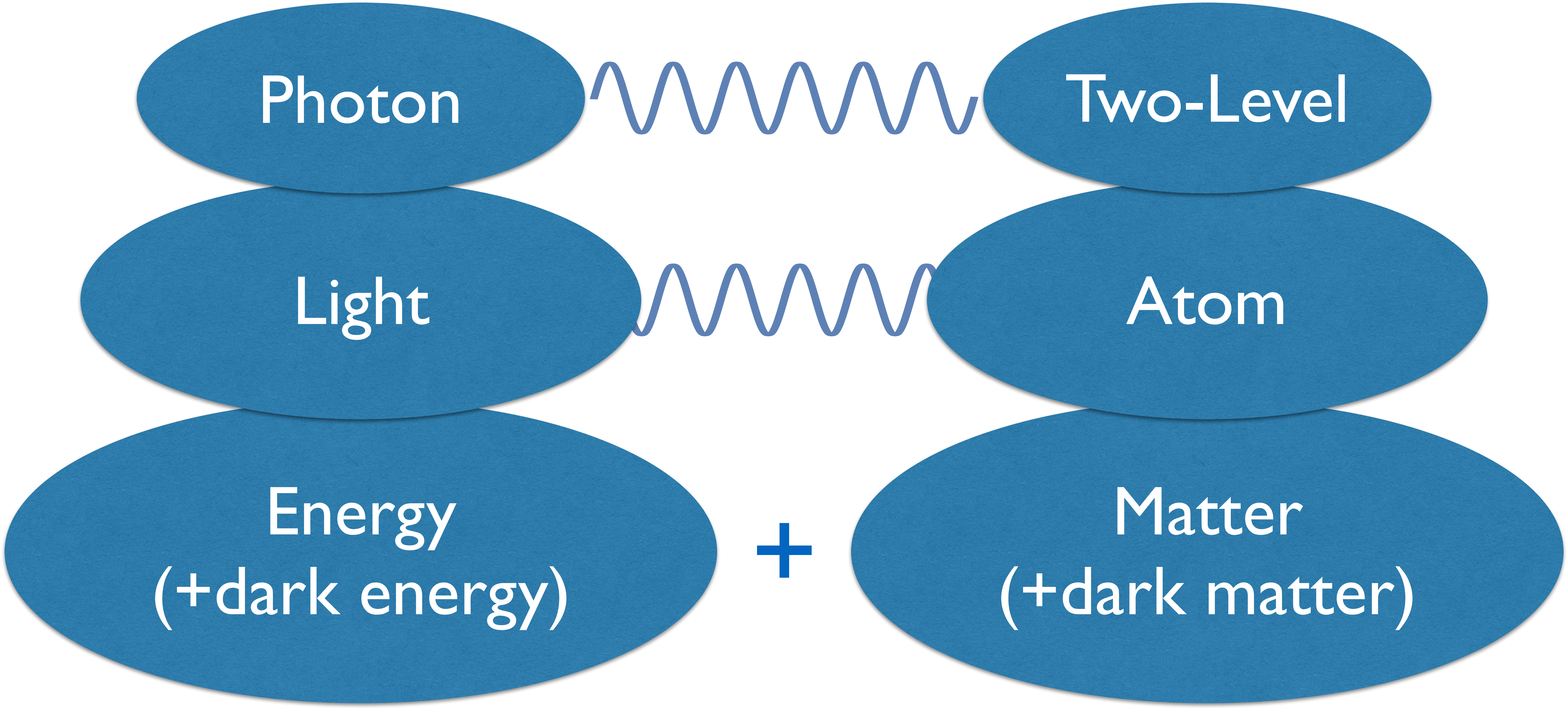
“Throw away all details not necessary” for understanding the fundamental ….

— C. Caves (1981) —

General Introduction

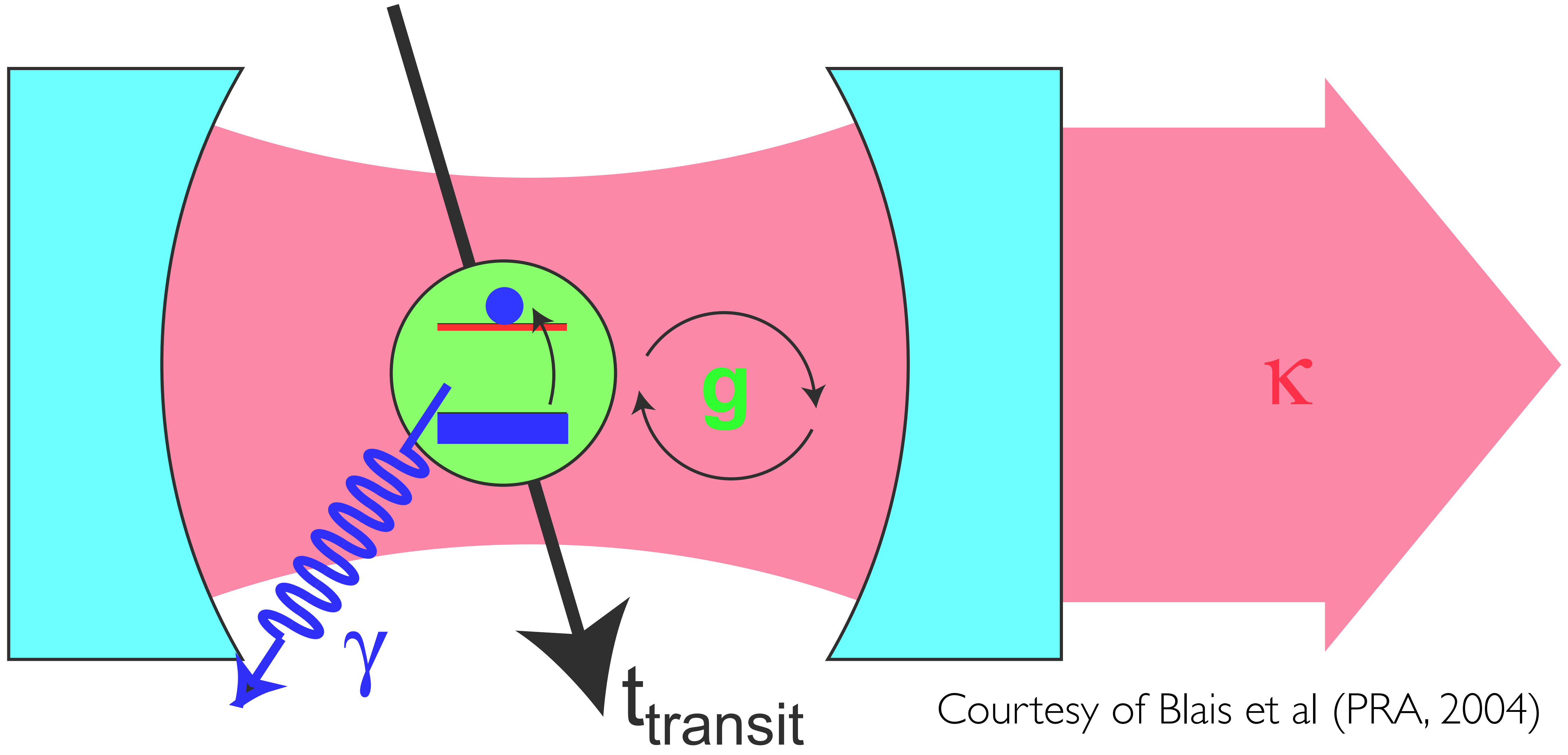
Background & Motivations

Light-Matter Interaction



Cavity QED

- Simple yet highly non-trivial.
- All essential features of light-matter interaction.



Courtesy of Blais et al (PRA, 2004)

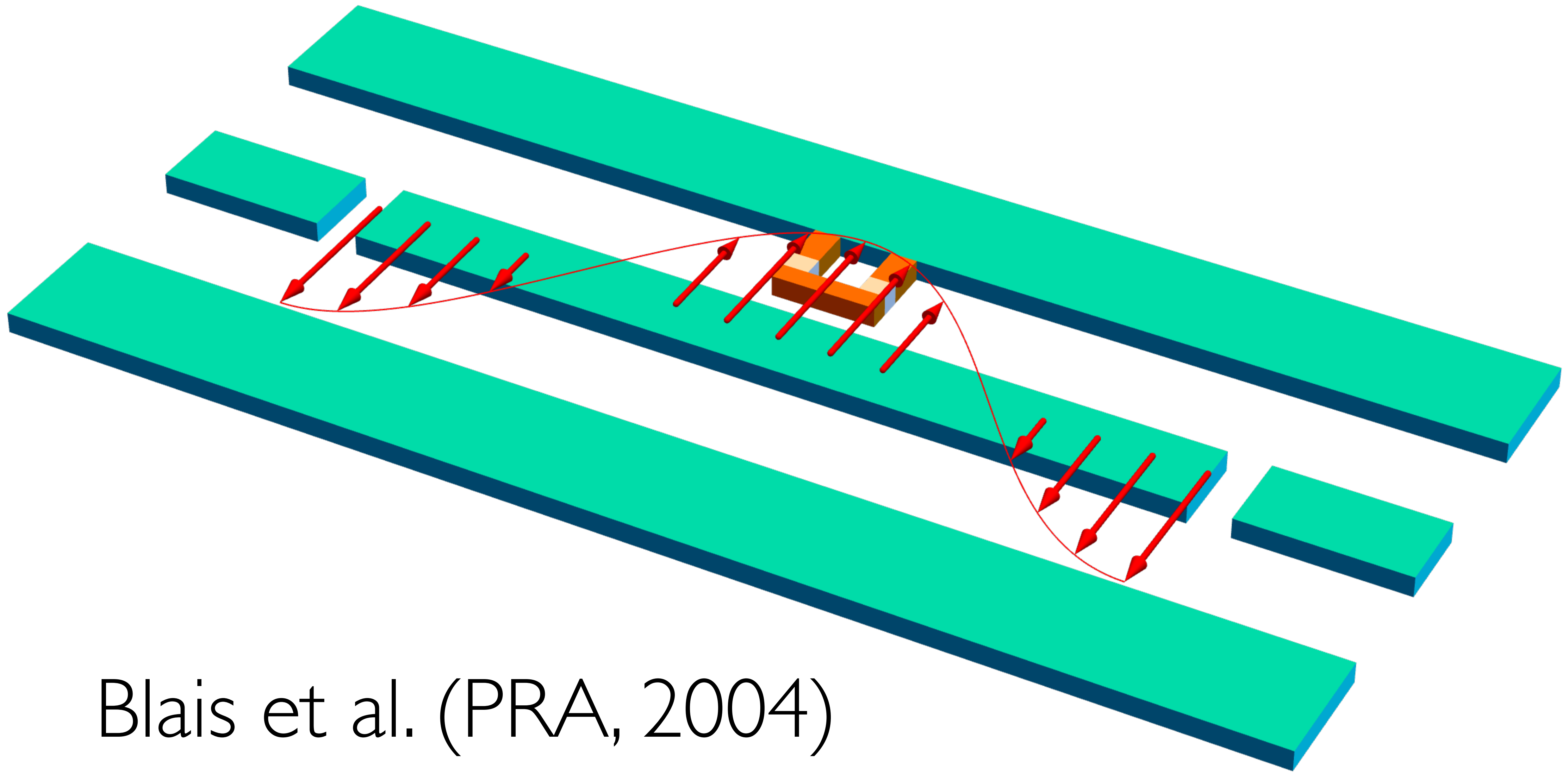
$$H = \omega a^{\dagger} a + g(a + a^{\dagger})\sigma^{x} + \frac{1}{2}\Omega\sigma^{z}$$

TWO LIMITATIONS

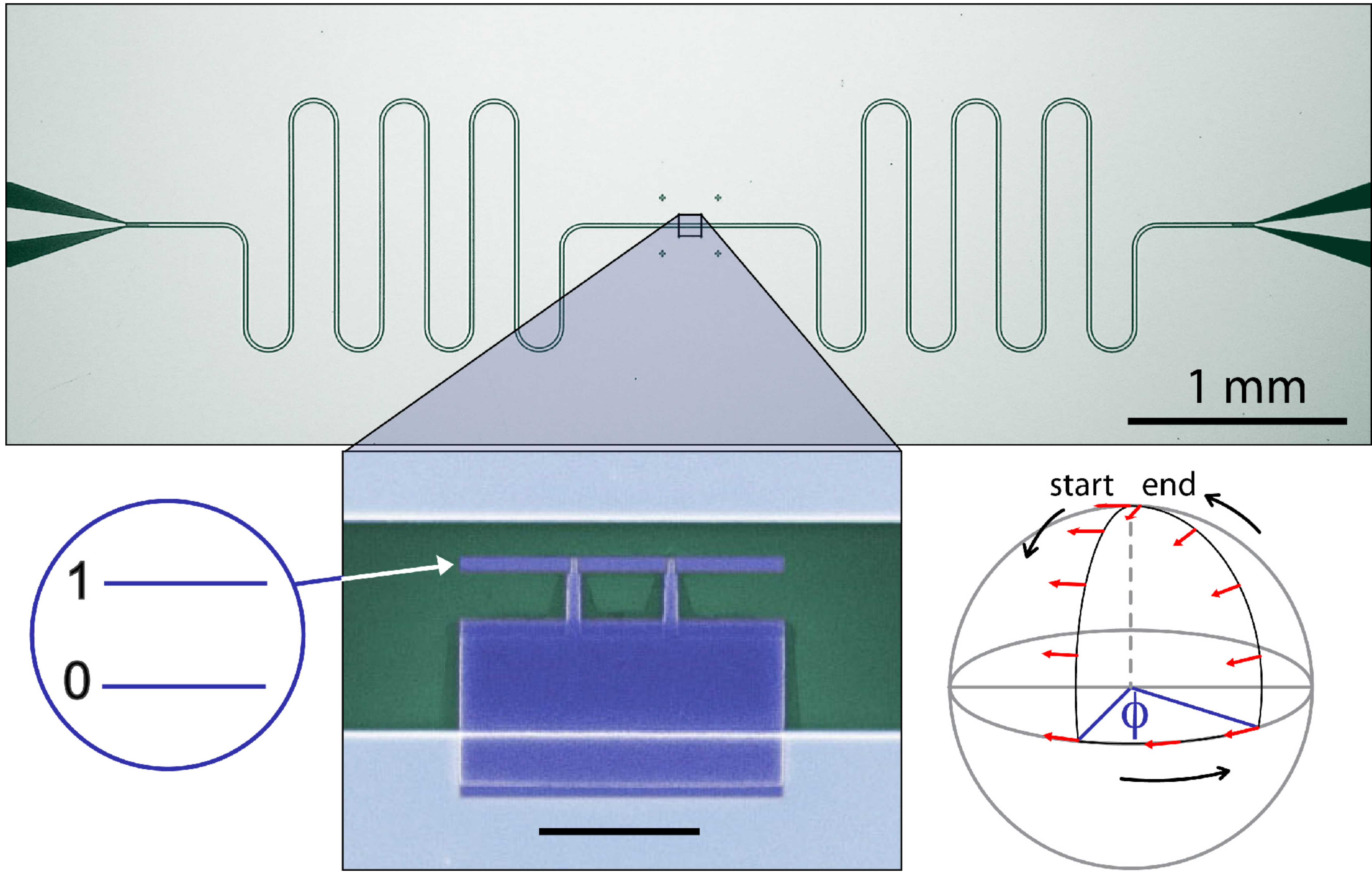
of the conventional cavity QED

1. The coupling is weak.
2. The qubit is not topological.

Circuit QED System



Blais et al. (PRA, 2004)



Wallraff et al. (Nature, 2004)

LIGHT & TOPOLOGICAL MATTER

What is the smallest unit (if any) of the topological matter?

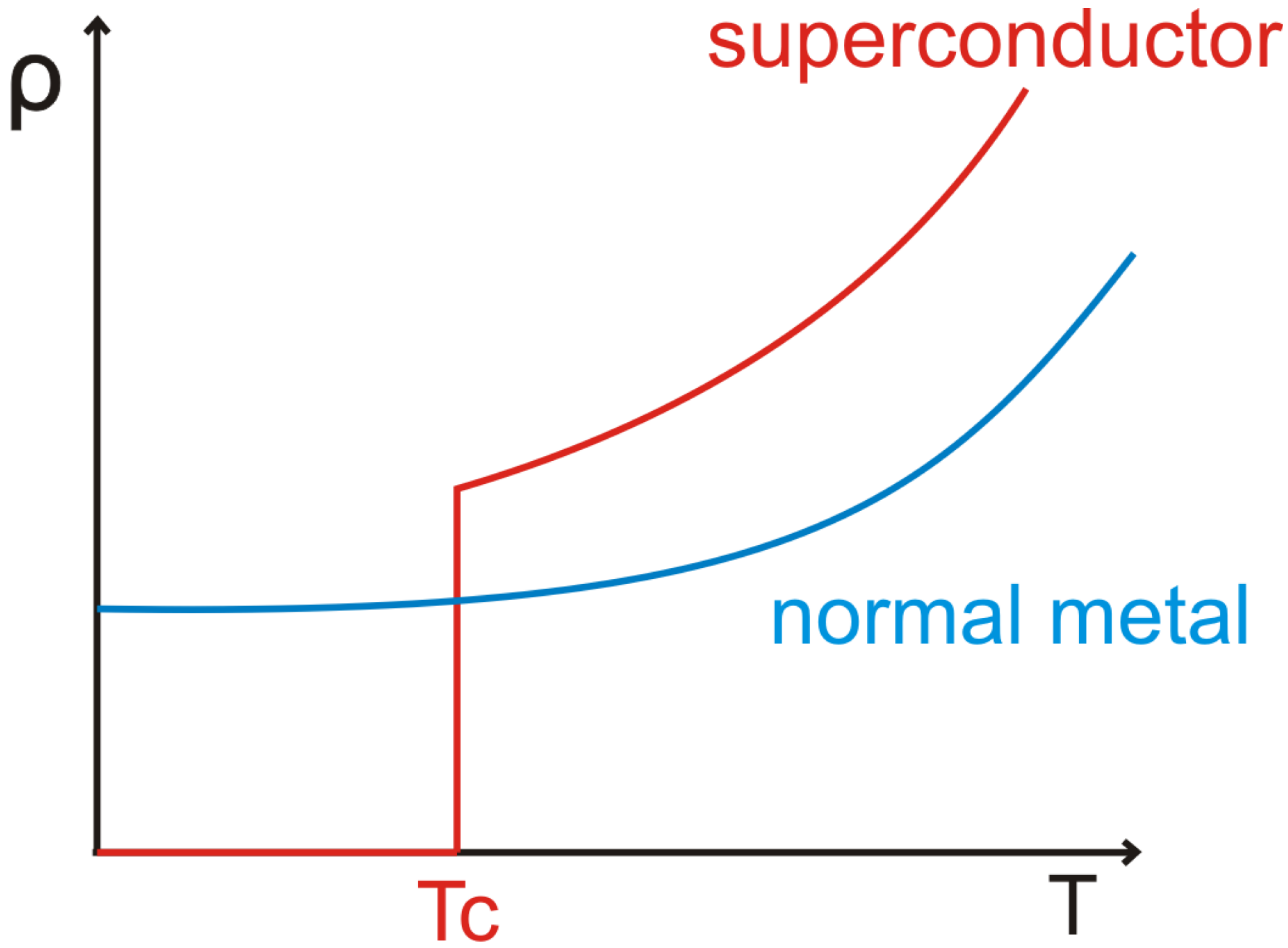
A simple yet quintessence-seizing model of topological matter?

1. To realize *topological qubits* based on Josephson junction arrays.
2. To achieve the *topological QED architecture* (with strong coupling).
3. To explore the *fundamental light-topological matter interaction*.

SUPERCONDUCTIVITY

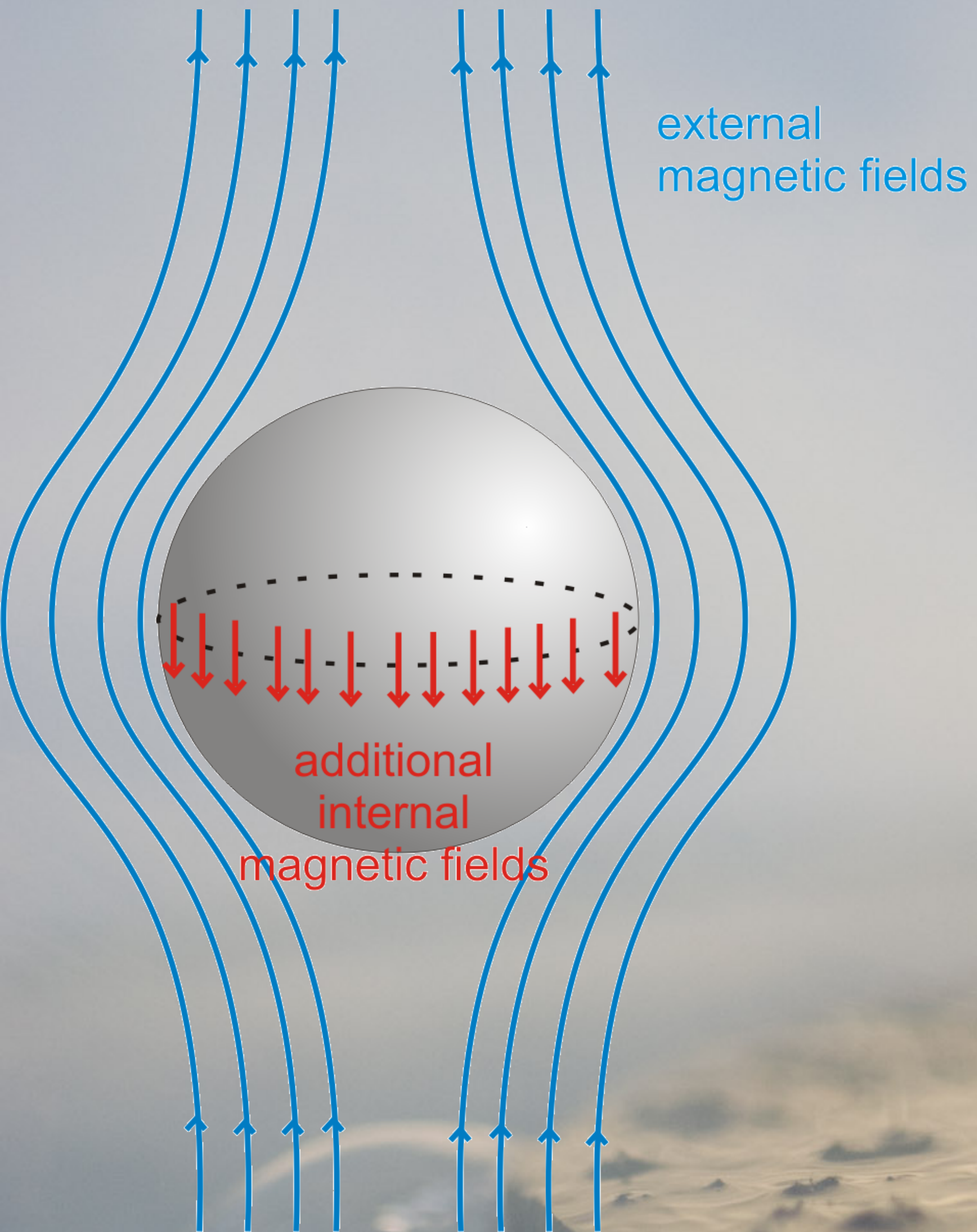
PHENOMENA

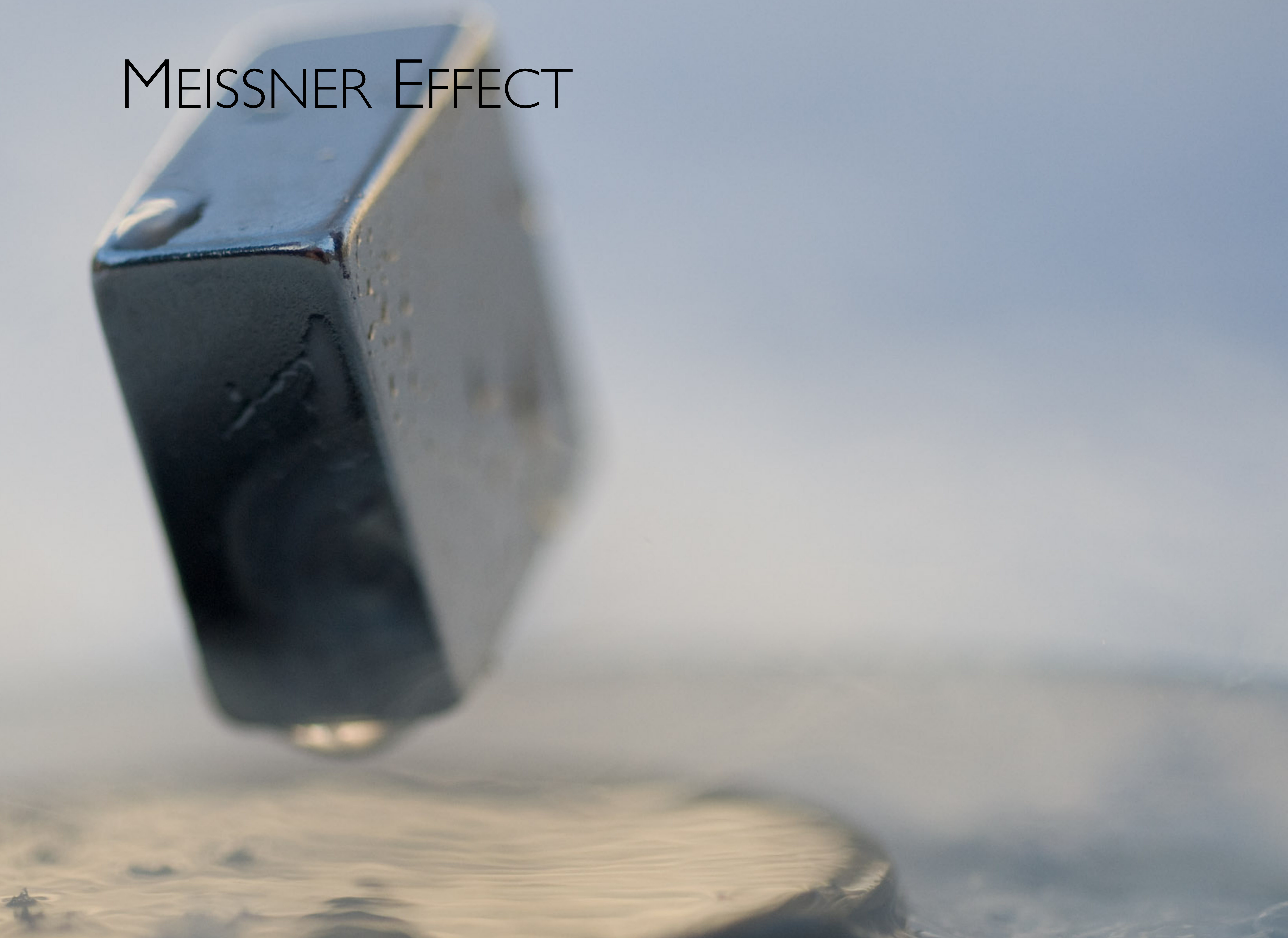
Perfect Conductivity



PERFECT DIAMAGNETISM

MEISSNER EFFECT



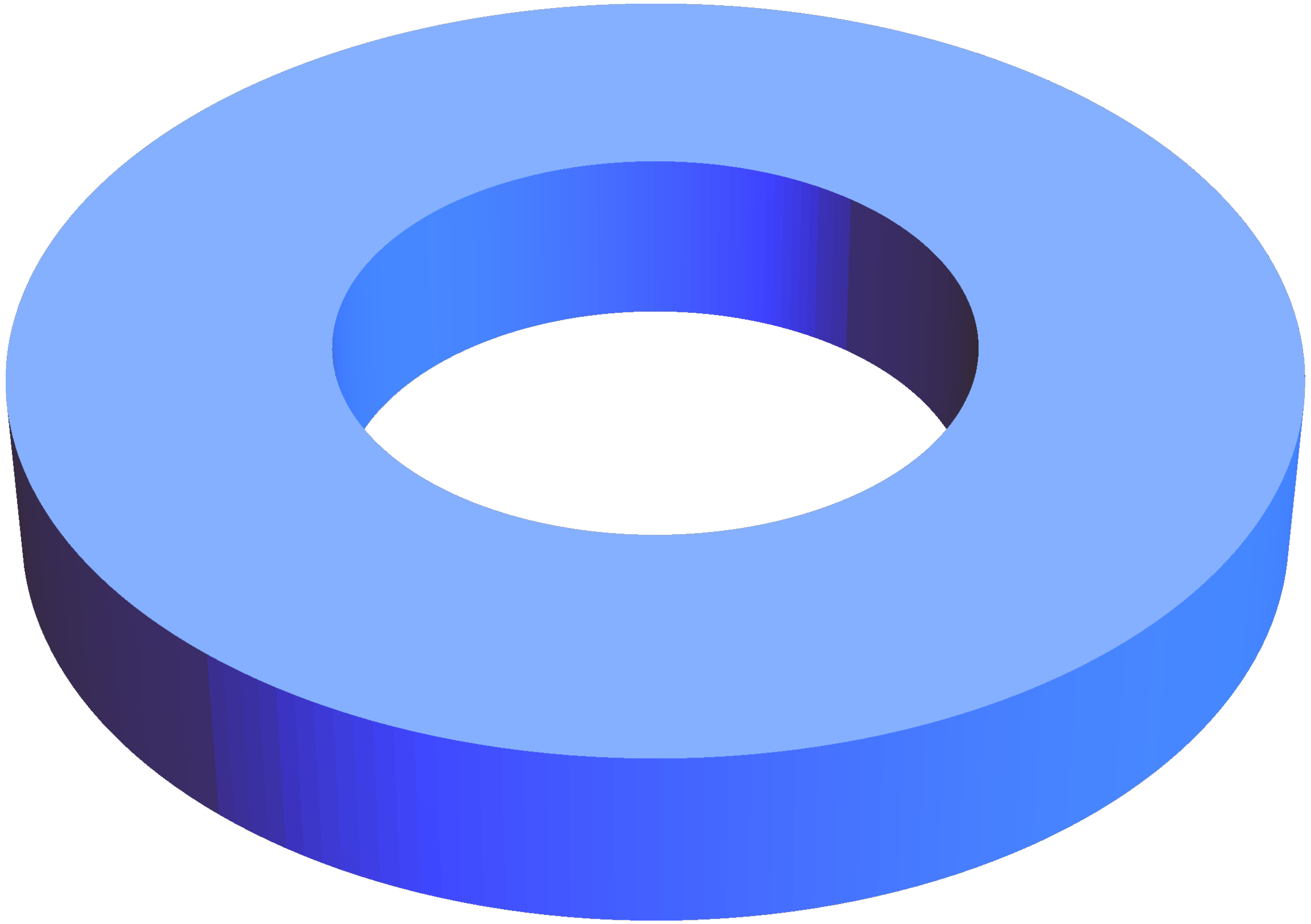


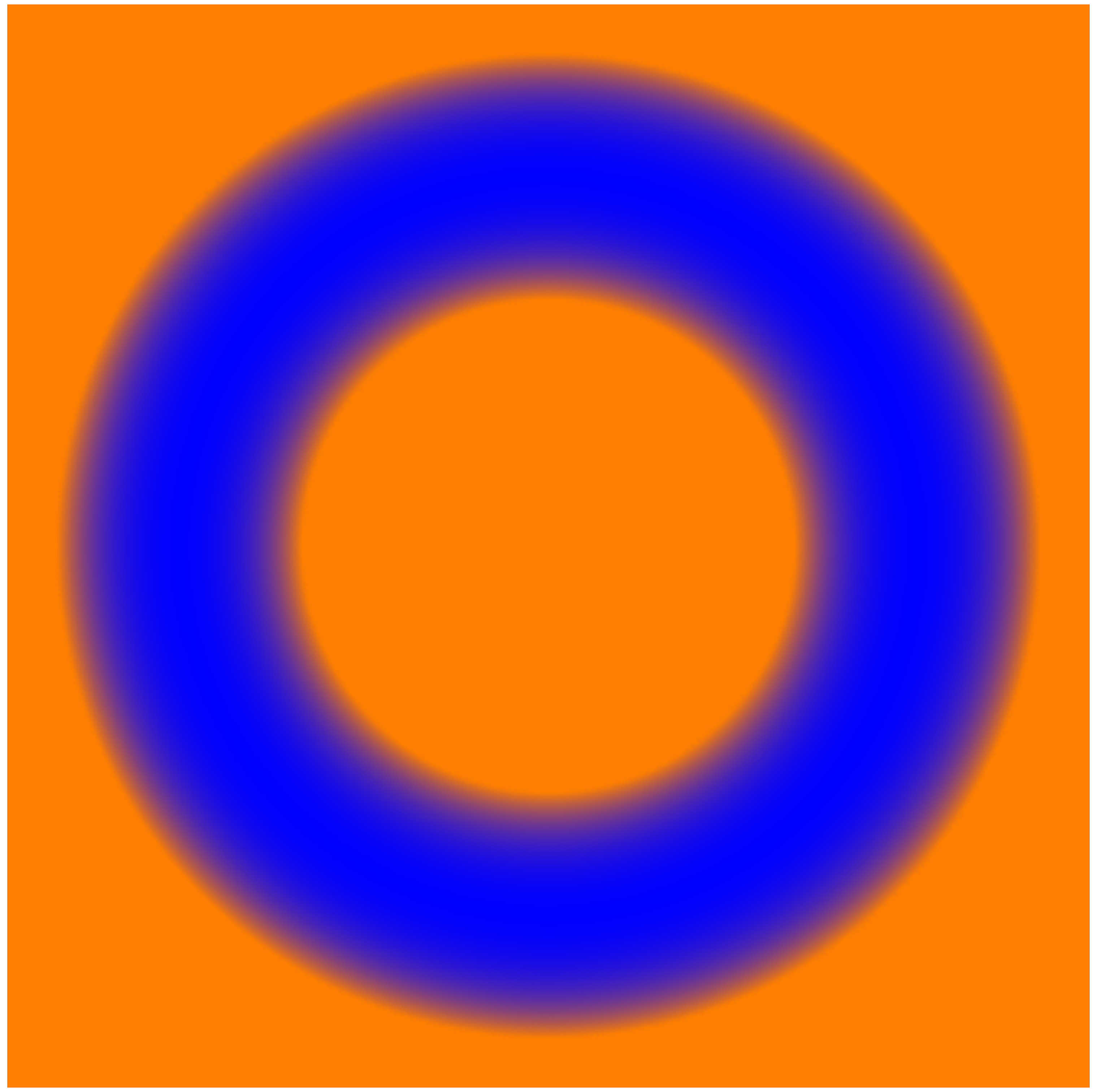
Ri Channel

Ri Channel

Flux Quantization

Macroscopically Quantum





SUPERCONDUCTIVITY

THEORY

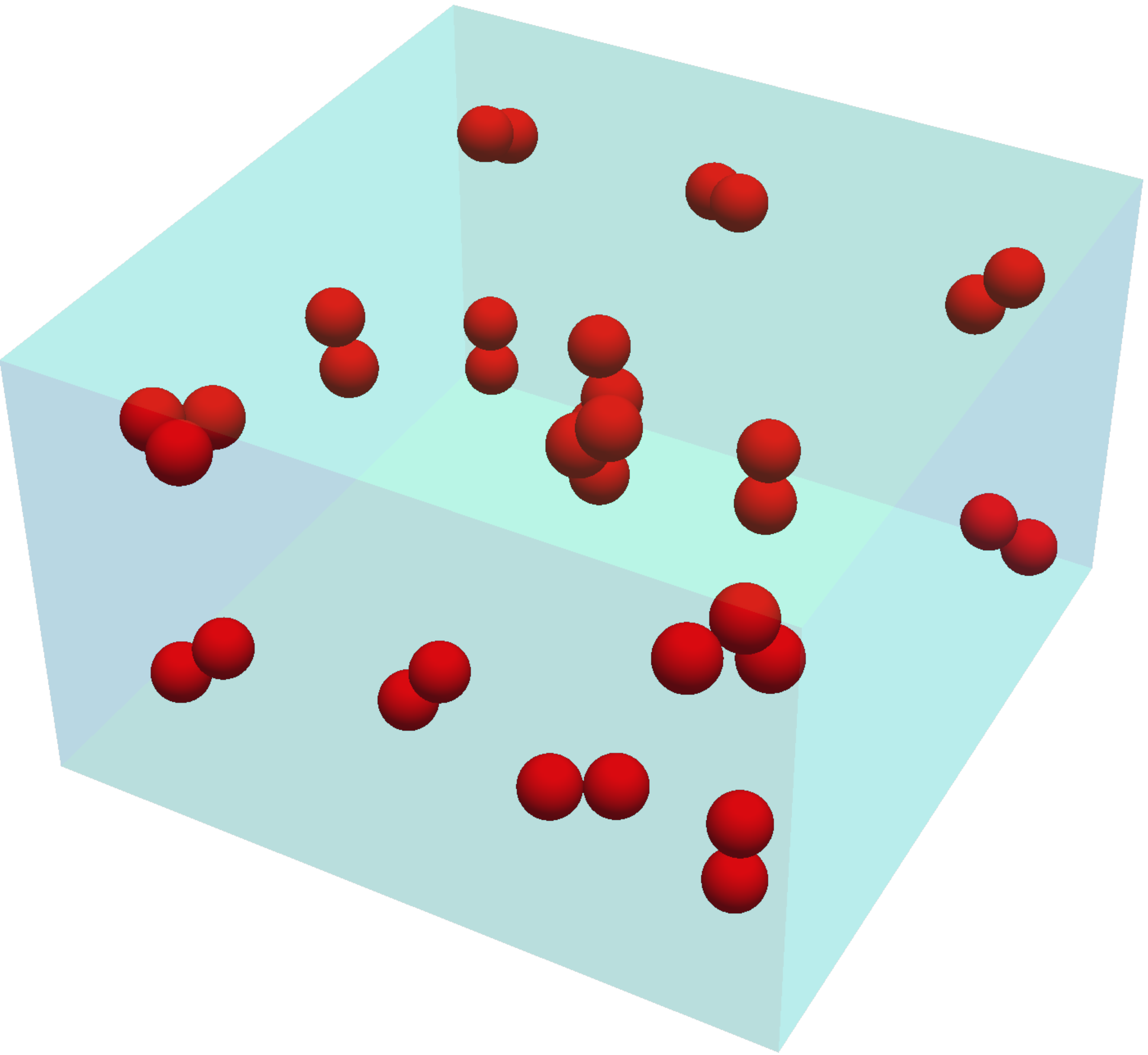
Two Approaches

Microscopic vs Macroscopic

- The BCS Theory
- The Ginzburg-Landau Theory

The BCS Theory

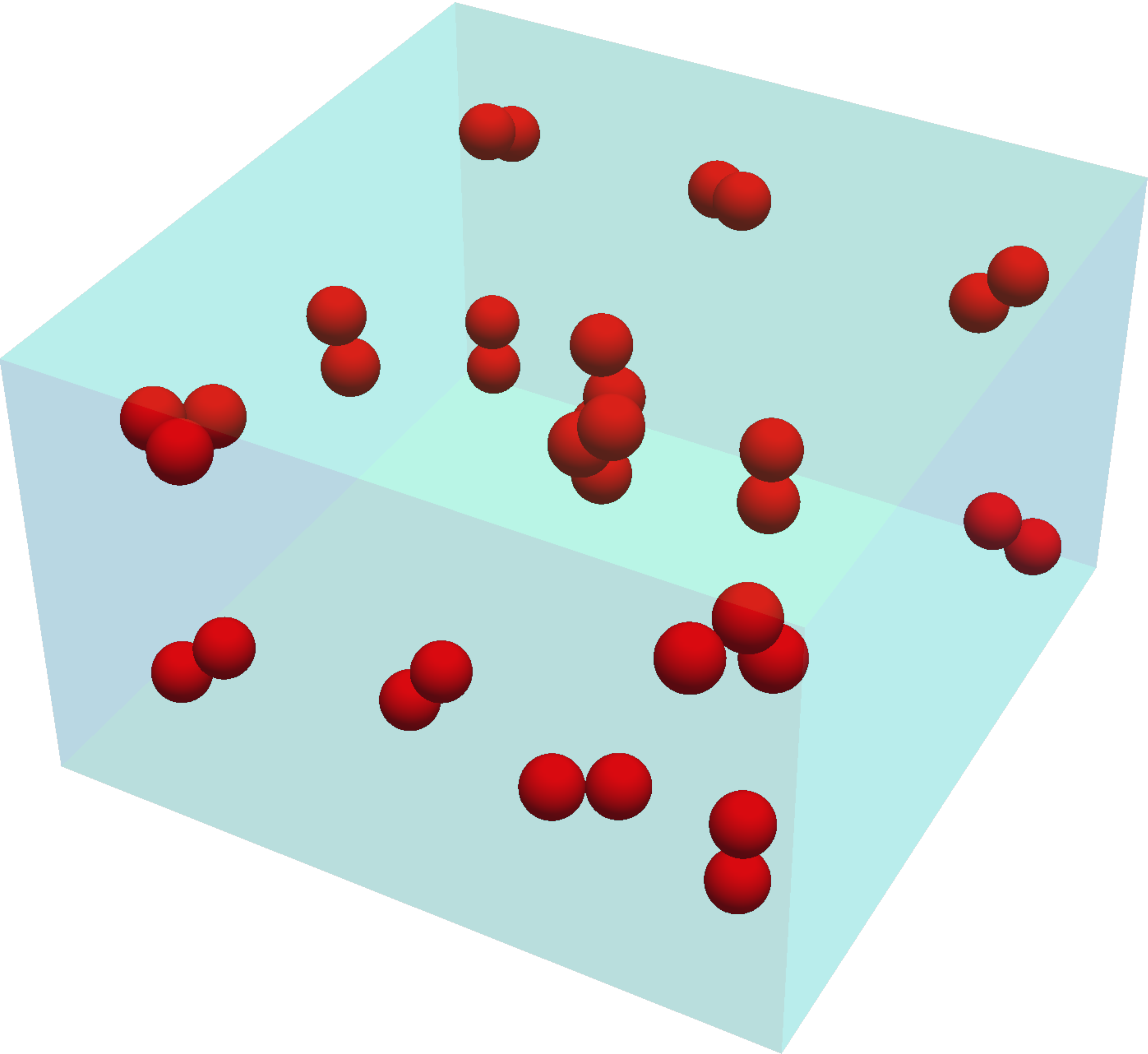
Cooper Pairs

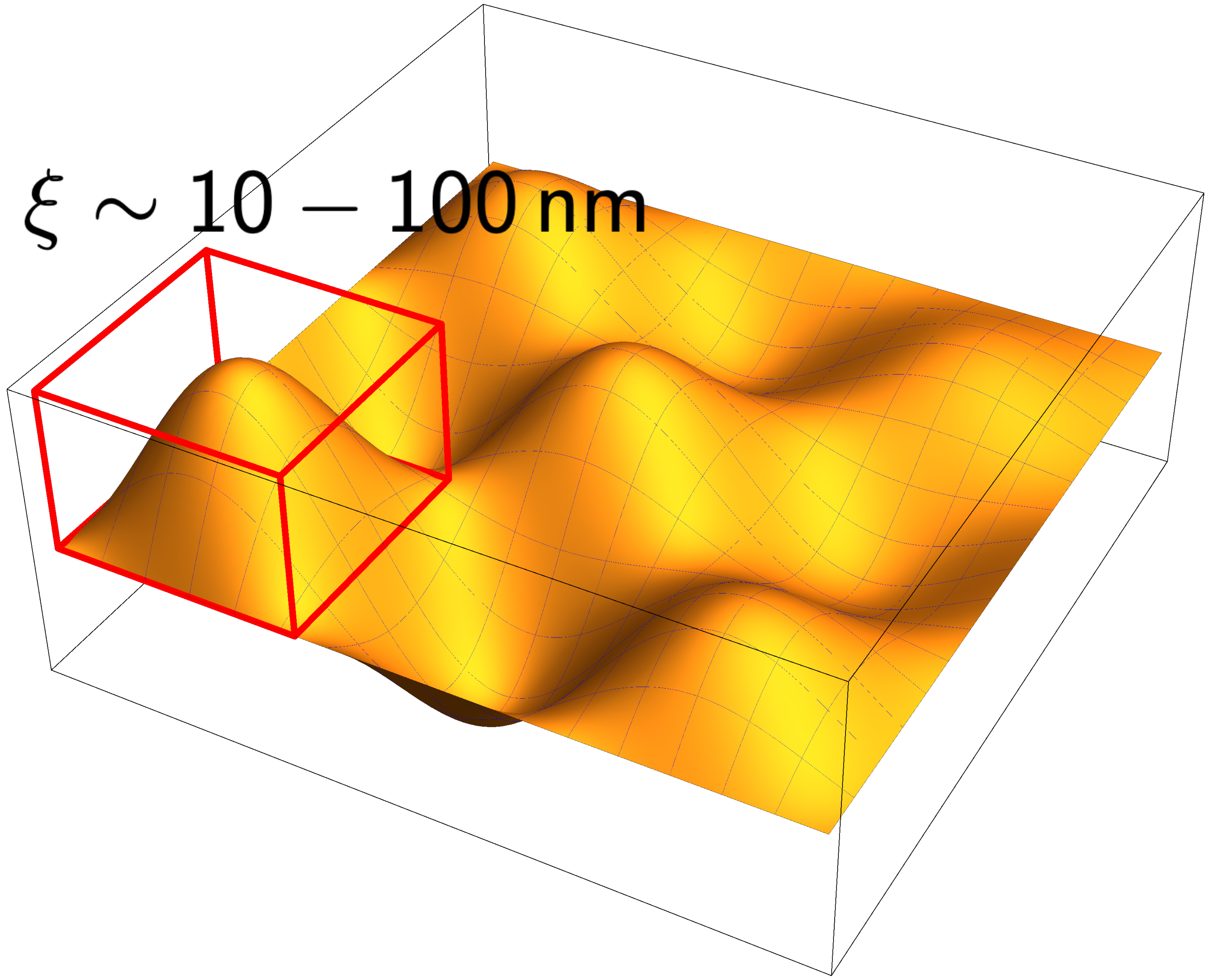


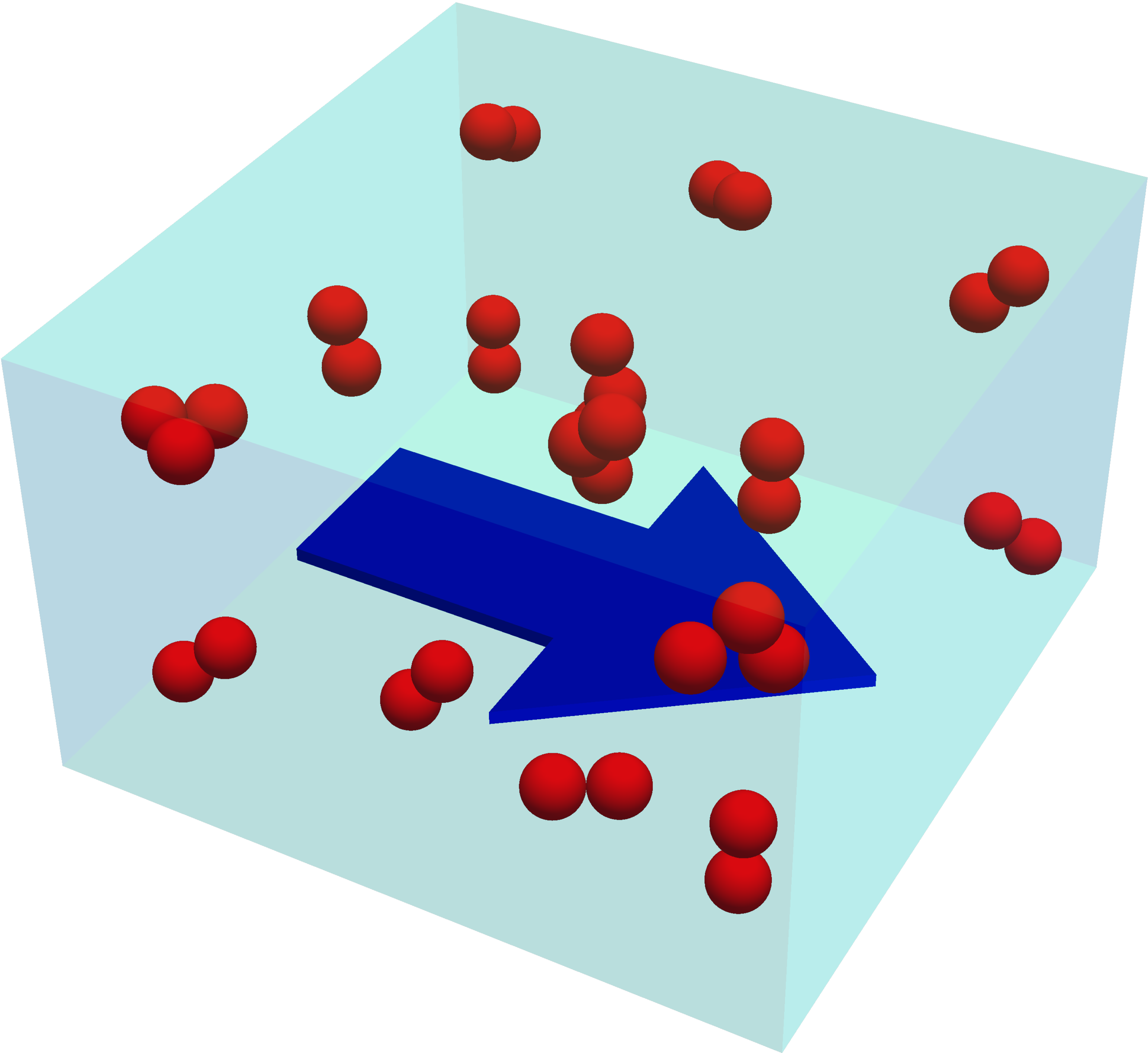
$$H = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} + \Delta \sum_{\boldsymbol{k}} \left(\underbrace{c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k},-\sigma}}_{\text{Cooper pairs}} + c_{-\boldsymbol{k},-\sigma} c_{\boldsymbol{k}\sigma} \right)$$

The Ginzburg-Landau Theory

Complex Order Parameter





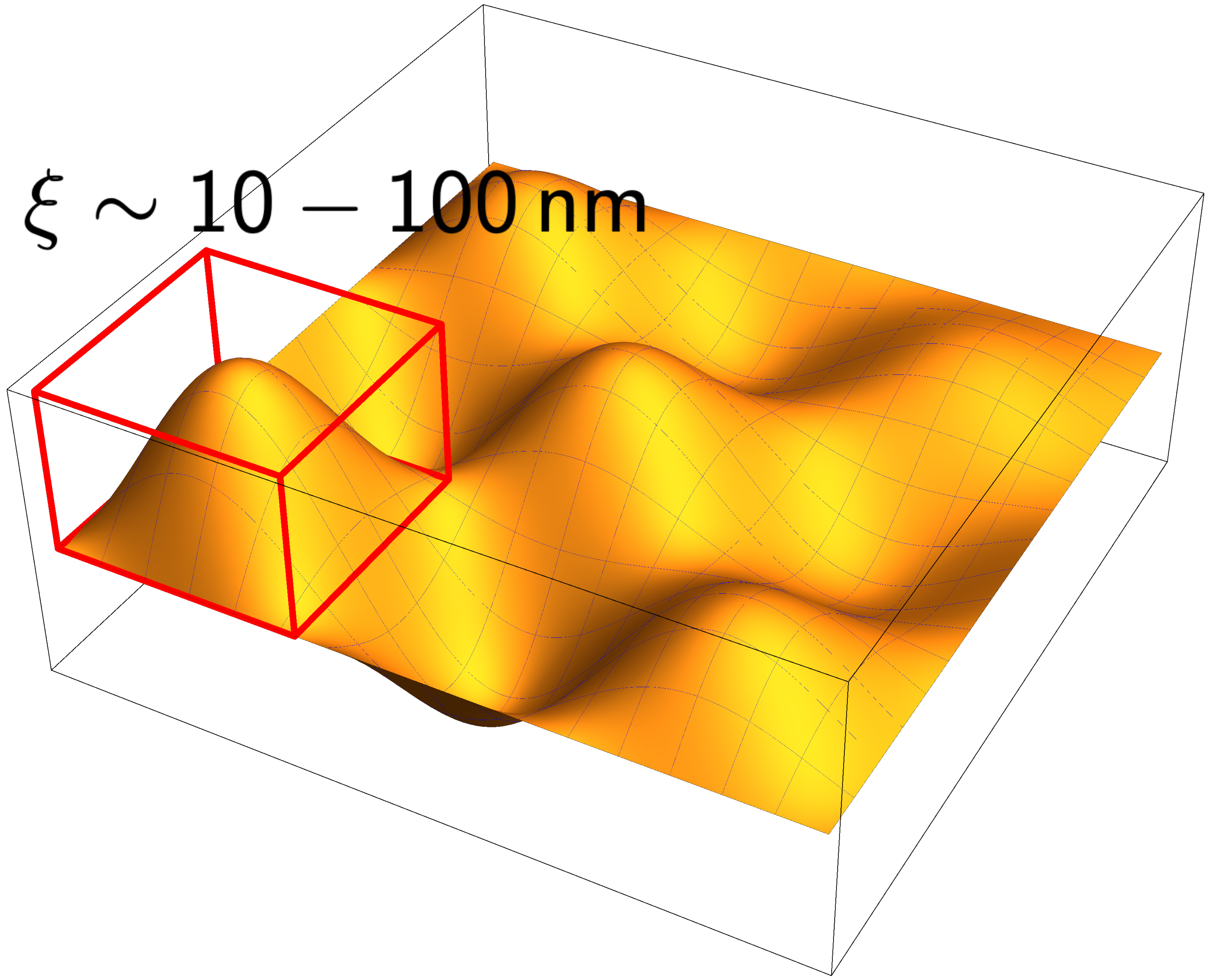


$$\Psi(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})}\,\exp[i\phi(\boldsymbol{r})]$$

$$\boldsymbol{J}_S(\boldsymbol{r}) \propto \boldsymbol{\nabla}\phi + 2\pi\boldsymbol{A}/\Phi_0$$

Complex Order Parameter

(Macroscopic Quantum Phenomena)

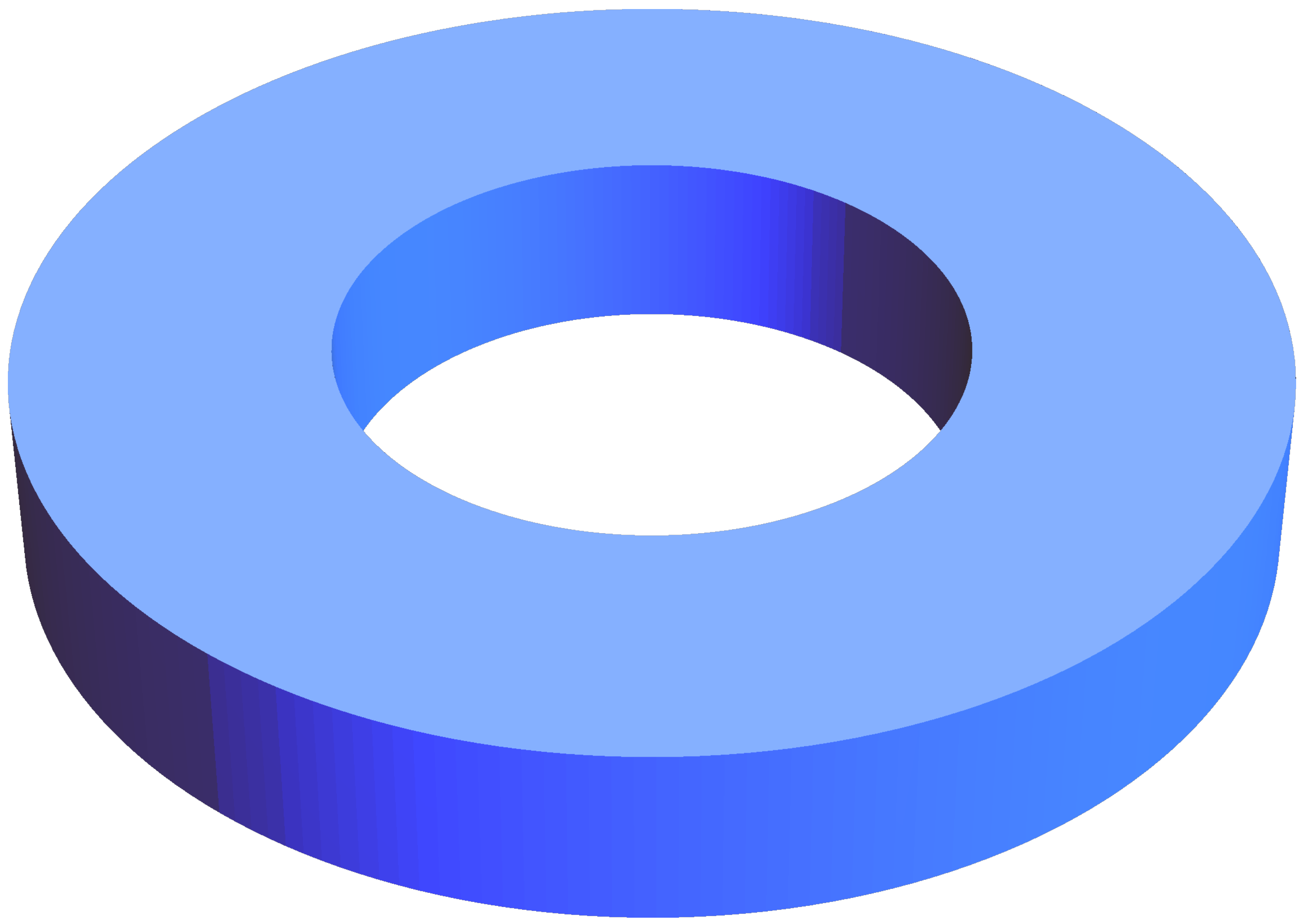


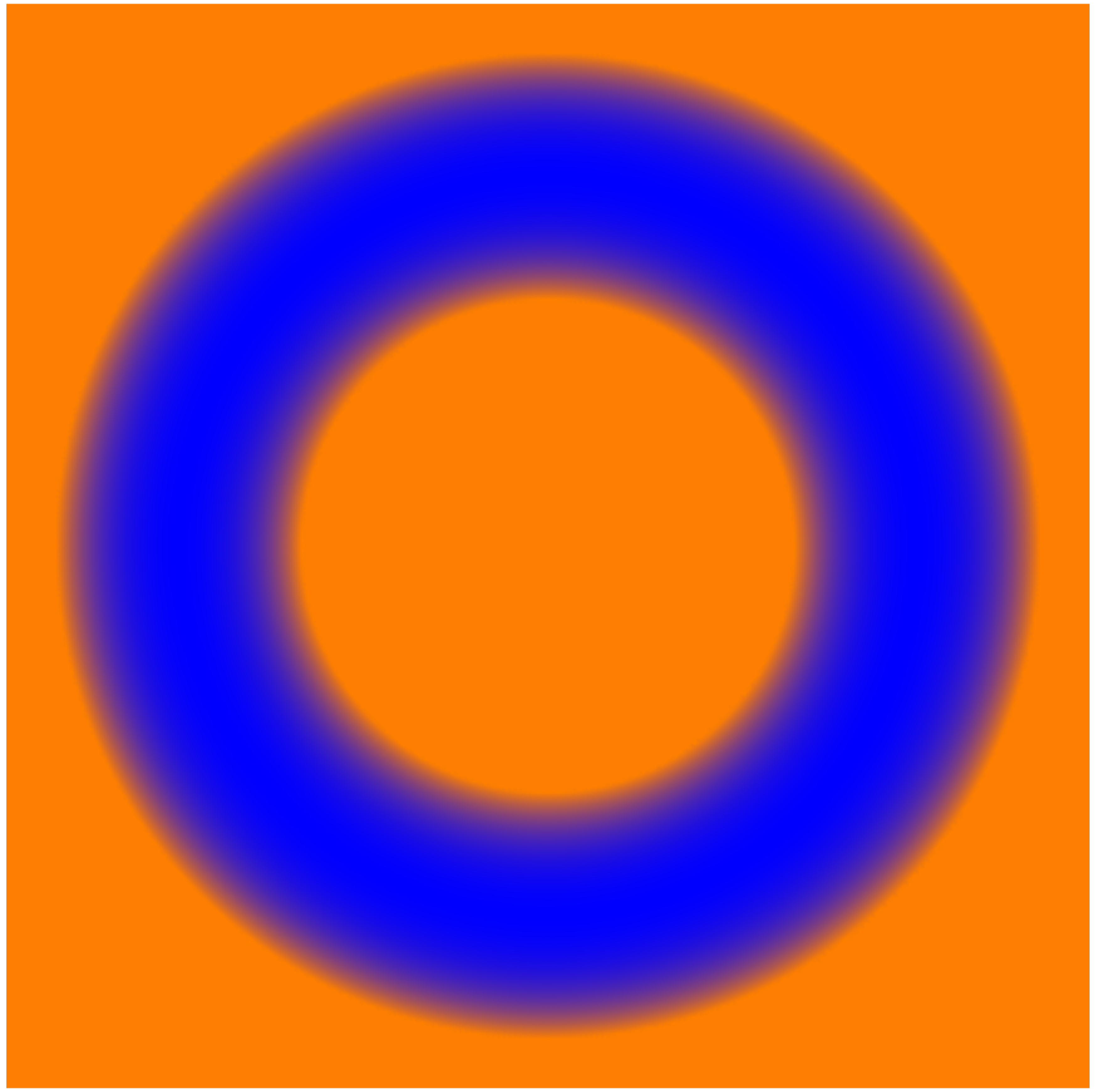
$$\Psi(\boldsymbol{r}) = \overbrace{\sqrt{\rho(\boldsymbol{r})}}^{\text{superfluid density}} \times \underbrace{\exp[i\phi(\boldsymbol{r})]}_{\text{super-current}}$$

$$\boldsymbol{J}(\boldsymbol{r}) = \boldsymbol{\nabla}\phi + 2\pi \boldsymbol{A}/\Phi_0$$

Flux Quantization

(Quantized Super-Current)





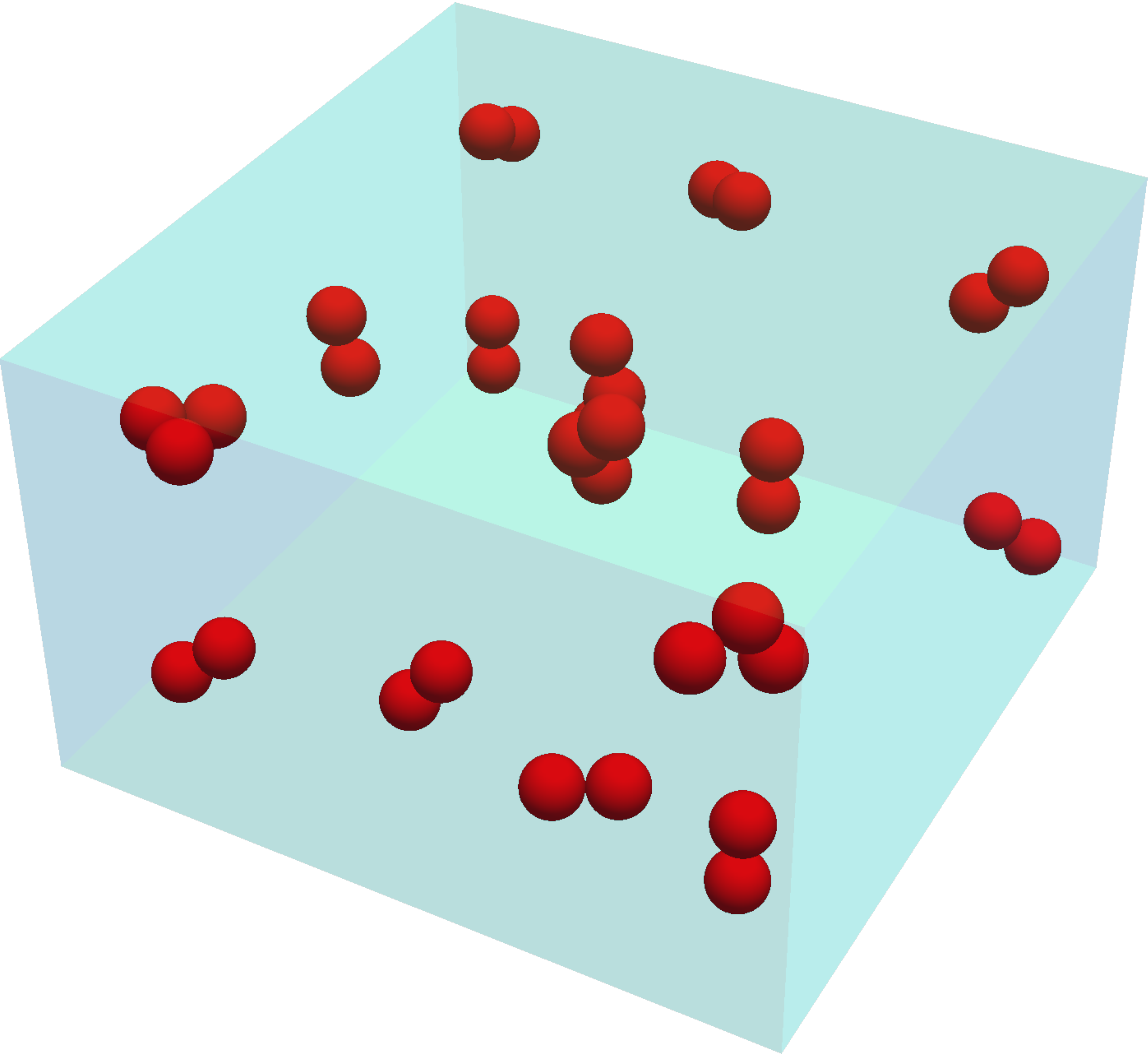
Single-valued-ness

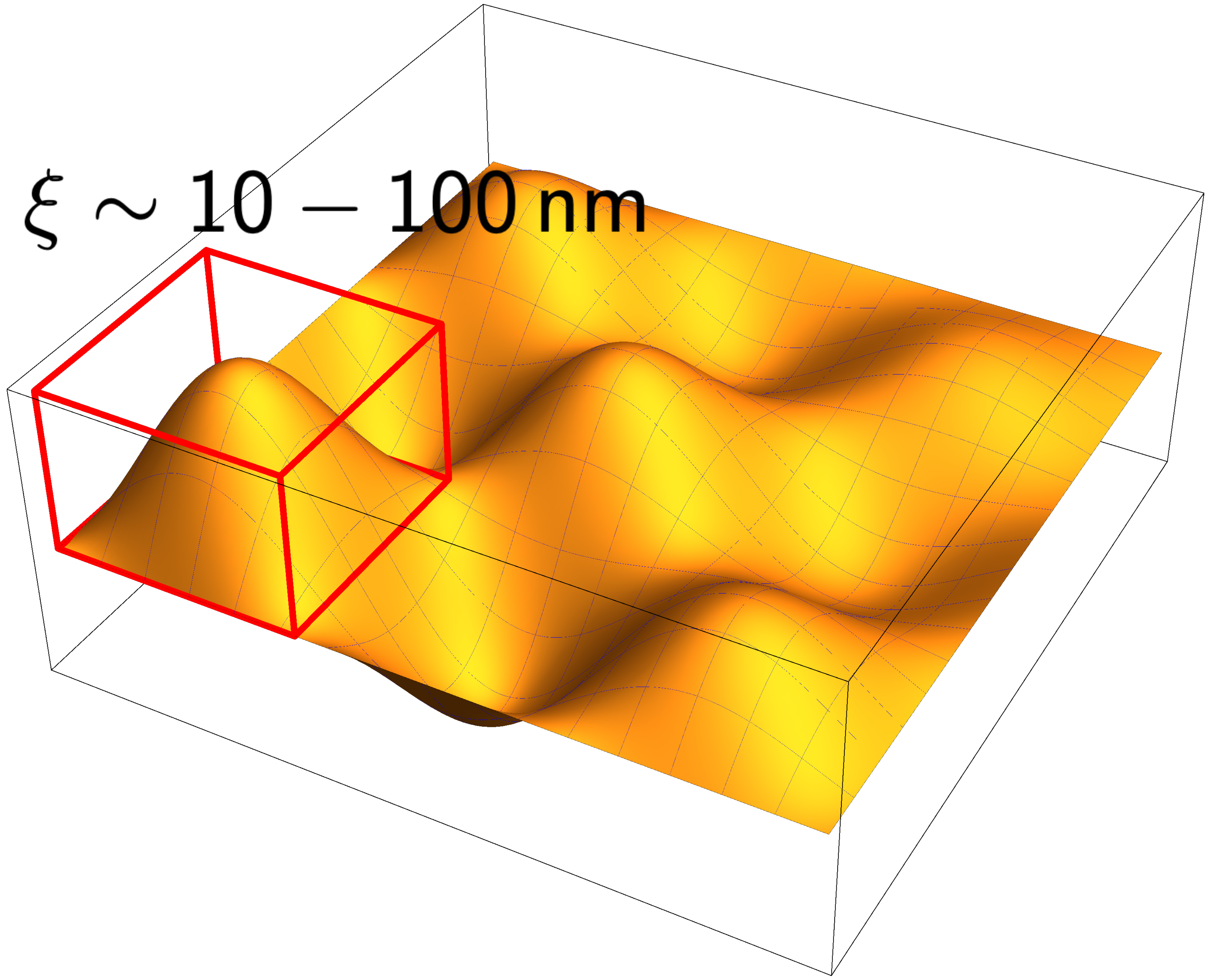
$$\frac{\Phi}{\Phi_0} \in \mathbb{Z}$$

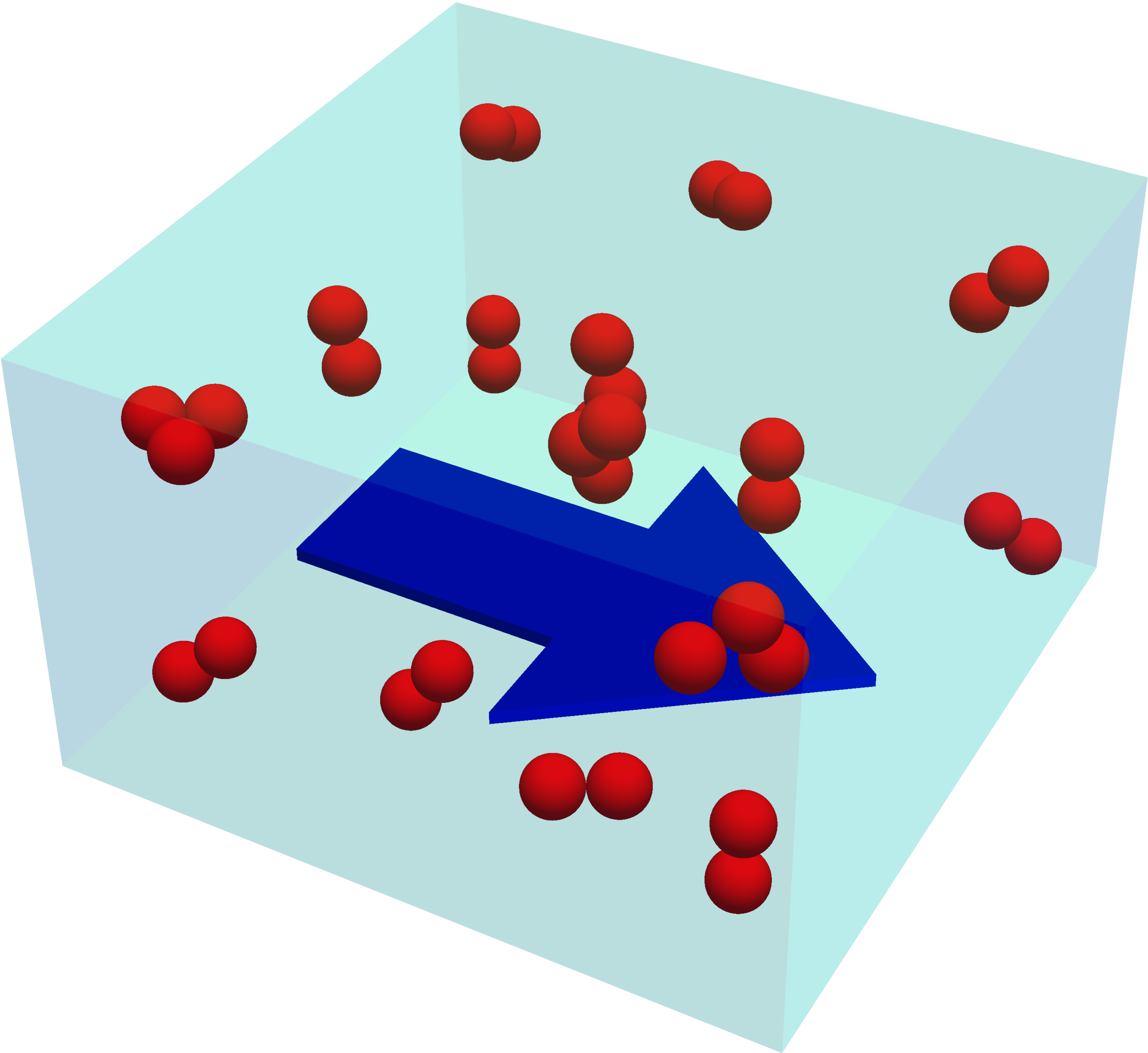
$$\Phi_0 := \frac{h}{2e}$$

Macroscopic Quantum Effects

Complex Order Parameter



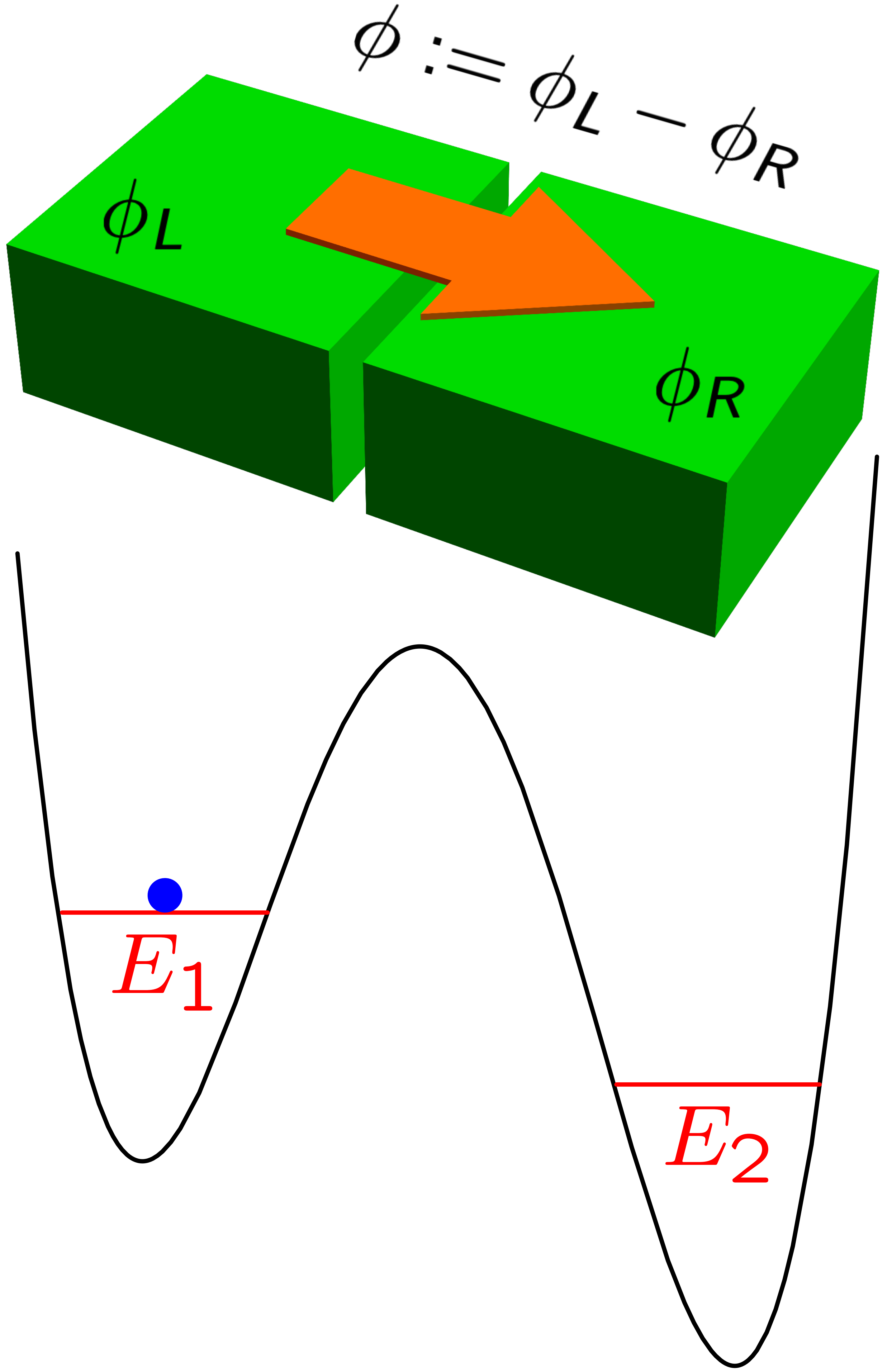




$$\Psi(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})}\,\exp[i\phi(\boldsymbol{r})]$$

$$\boldsymbol{J}_S(\boldsymbol{r}) \propto \boldsymbol{\nabla}\phi + 2\pi \boldsymbol{A}/\Phi_0$$

JOSEPHSON JUNCTIONS



$$i\hbar\dot{\Psi}_L = E_L\Psi_L - J\Psi_R\,, \quad \Psi_L = \sqrt{N_L}e^{i\phi_L}$$
$$i\hbar\dot{\Psi}_R = E_R\Psi_R - J\Psi_L\,, \quad \Psi_R = \sqrt{N_R}e^{i\phi_R}$$

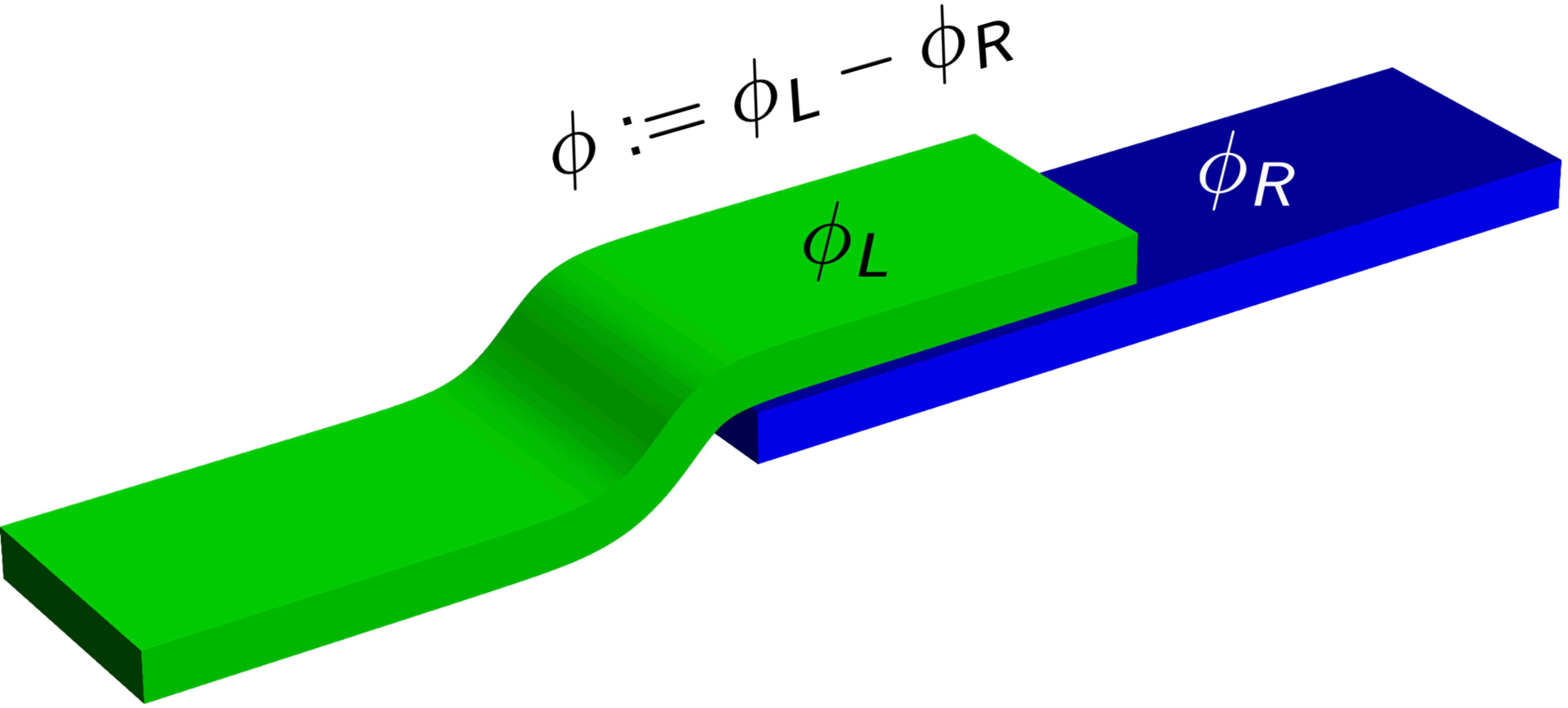
- DC Josephson Effect

$$I_S = \dot{N}_L = -\dot{N}_R = I_J \sin(\phi)$$

- AC Josephson Effect

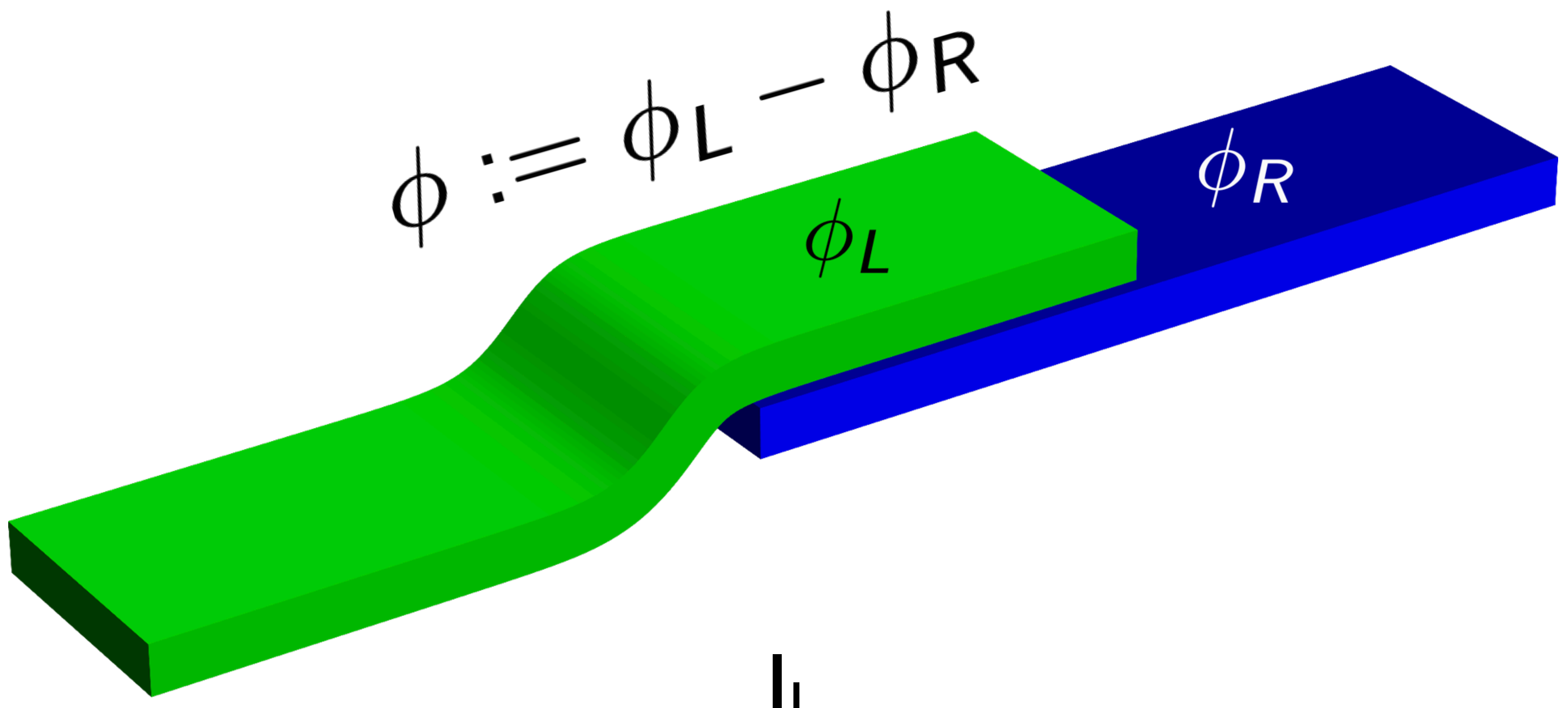
$$\hbar\dot{\phi} = E_L - E_R = 2eV$$

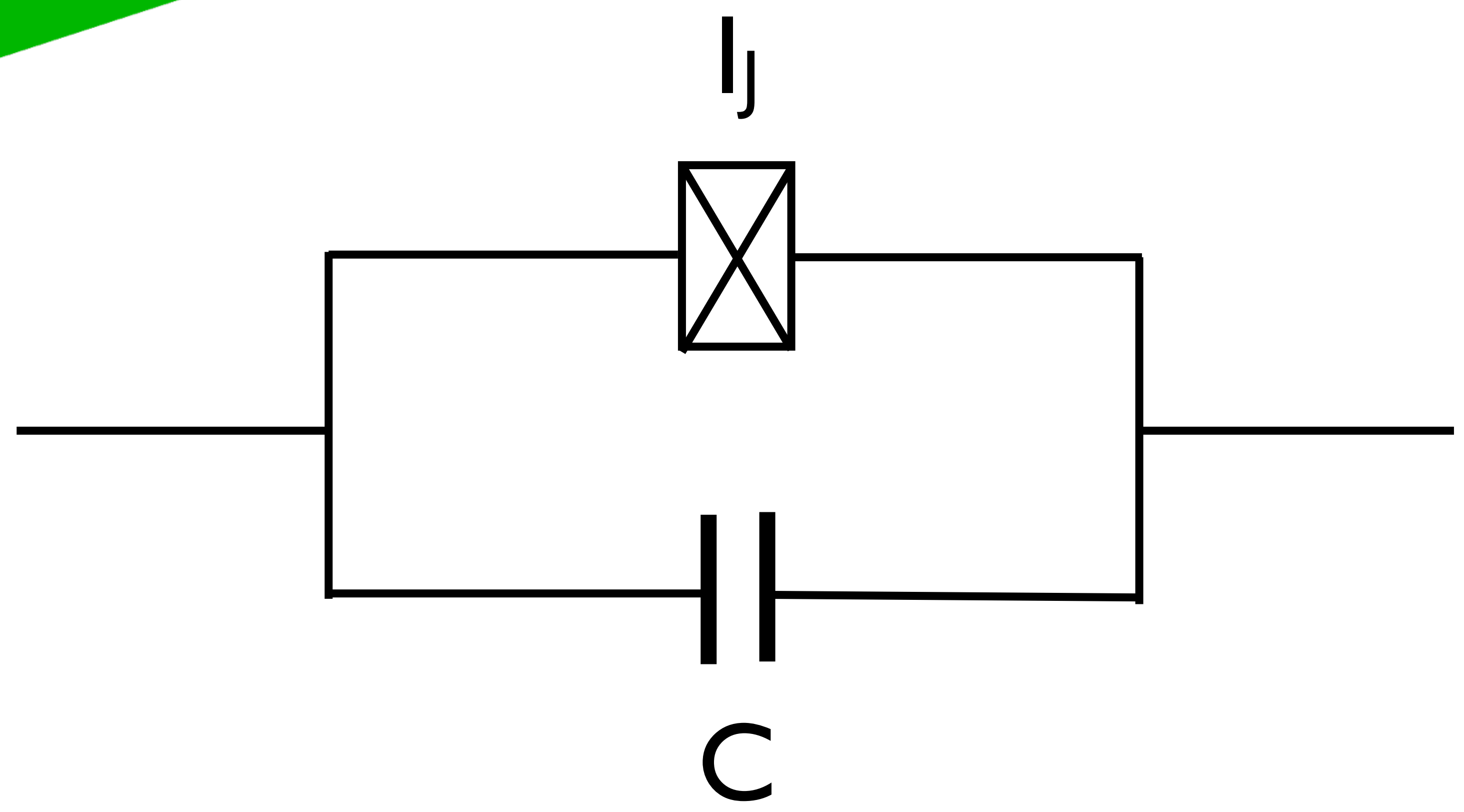
Josephson Junction



- DC Josephson Effect
  $I_S = I_J \sin(\phi)$
- AC Josephson Effect
  $\dot{\phi} = 2eV/\hbar$

Small Josephson Junction





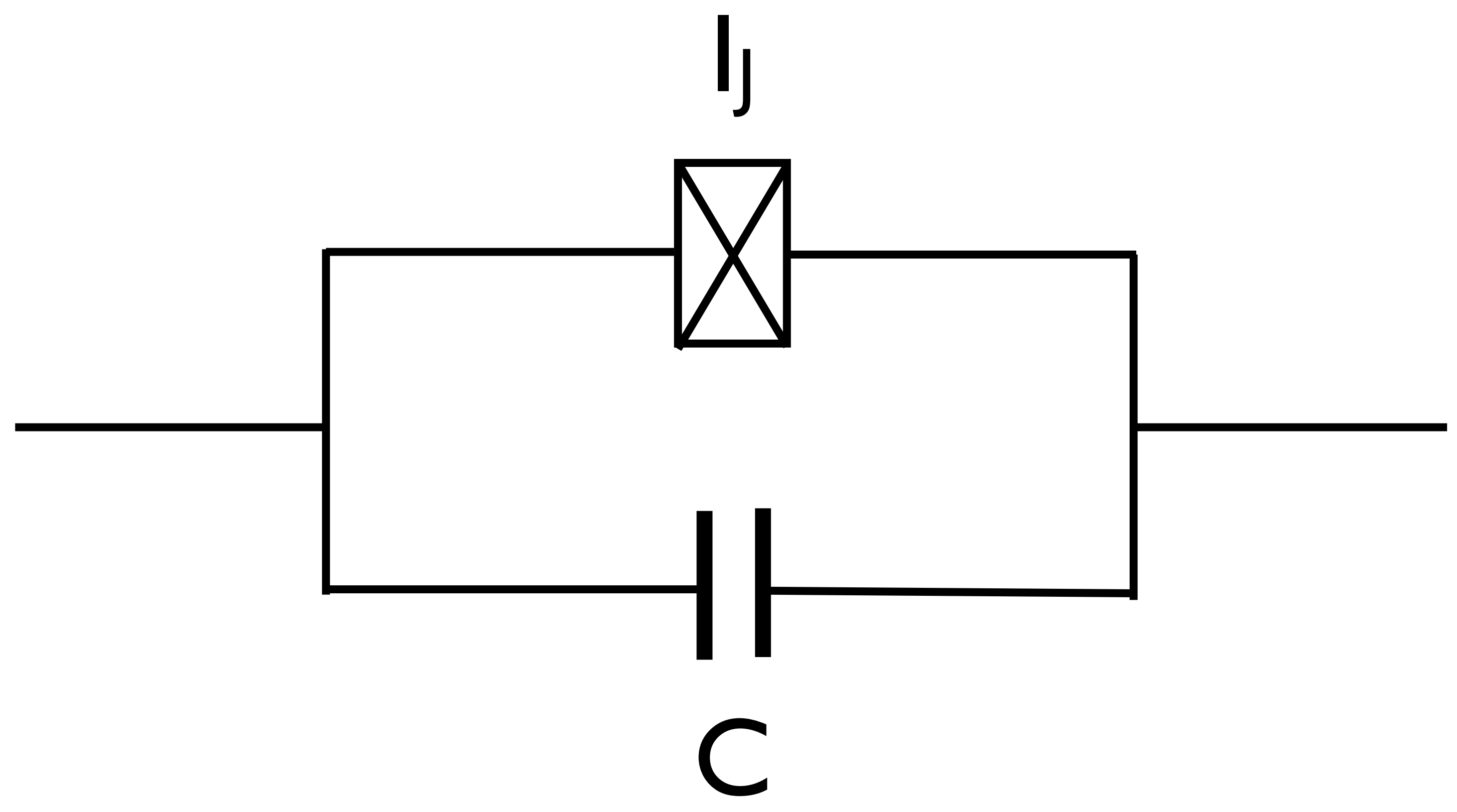
$$\dot{\phi} = \frac{2eV}{\hbar} = \frac{2e}{\hbar}\frac{Q}{C}$$

$$\dot{Q} = -I_J \sin(\phi)$$

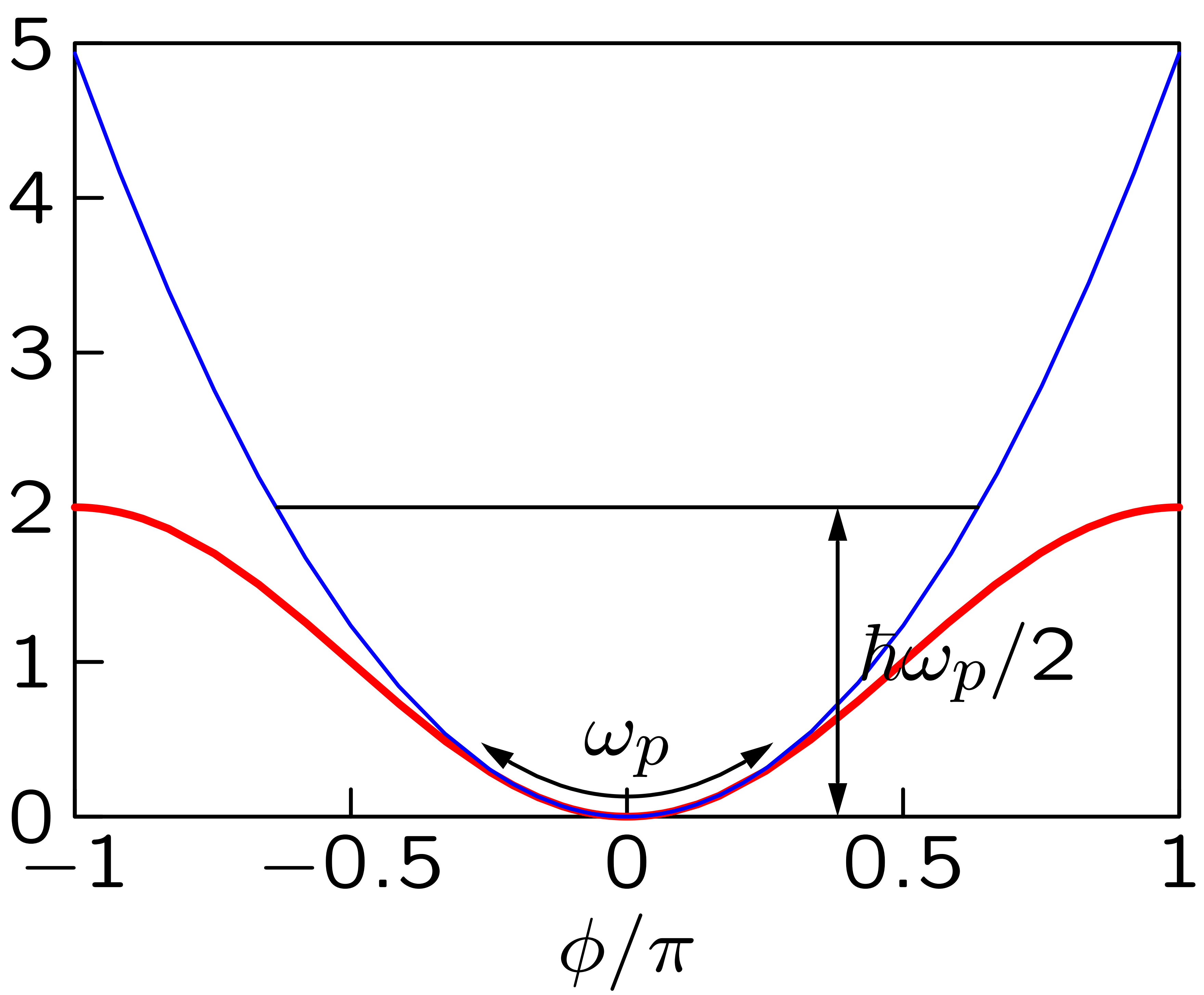
$$\ddot{\phi} = -\omega_p^2 \sin(\phi)$$

$$\omega_p^2 := \frac{2e\, I_J}{C}$$

Macroscopic Quantization



$$\hat{H} = E_C \hat{n}^2 - E_J \cos\hat{\phi}\,, \qquad [\hat{\phi}, \hat{n}] = 1$$



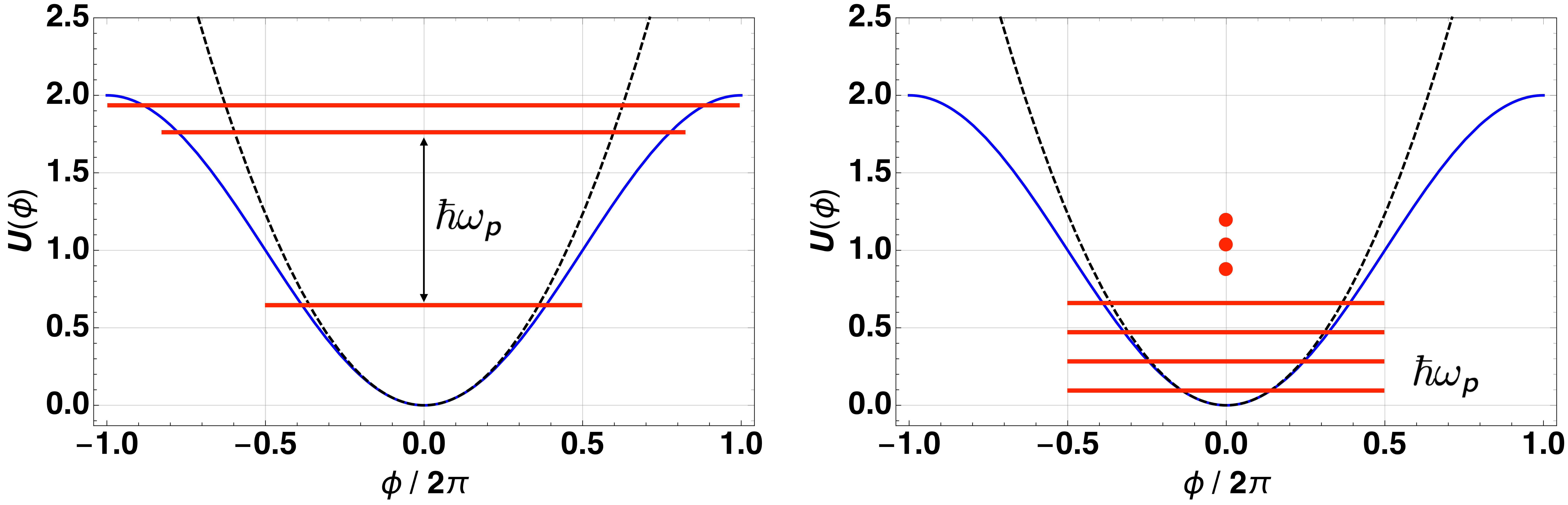
$$E_C := \frac{(2e)^2}{2C}$$

$$E_J := \frac{\hbar I_J}{2e} = \frac{I_J \Phi_0}{2\pi}$$

$$\hbar\omega_p := \sqrt{2E_C E_J}$$

Two Extremes

$E_C \gg E_J$ | $E_C \ll E_J$

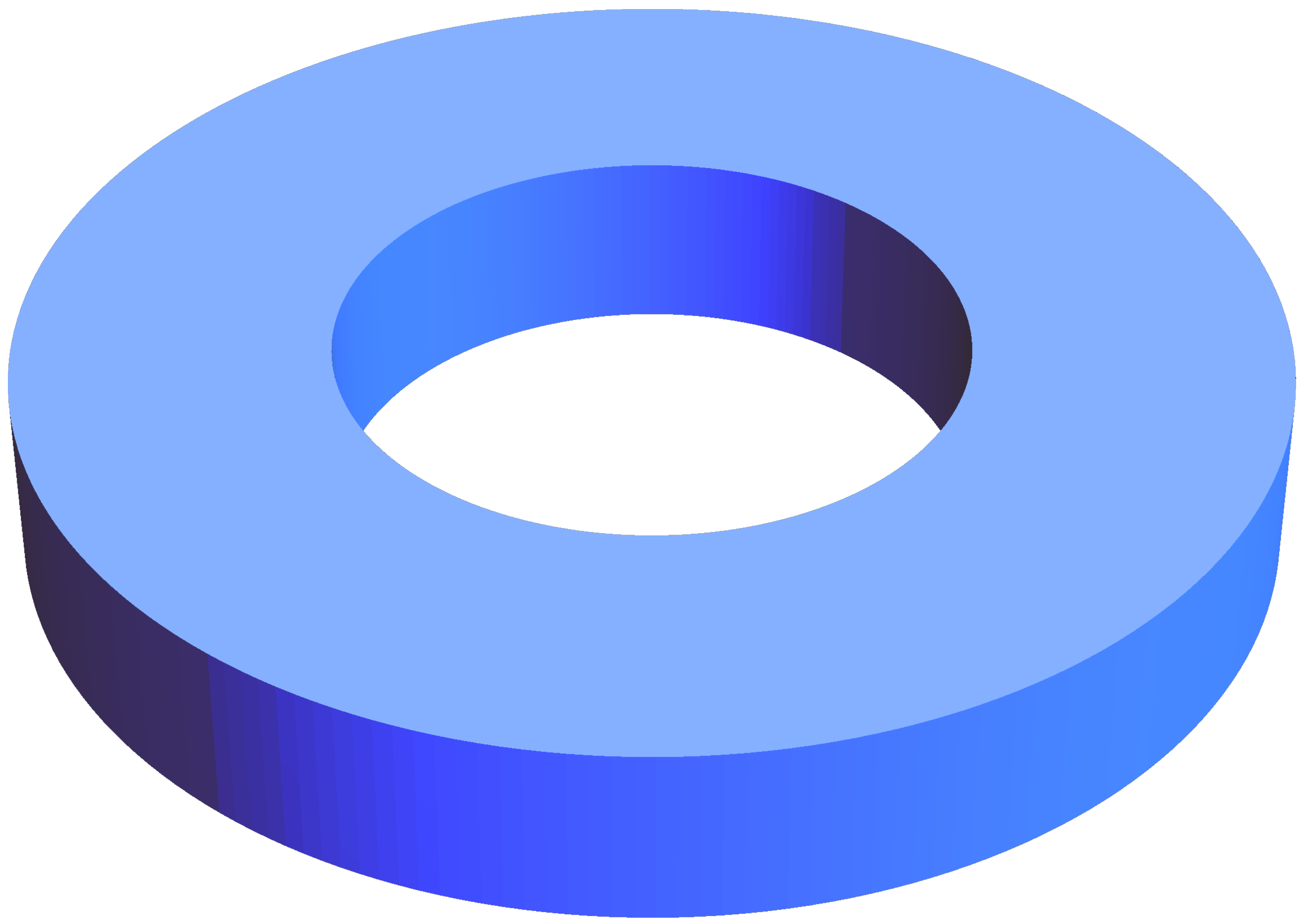


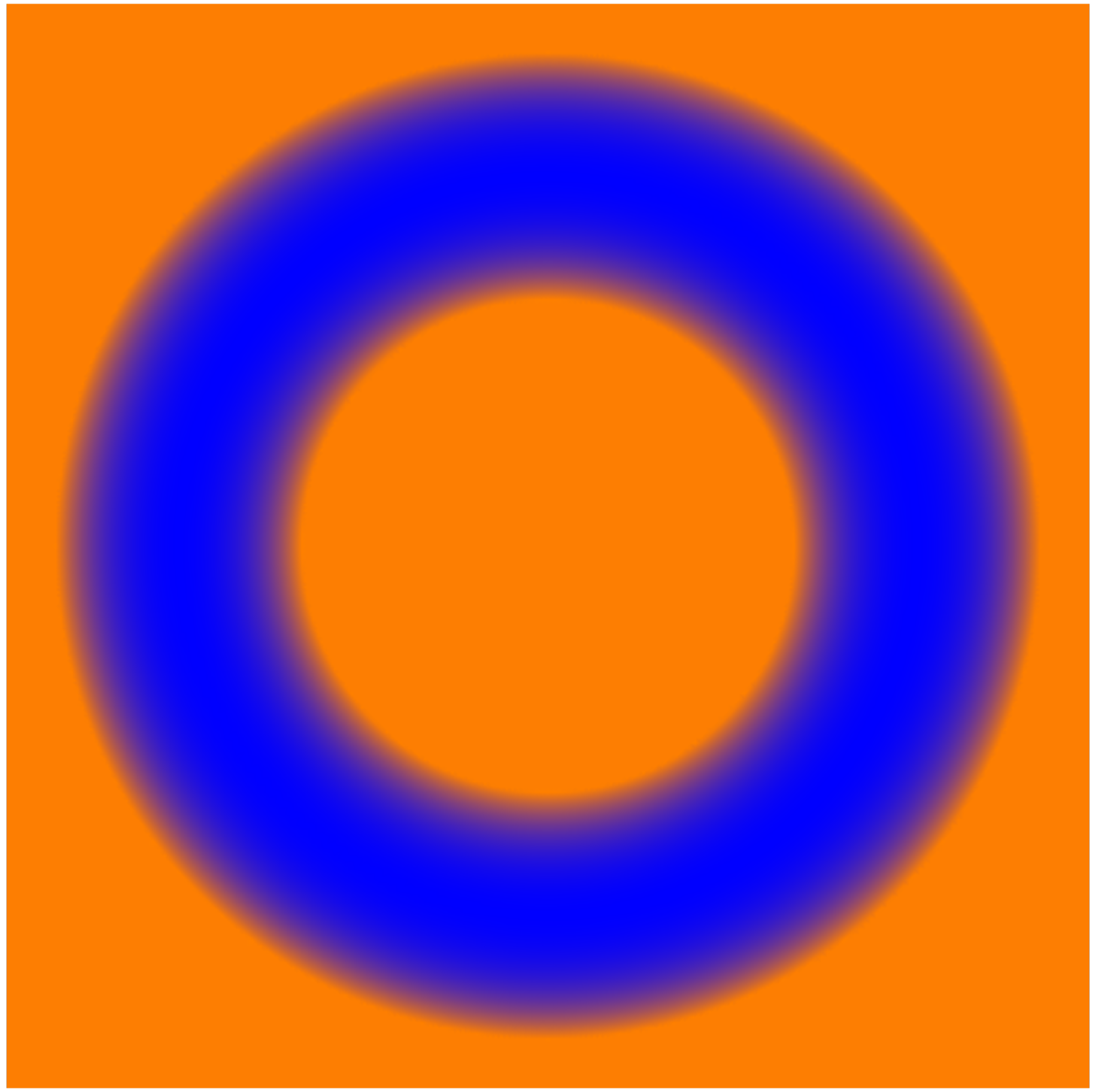
Josephson Junction Circuits

Likharev, Dynamics of Josephson … (1986)

Quantized Flux

(Quantized Super-Current)





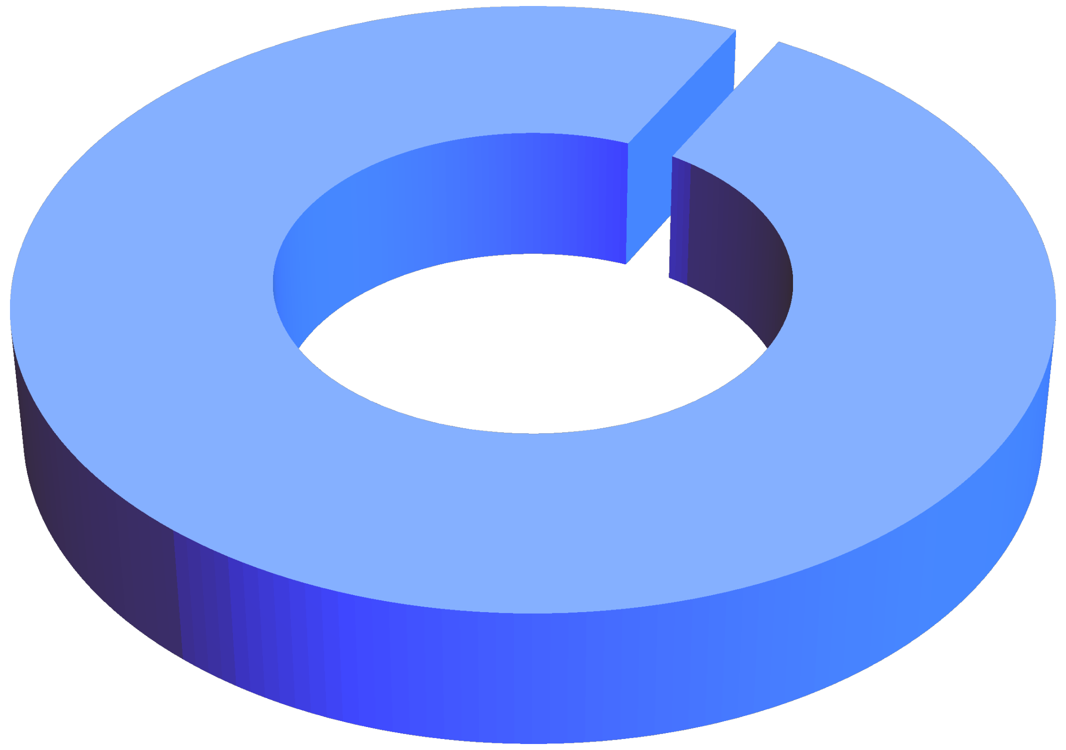
Single-valued-ness

$$\frac{\Phi}{\Phi_0} \in \mathbb{Z}$$

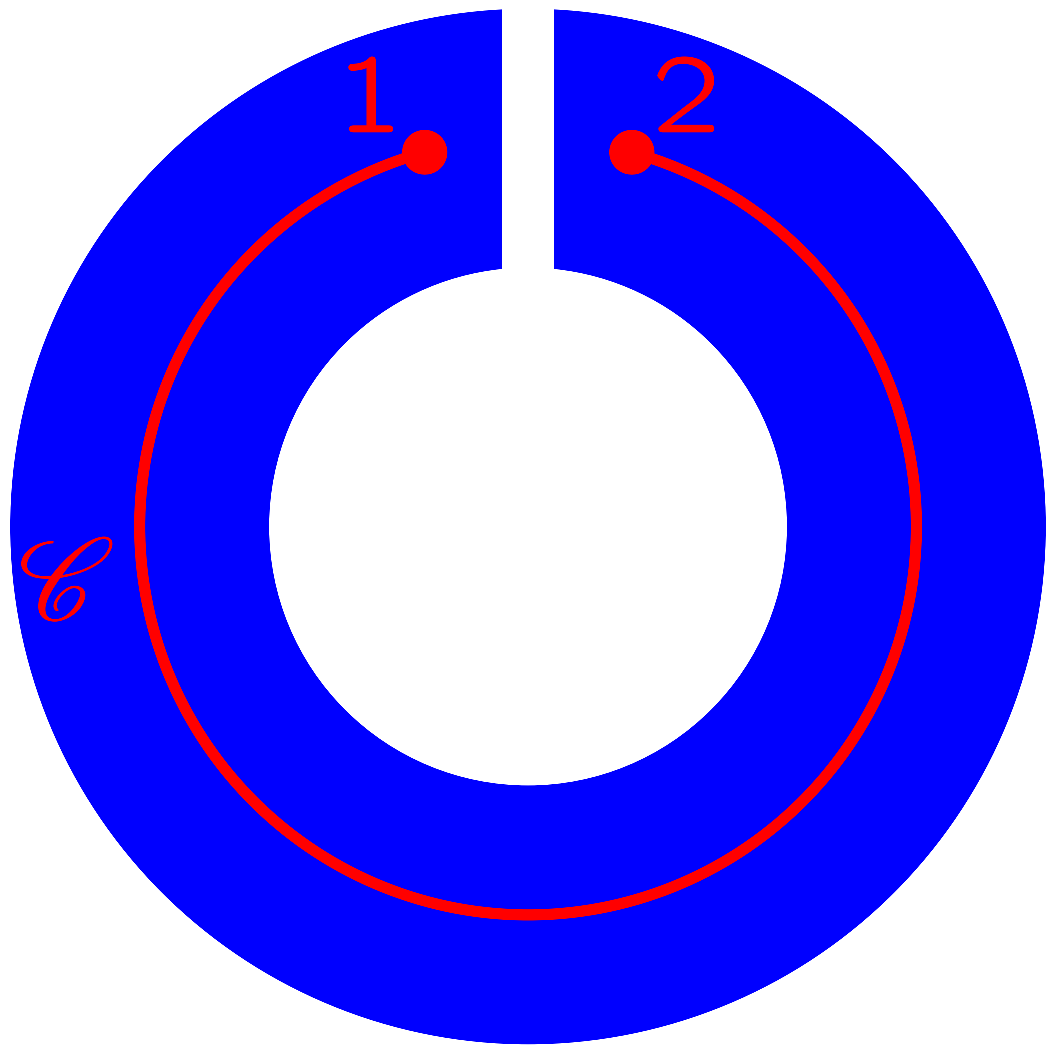
$$\Phi_0 := \frac{h}{2e}$$

Ring with a Josephson Junction

(RF-SQUID)

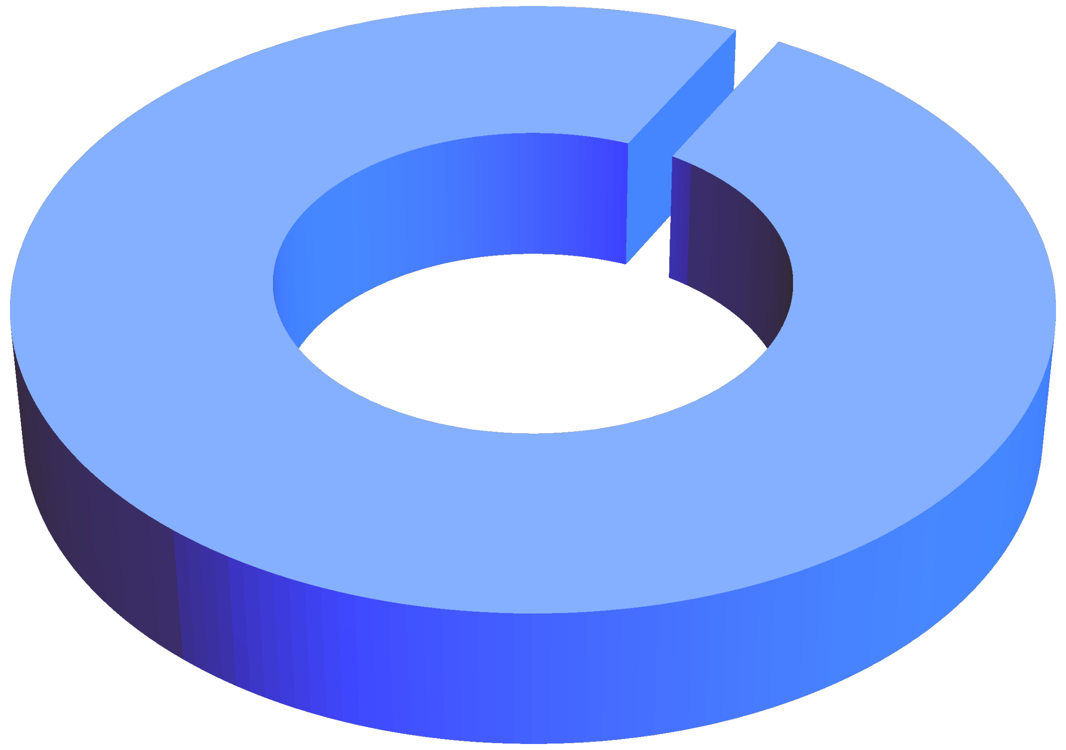


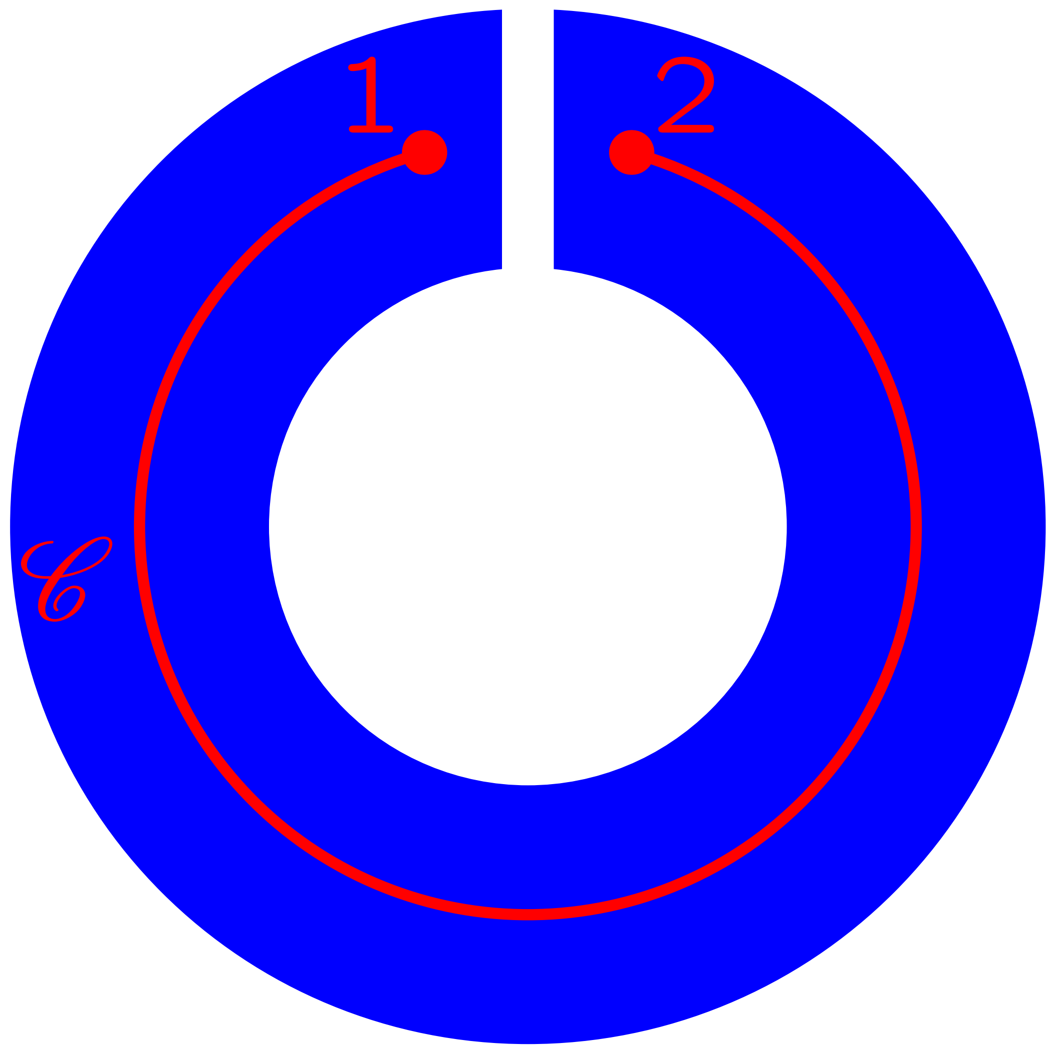
$$\int_{\mathscr{C}} d\boldsymbol{\ell} \cdot \boldsymbol{J}_S = (-2e\rho_S)\frac{\hbar}{m}\int_{\mathscr{C}} d\boldsymbol{\ell} \cdot \left(\boldsymbol{\nabla}\phi + \frac{2\pi}{\Phi_0}\boldsymbol{A}\right) = 0$$



Ring with a Josephson Junction

(RF-SQUID)





$$\int_{\mathscr{C}} d\boldsymbol{\ell} \cdot \boldsymbol{J}_S = (-2e\rho_S)\frac{\hbar}{m}\int_{\mathscr{C}} d\boldsymbol{\ell} \cdot \left(\boldsymbol{\nabla}\phi + \frac{2\pi}{\Phi_0}\boldsymbol{A}\right) = 0$$

$$\varphi := \phi_2 - \phi_1 + \frac{2\pi}{\Phi_0}\int_1^2 d\boldsymbol{\ell} \cdot \boldsymbol{A} = -2\pi\frac{\Phi}{\Phi_0}$$

Single-Junction Flux Qubit

- Supercurrent

$$I_S = -I_J \sin(2\pi\Phi/\Phi_0)$$

- Displacement current $\frac{Q}{C} = V = \frac{\hbar\dot{\varphi}}{2e} = -\dot{\Phi}$

$$I_D = -C\ddot{\Phi}$$

- Equation of motion $\Phi = \Phi_{\text{ext}} + \Phi_{\text{ind}} = \Phi_{\text{ext}} + L(I_S + I_D)$

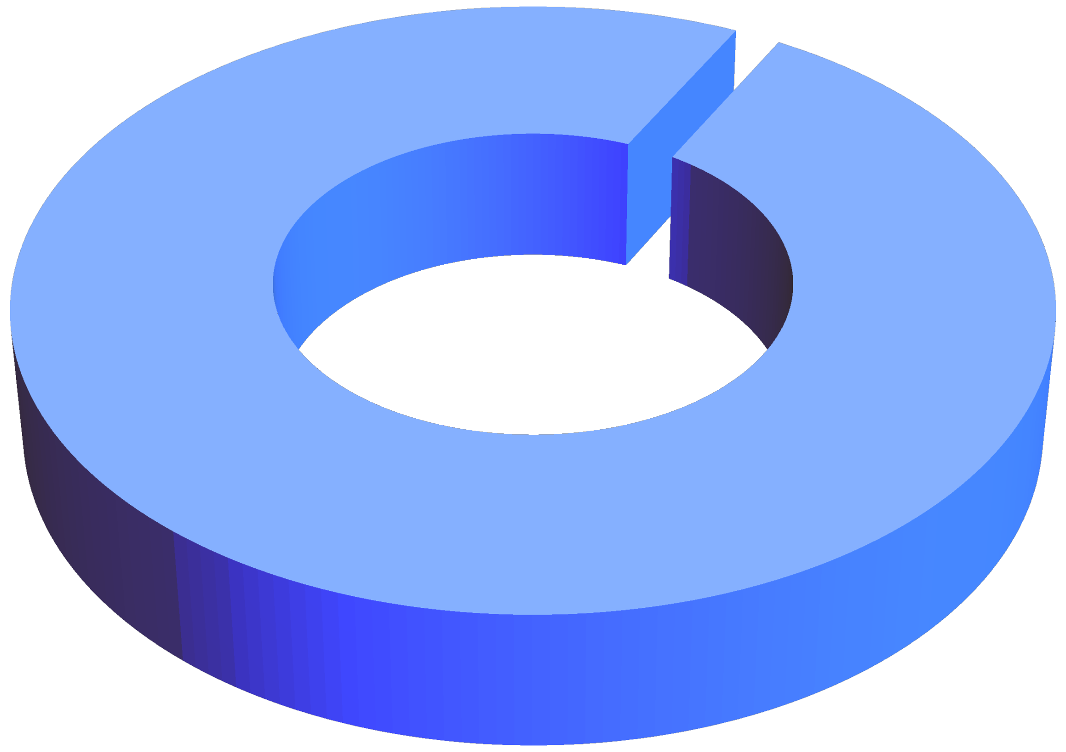
$$\ddot{\Phi} = -\frac{1}{LC}(\Phi - \Phi_{\text{ext}}) - \frac{I_J}{C}\sin(2\pi\Phi/\Phi_0)$$

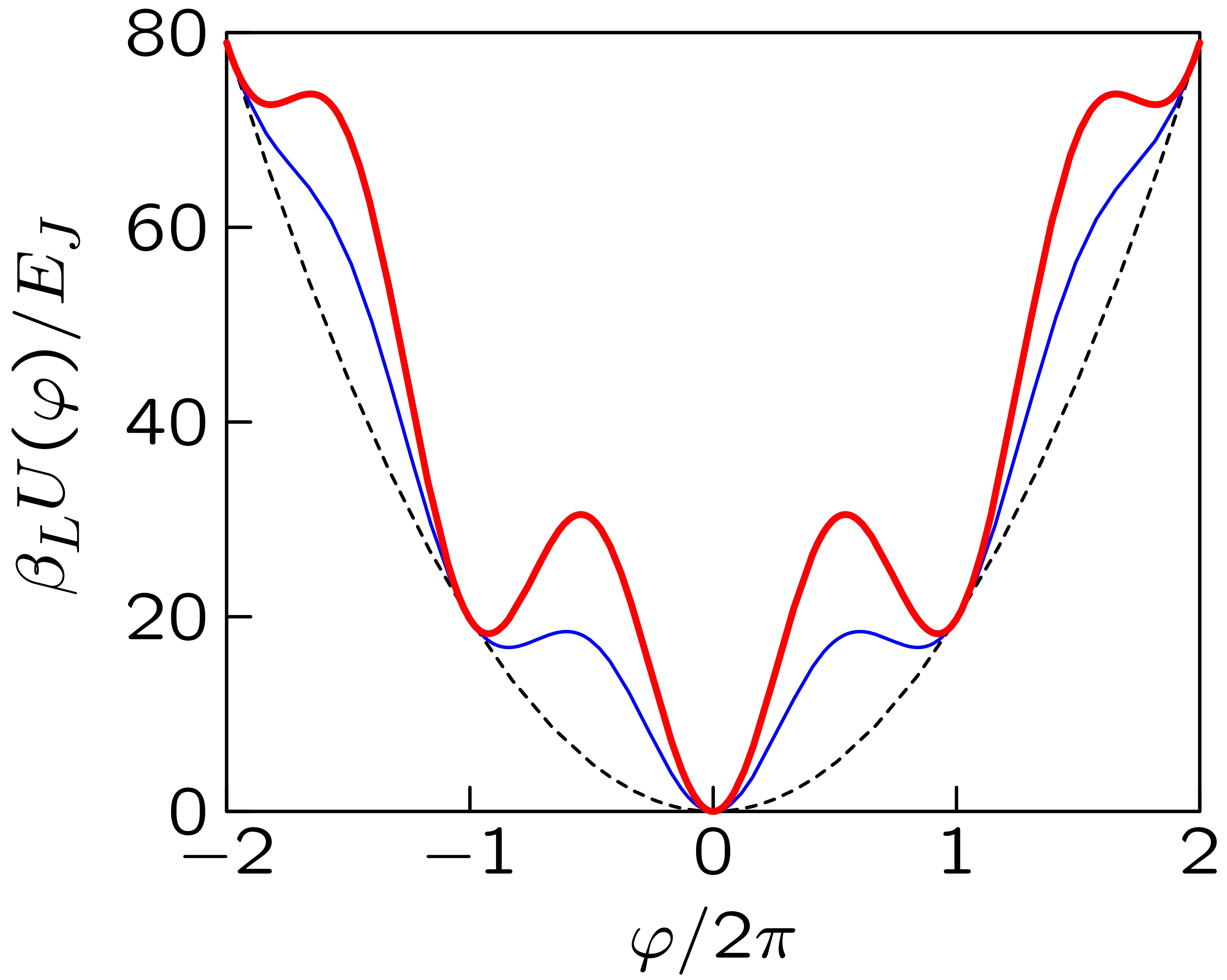
- Lagrangian
- Hamiltonian

Single-Junction Flux Qubit

$$\mathscr{L} = \frac{C}{2}\dot{\Phi}^2 - U(\Phi)\,,$$

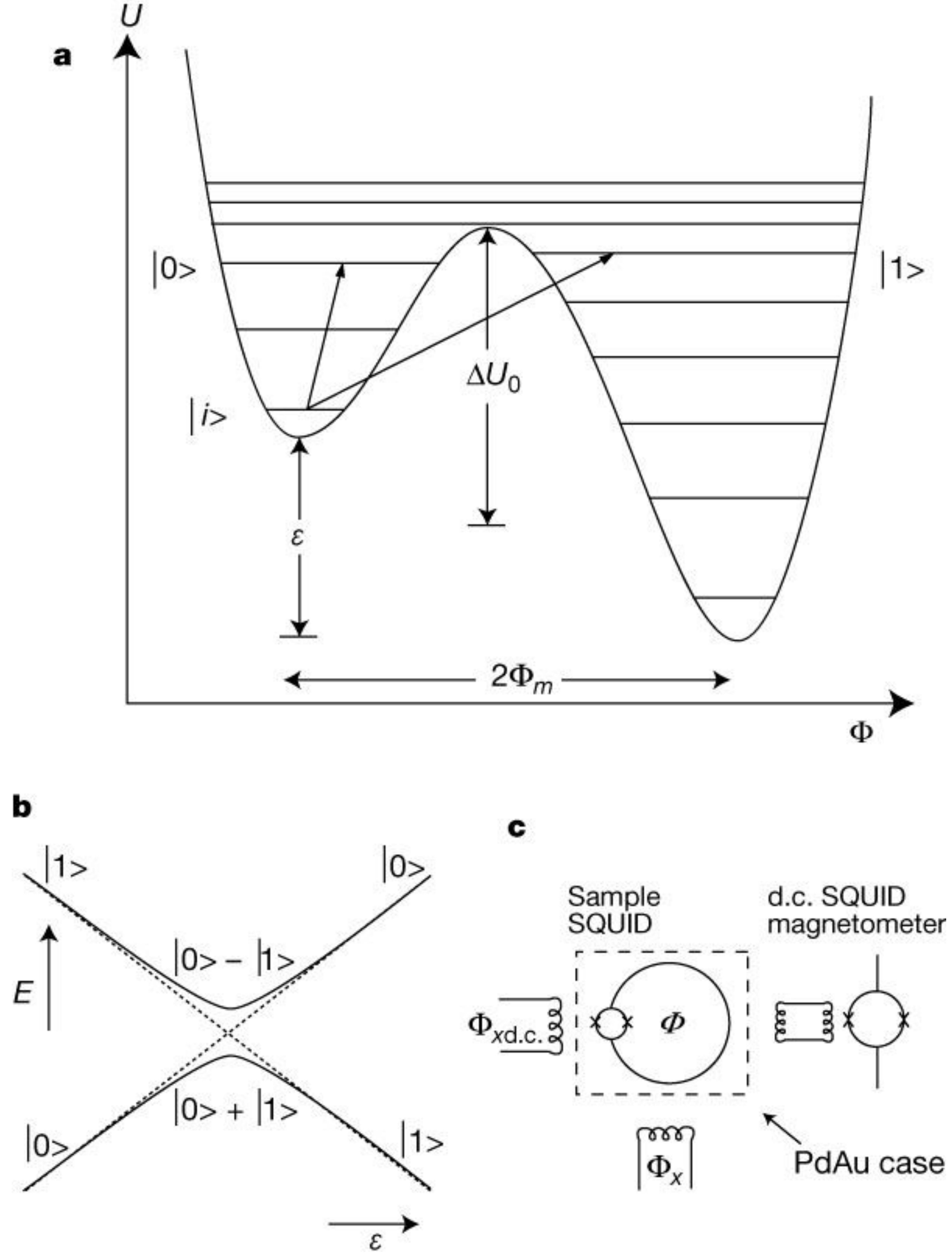
$$U(\Phi) = \frac{1}{2L}(\Phi - \Phi_{\text{ext}})^2 - E_J \cos(2\pi\Phi/\Phi_0)$$

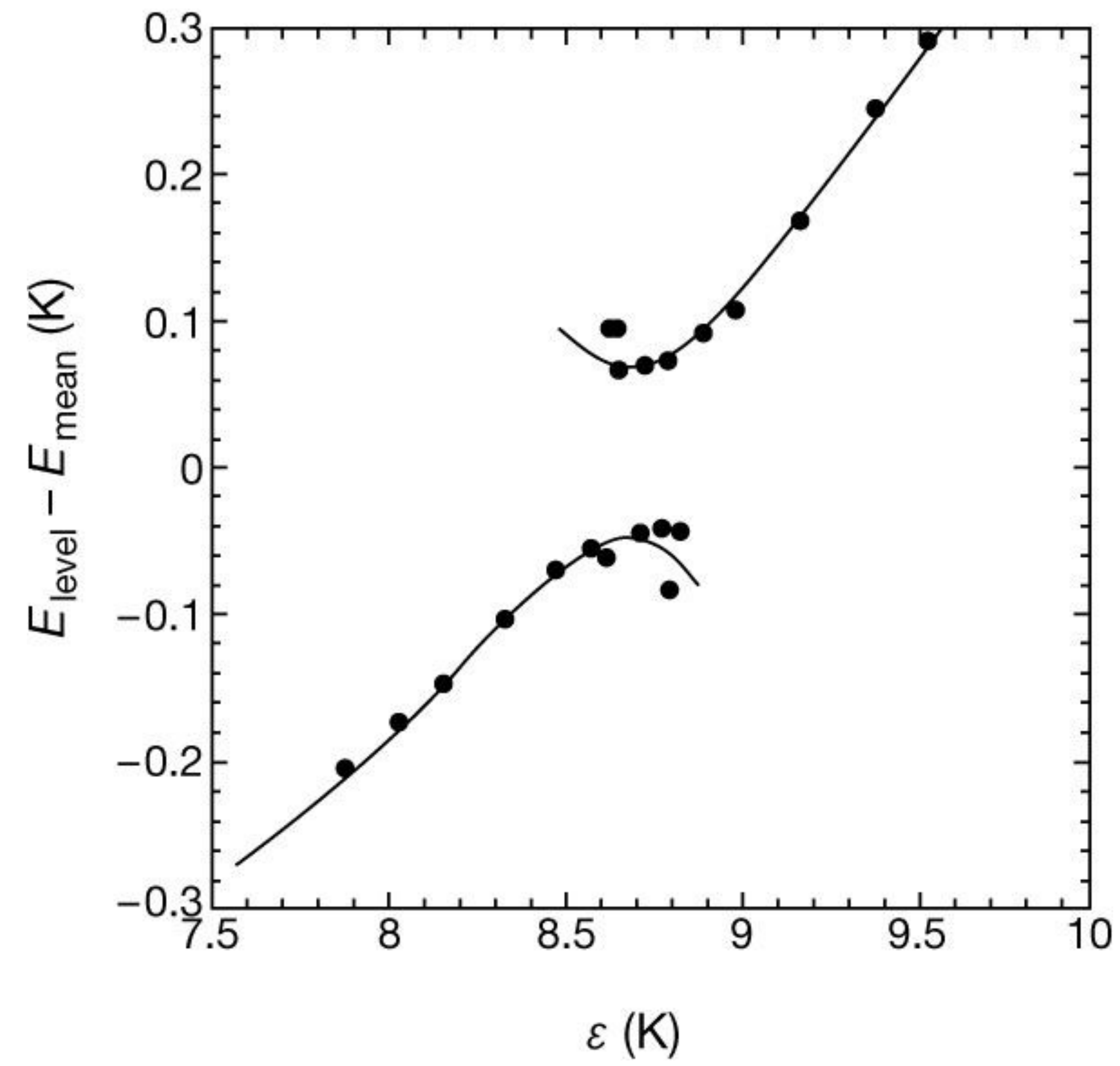




MACROSCOPIC QUANTUM TUNNELING

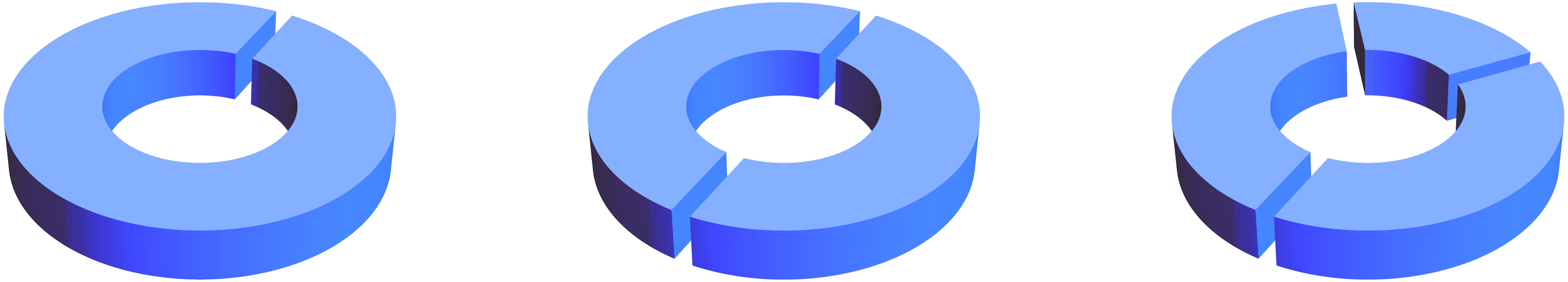
Friedman et al. (Nature, 2000)

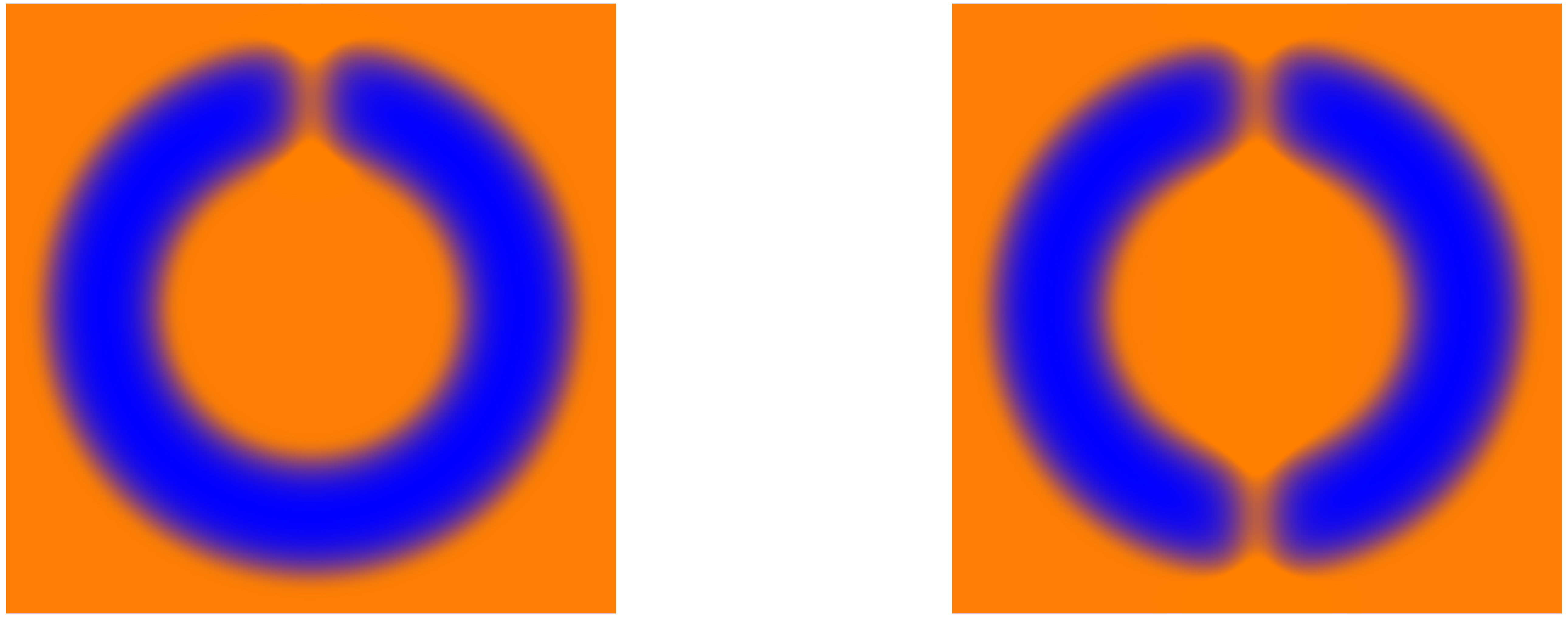




SQUIDs

(Superconducting Quantum Interference Devices)

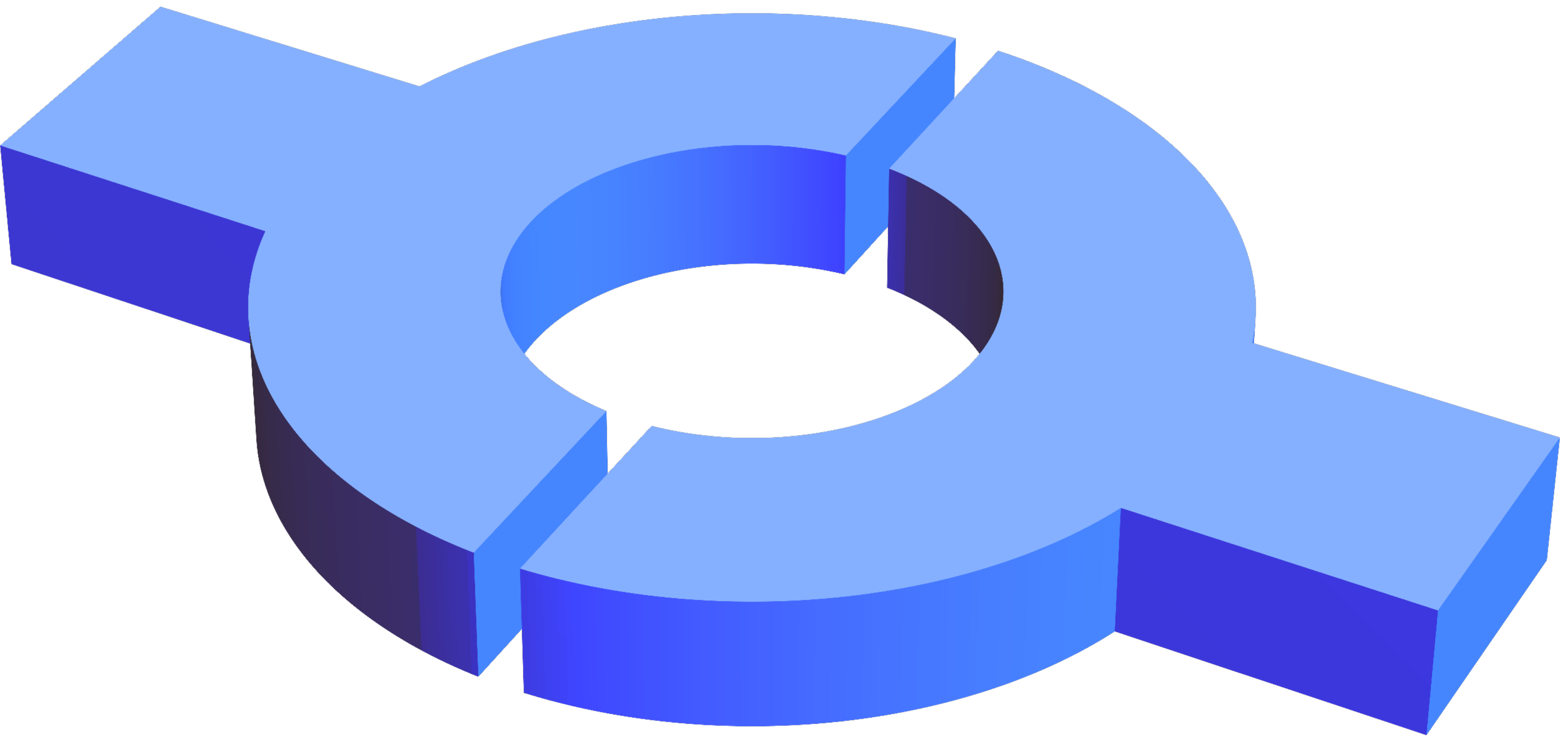




$$\sum_j \varphi_j + 2\pi\Phi/\Phi_0 = 0$$

TUNABLE JOSEPHSON JUNCTION

DC SQUID



$$I_S = I_{S1} + I_{S2} = I_{J1}\sin(\varphi_1) + I_{J2}\sin(\varphi_2)$$

$$\varphi_1 - \varphi_2 + 2\pi\Phi/\Phi_0 = 0$$

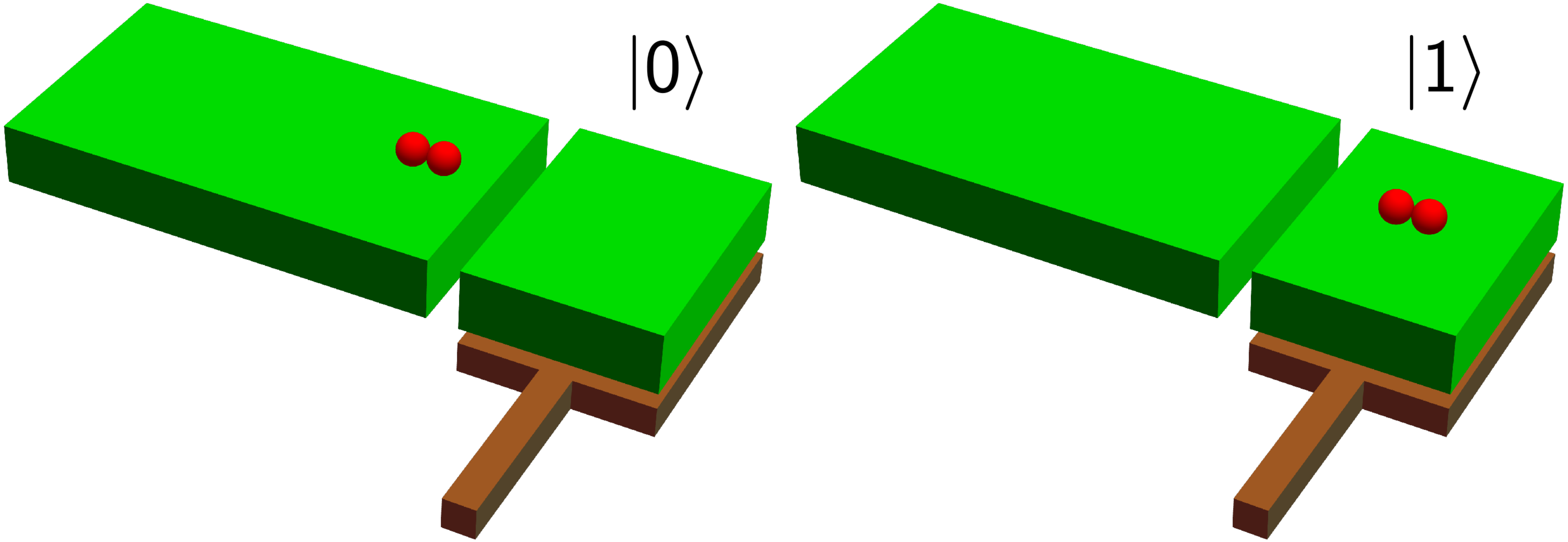
$$I_S = \overbrace{2I_J\cos(\pi\Phi/\Phi_0)}^{I_J^{\text{eff}}}\sin(\varphi_1 - \pi\Phi/\Phi_0)$$

Superconducting Qubits

Charge, Phase & Transmon Qubits

Quantized Charge

(Quantized Number of Cooper Pairs)

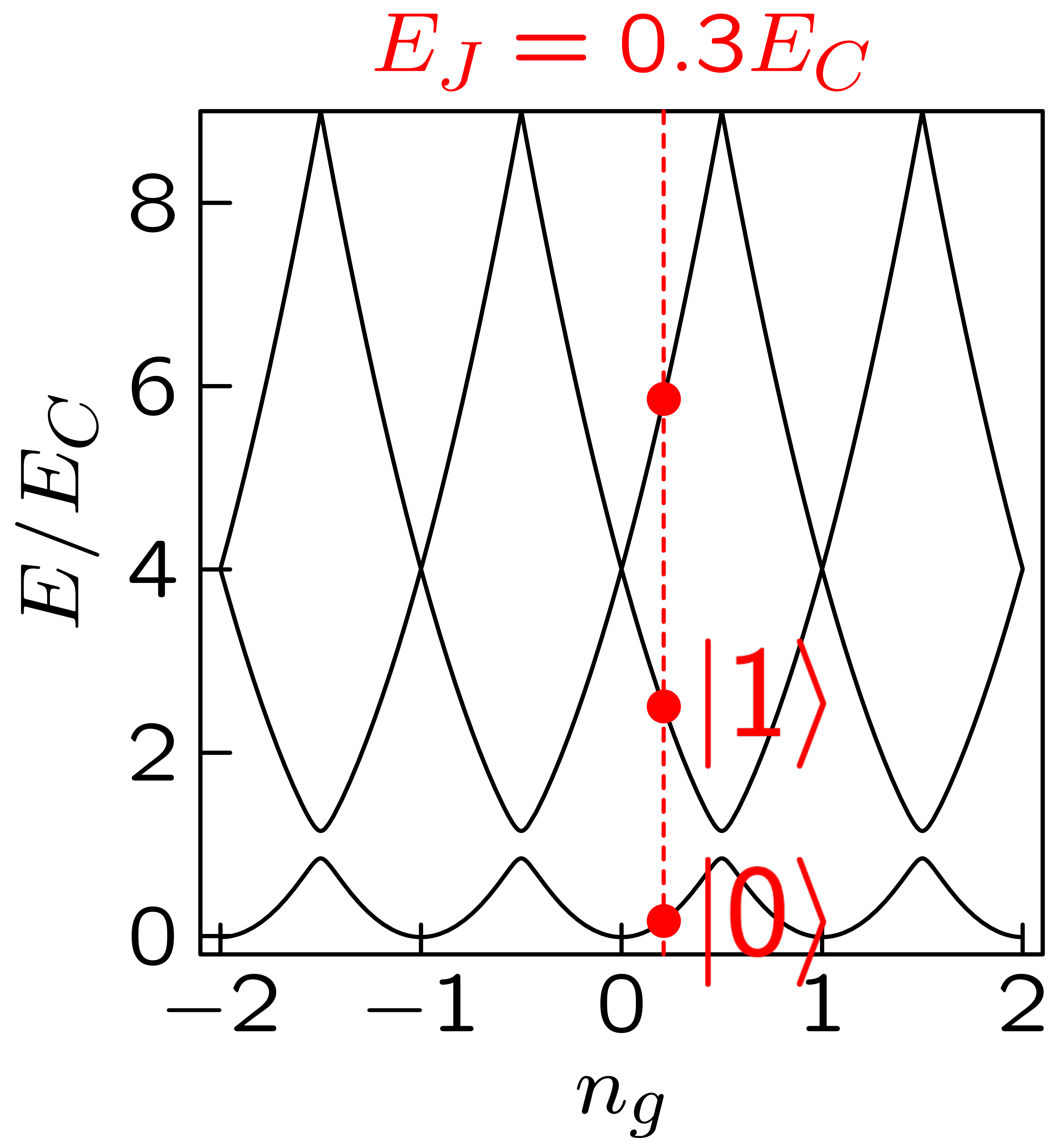


$$H_{\mathrm{qubit}} = \frac{1}{2}\Omega\sigma^z$$

$$\Omega/2\pi \sim 5\,\mathrm{GHz}$$

Bouchiat *et al.* (Phys. Scr., 1998)
Nakamura *et al.* (Nature, 1999)

Quantized Charge



$$[\phi, n] = i$$

$$H = \overbrace{E_C(n - n_g)^2}^{\text{dominant}} - E_J \cos\phi$$

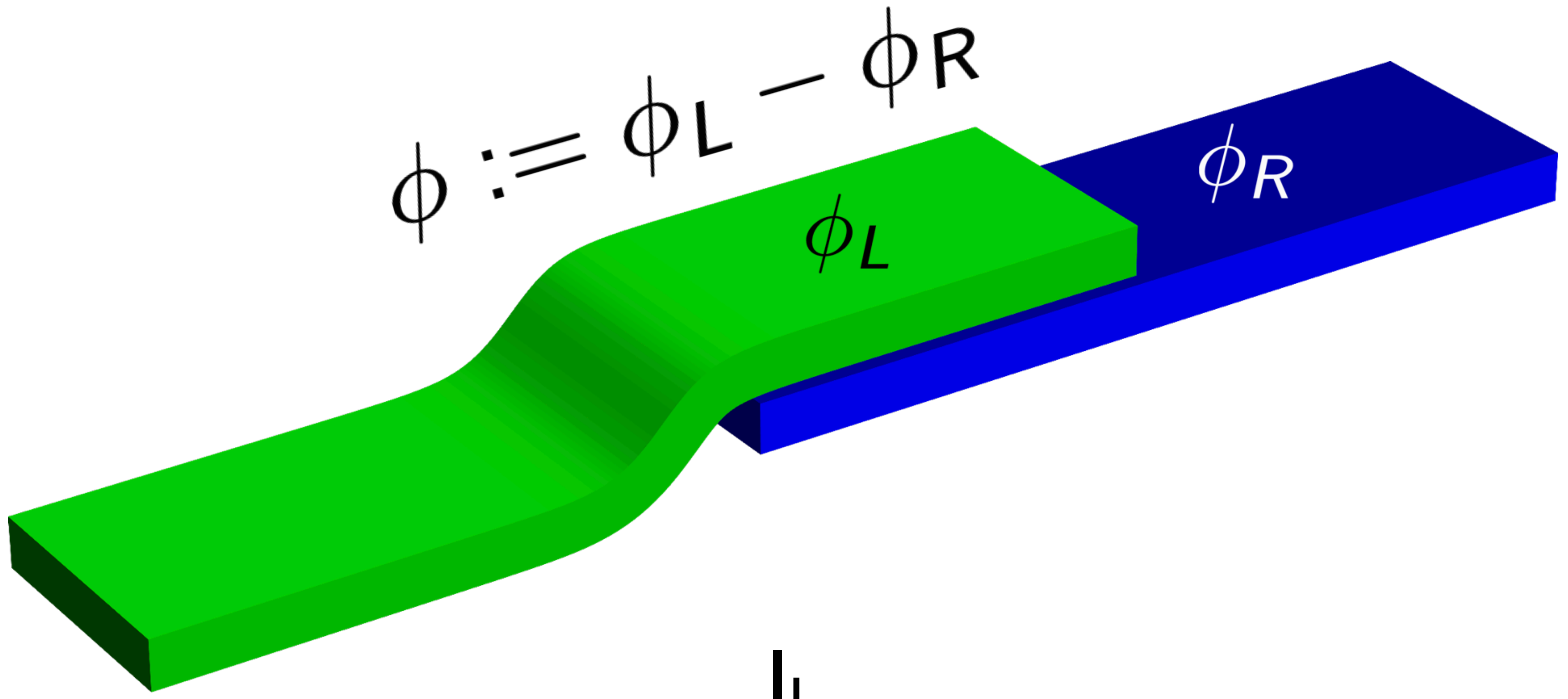
$$H \approx E_C(n_g - 1)\sigma^z - \frac{1}{2}E_J\sigma^x$$

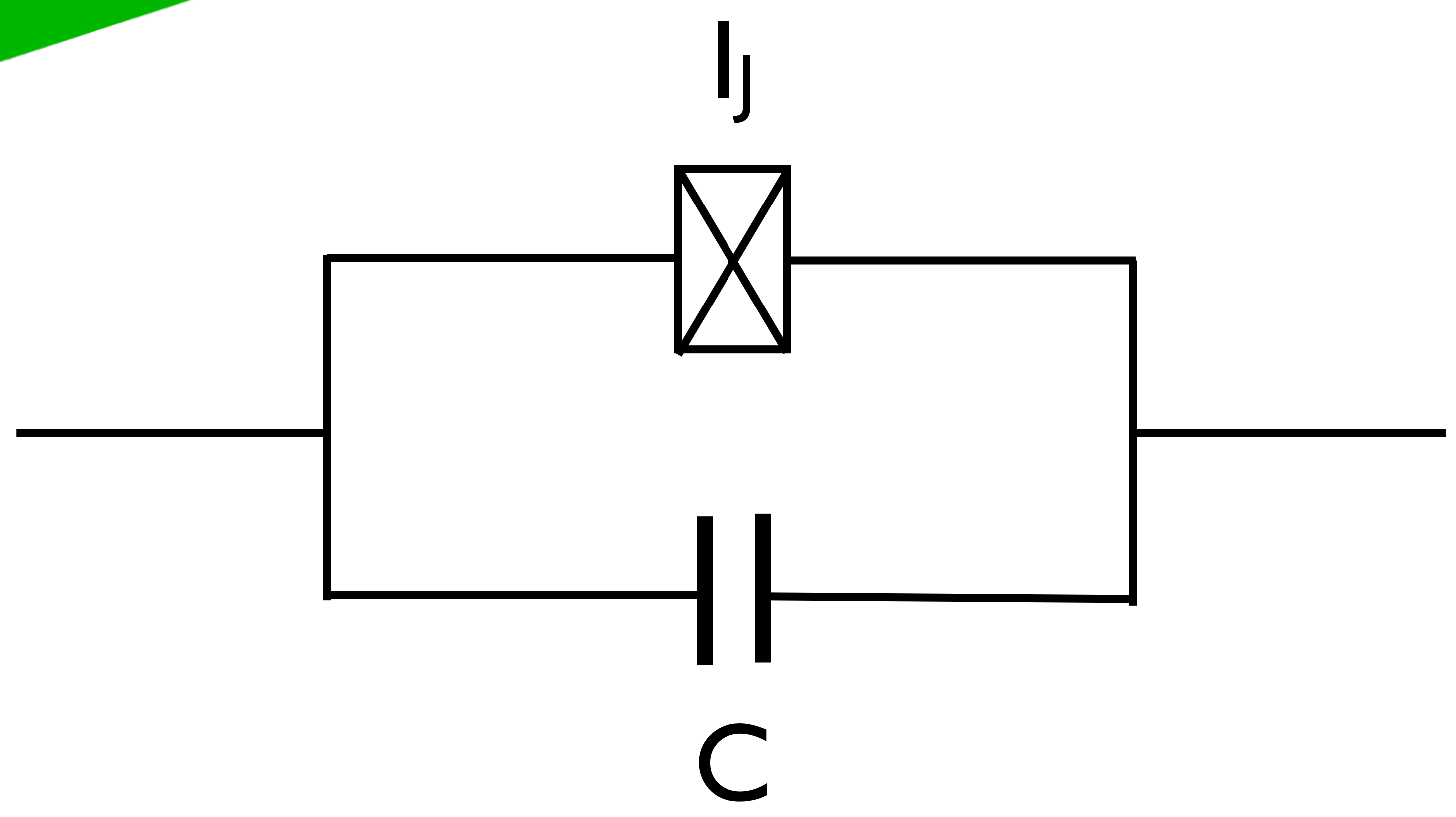
PROS & CONS

- All electric
- Simple and fast
- Too sensitive to environment (substrates, etc.) charges
- Short decoherence time

Phase Qubit

Biased Anharmonic Oscillator



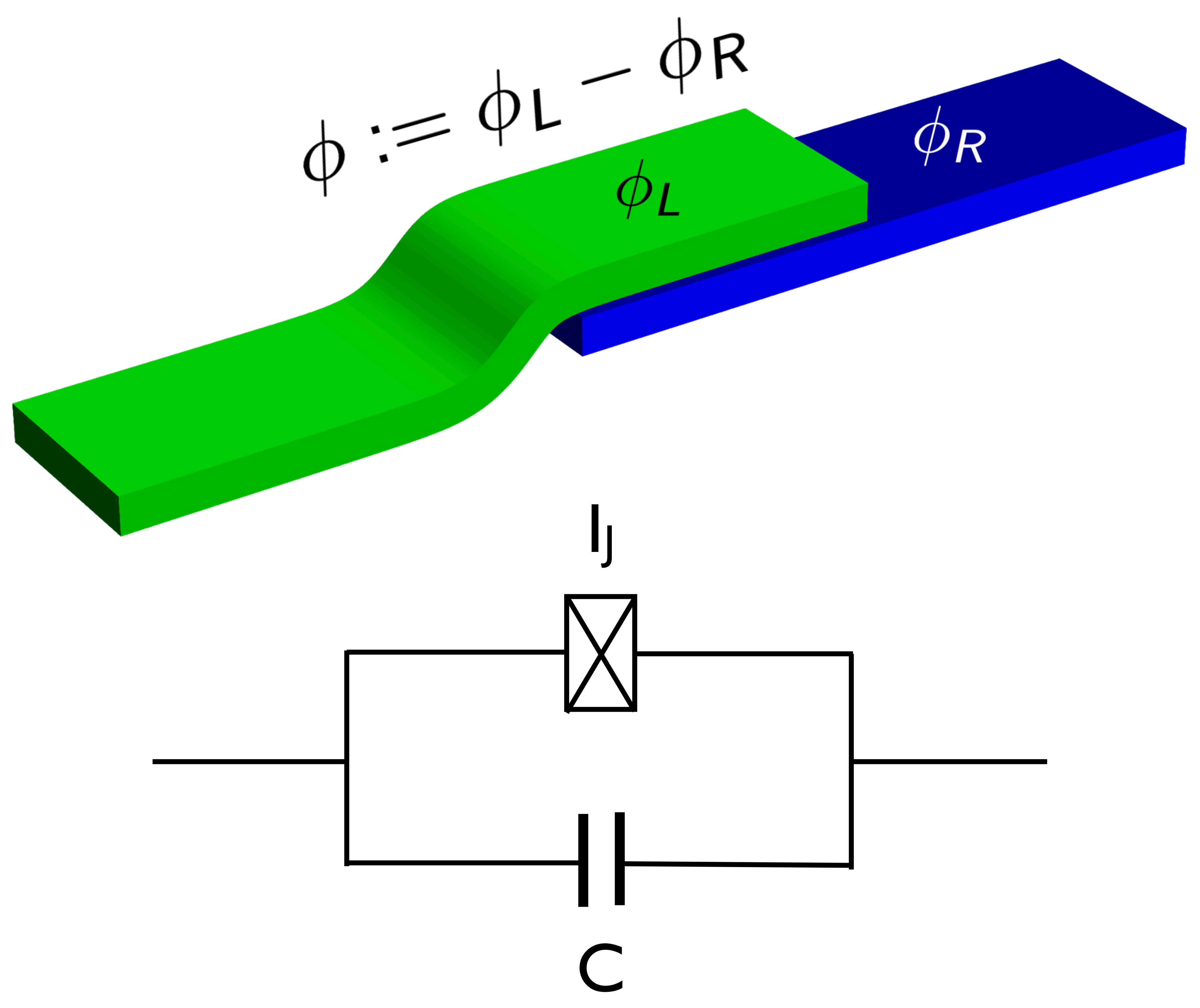


$$\frac{E_J}{E_C} \sim 10^4$$

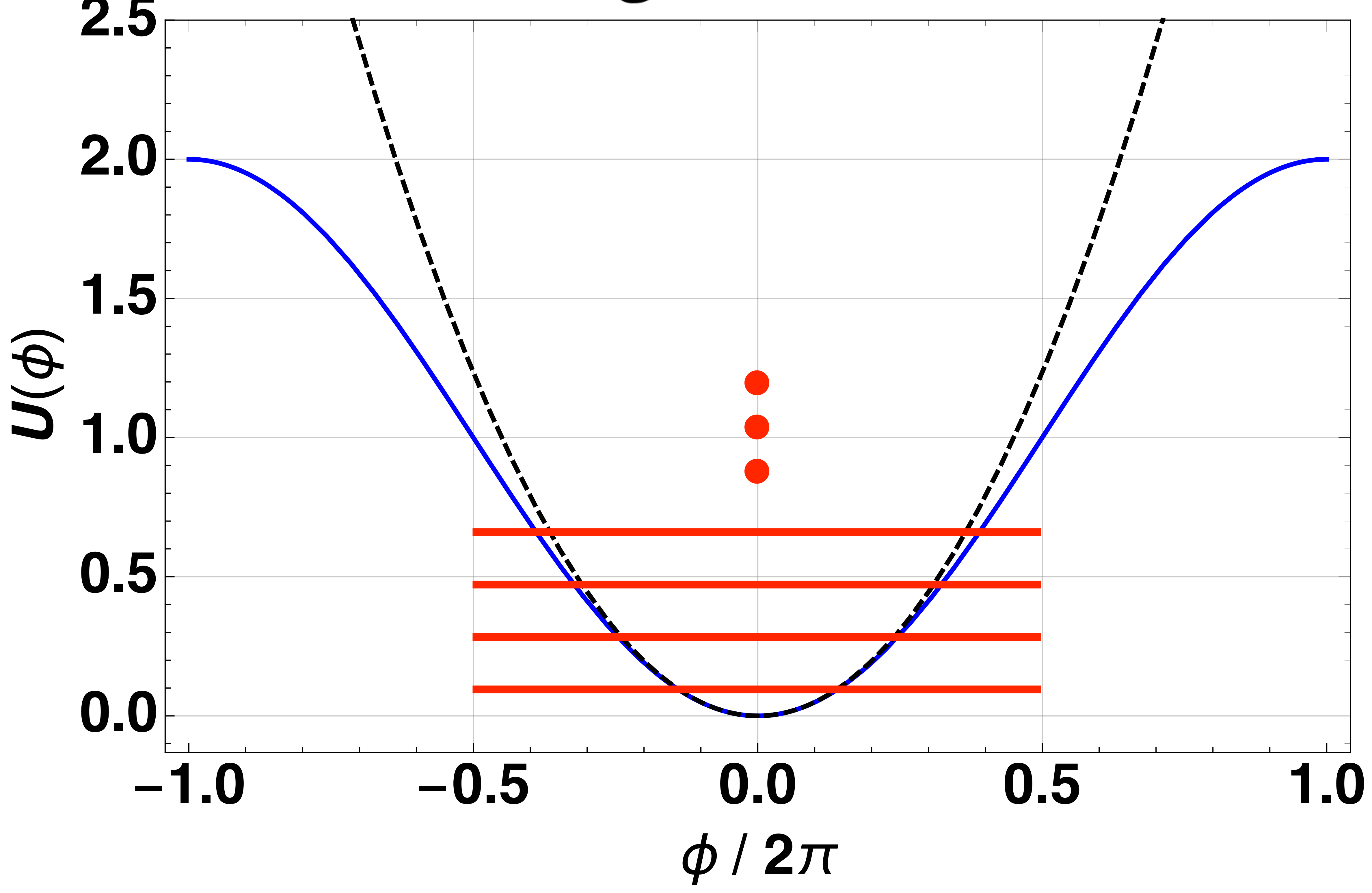
$$\hat{H} = E_C \hat{n}^2 - E_J \cos\hat{\phi}\,, \qquad [\hat{\phi}, \hat{n}] = 1$$

Phase Qubit

Biased Anharmonic Oscillator

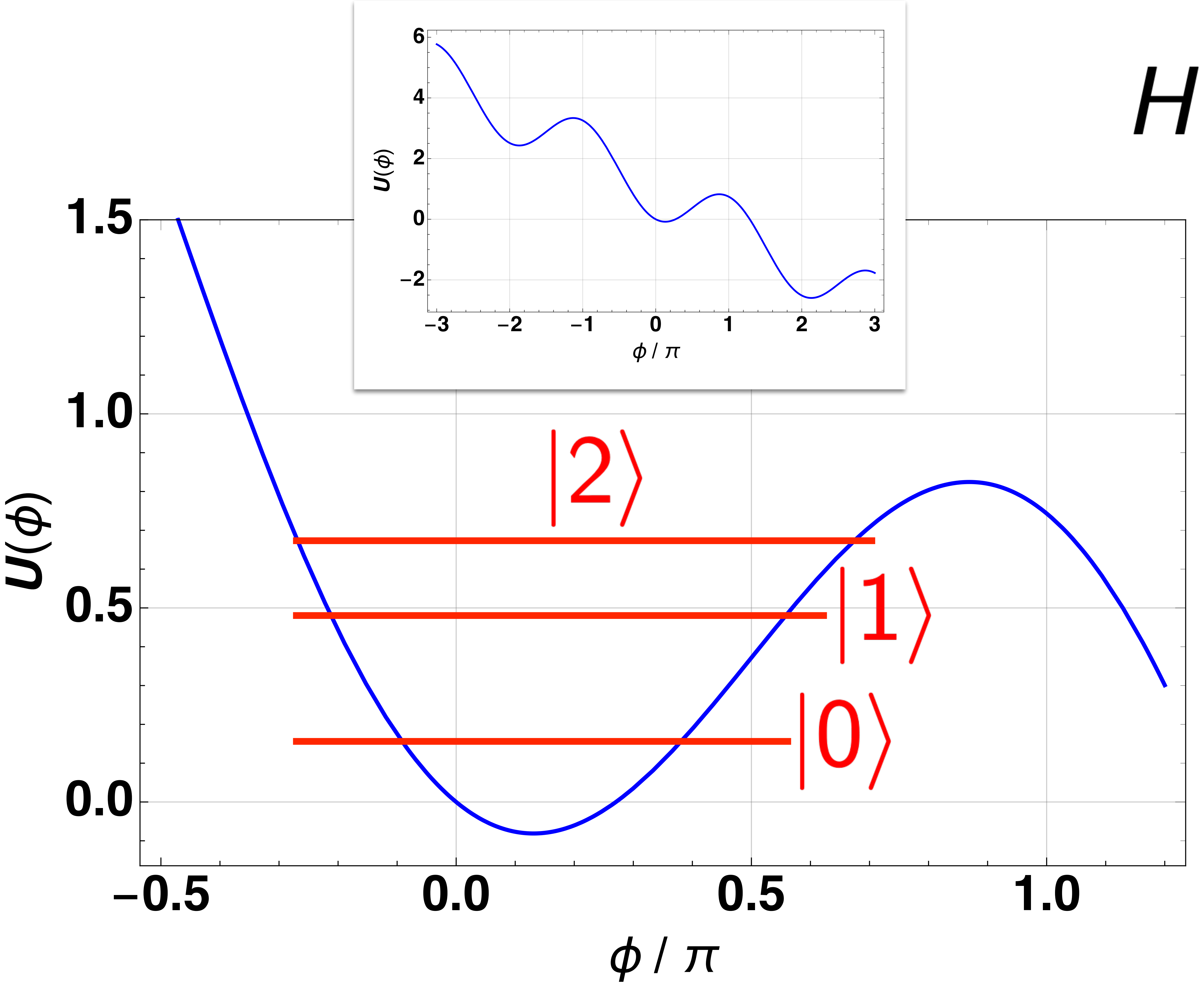


$$\frac{E_J}{E_C} \sim 10^4$$



Phase Qubit

Biased Anharmonic Oscillator

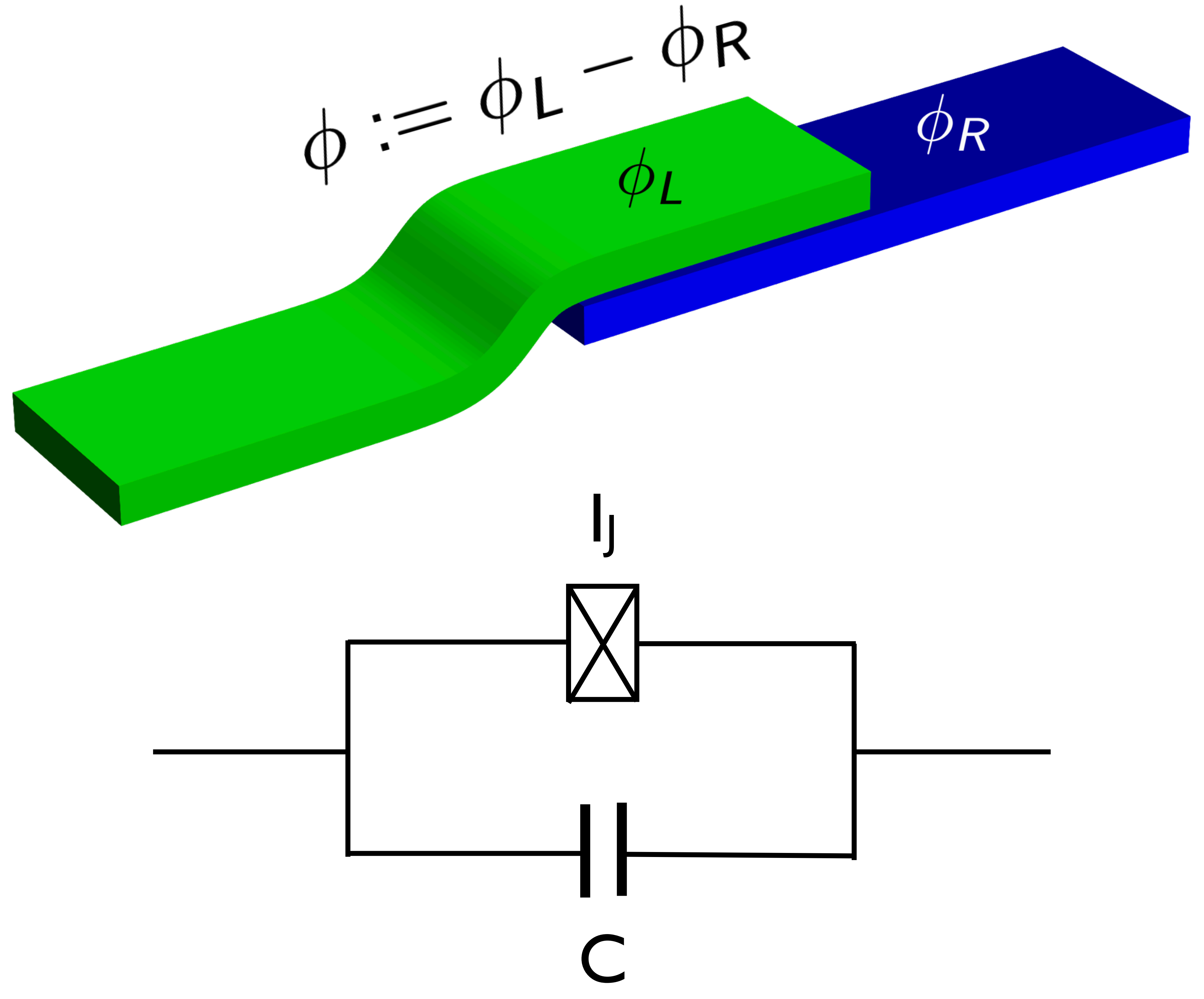


$$H = \underbrace{E_C(n - n_g)^2}_{\text{perturbation}} - E_J \cos\phi - I\phi$$

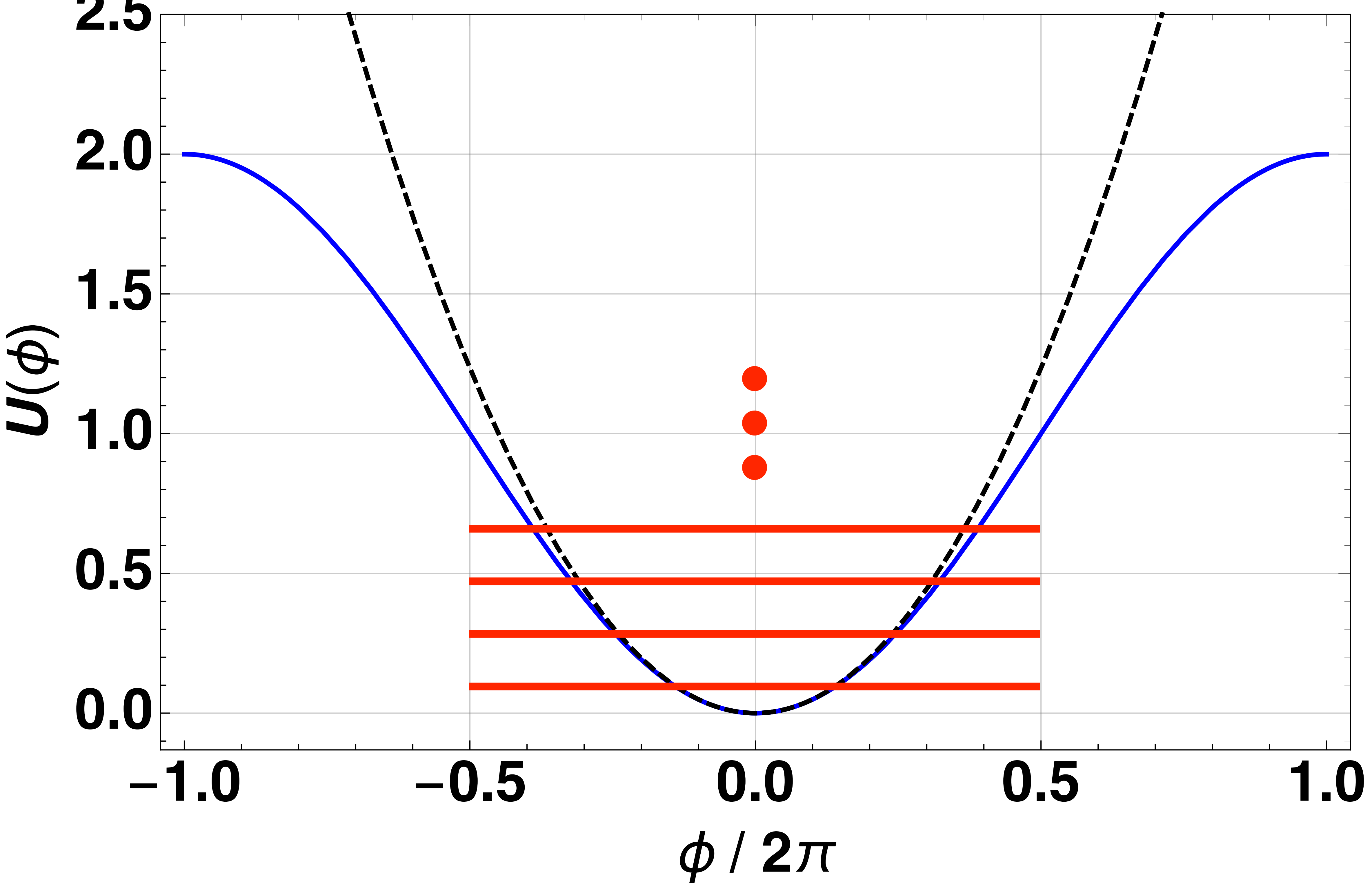
Martinis *et al.* (PRL, 2002)
Steffen *et al.* (PRL, 2006)

Transmon Qubit

(Unbiased) Anharmonic Oscillator



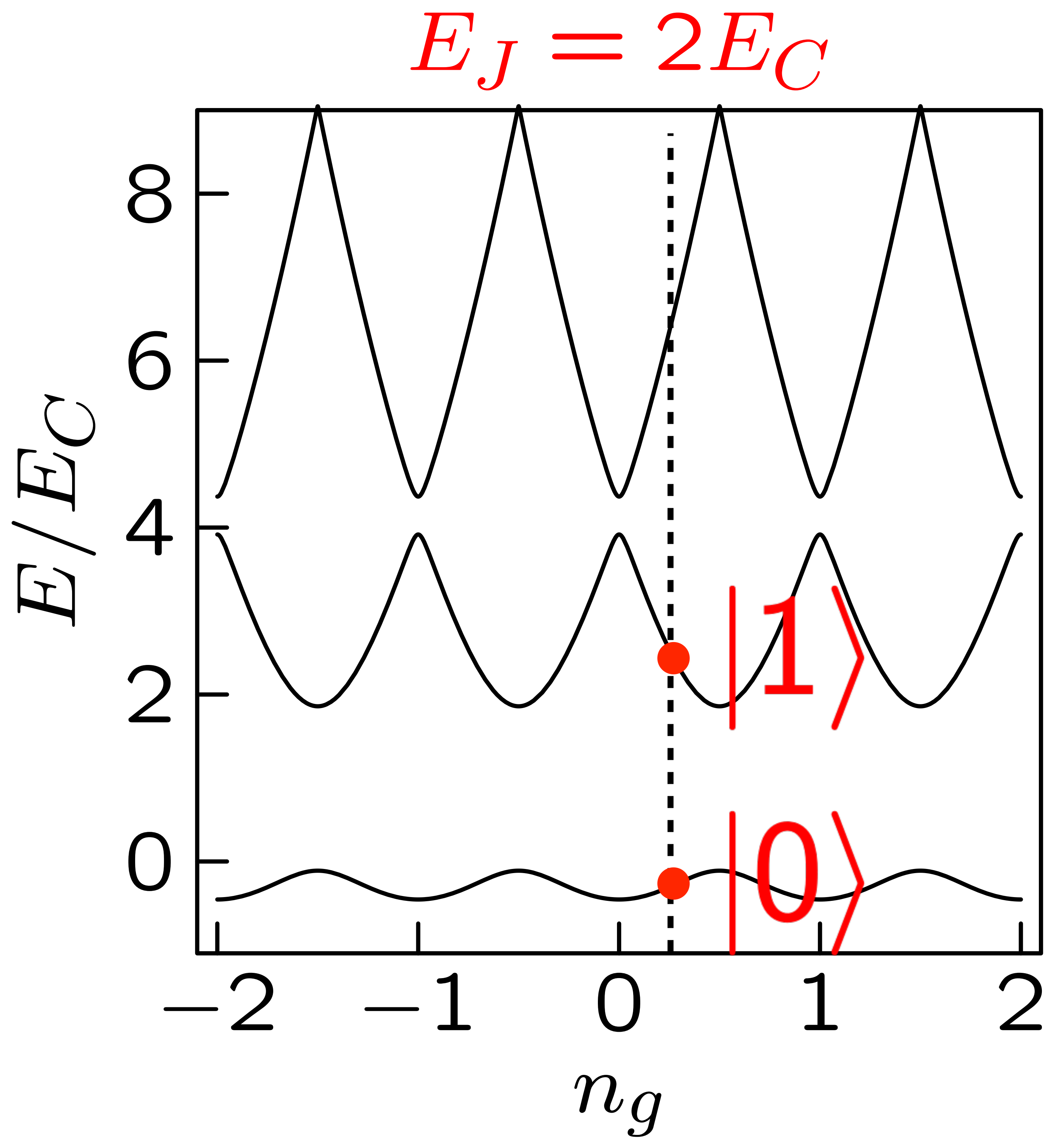
$$\frac{E_J}{E_C} \simeq 1 - 100$$



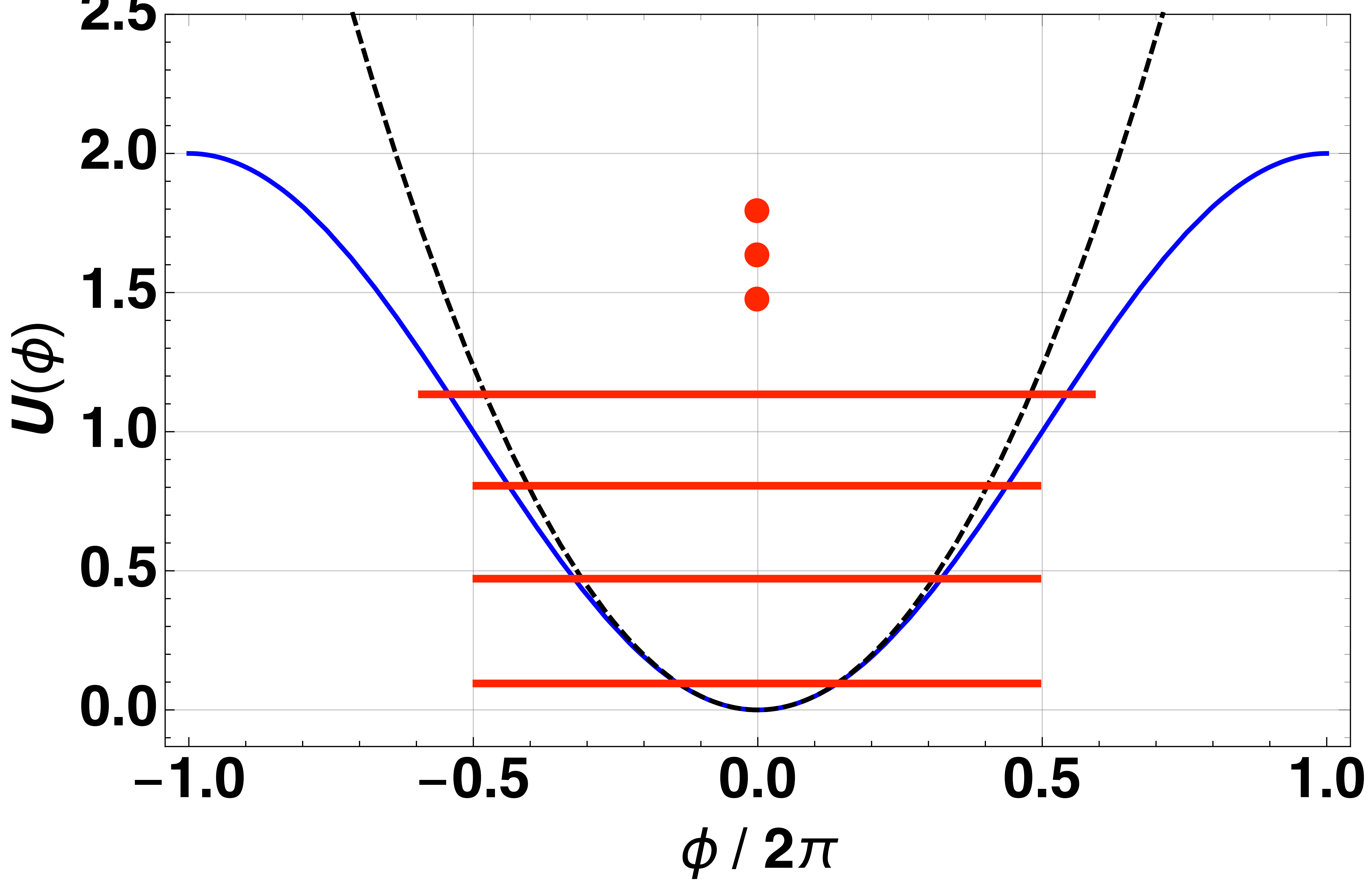
Quantronium / Transmon Qubit

(Unbiased) Anharmonic Oscillator

Cottet (PhD, 2002)
Vion *et al.* (Science, 2002)



$$\frac{E_J}{E_C} \simeq 1 - 100$$

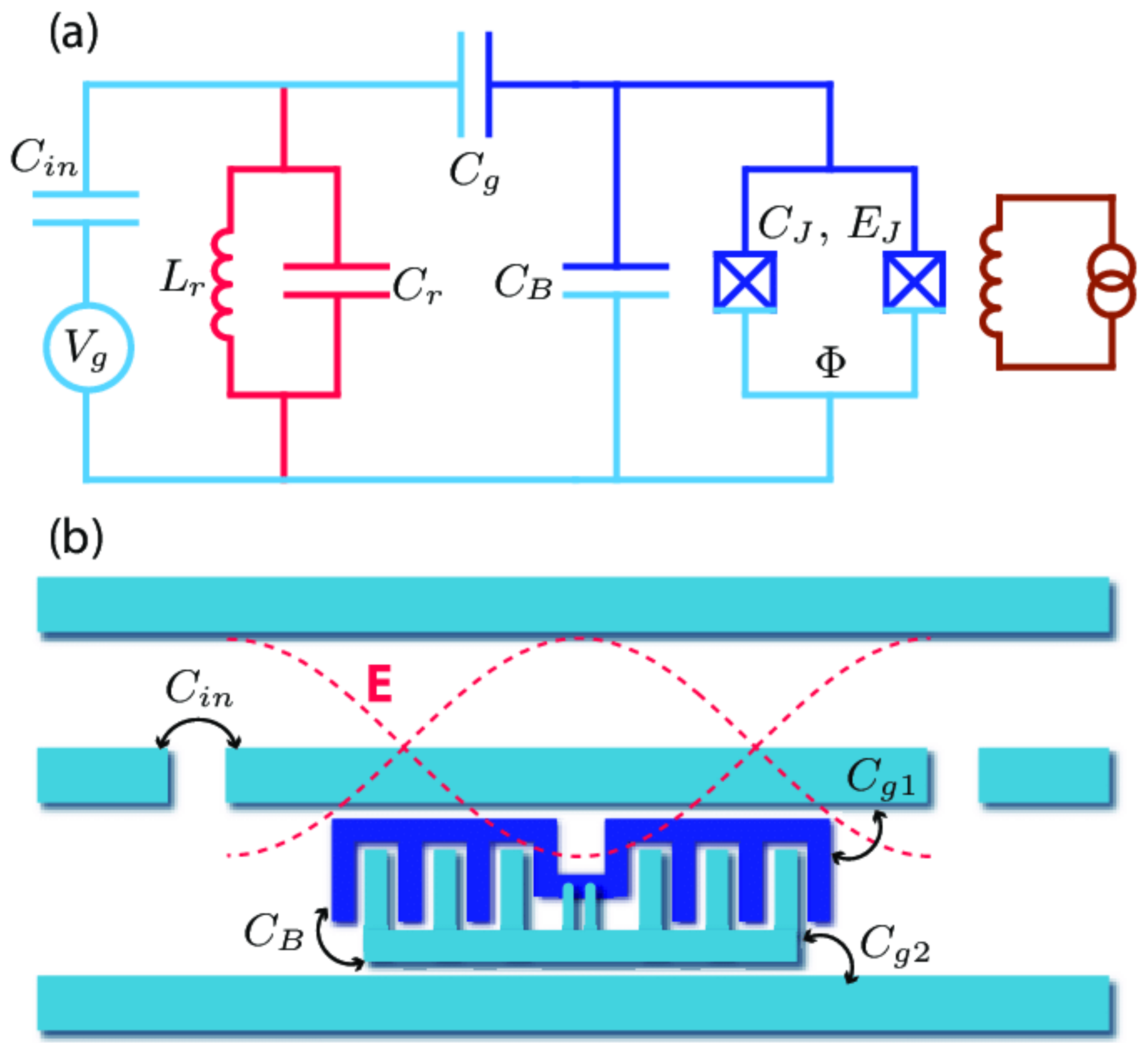


Transmon Qubit

Anharmonic Oscillator

$$\frac{E_J}{E_C} \simeq 100$$

Houck et al. (Nature, 2007)
Koch *et al.* (PRA, 2002)



Still significant through large gate capacitance

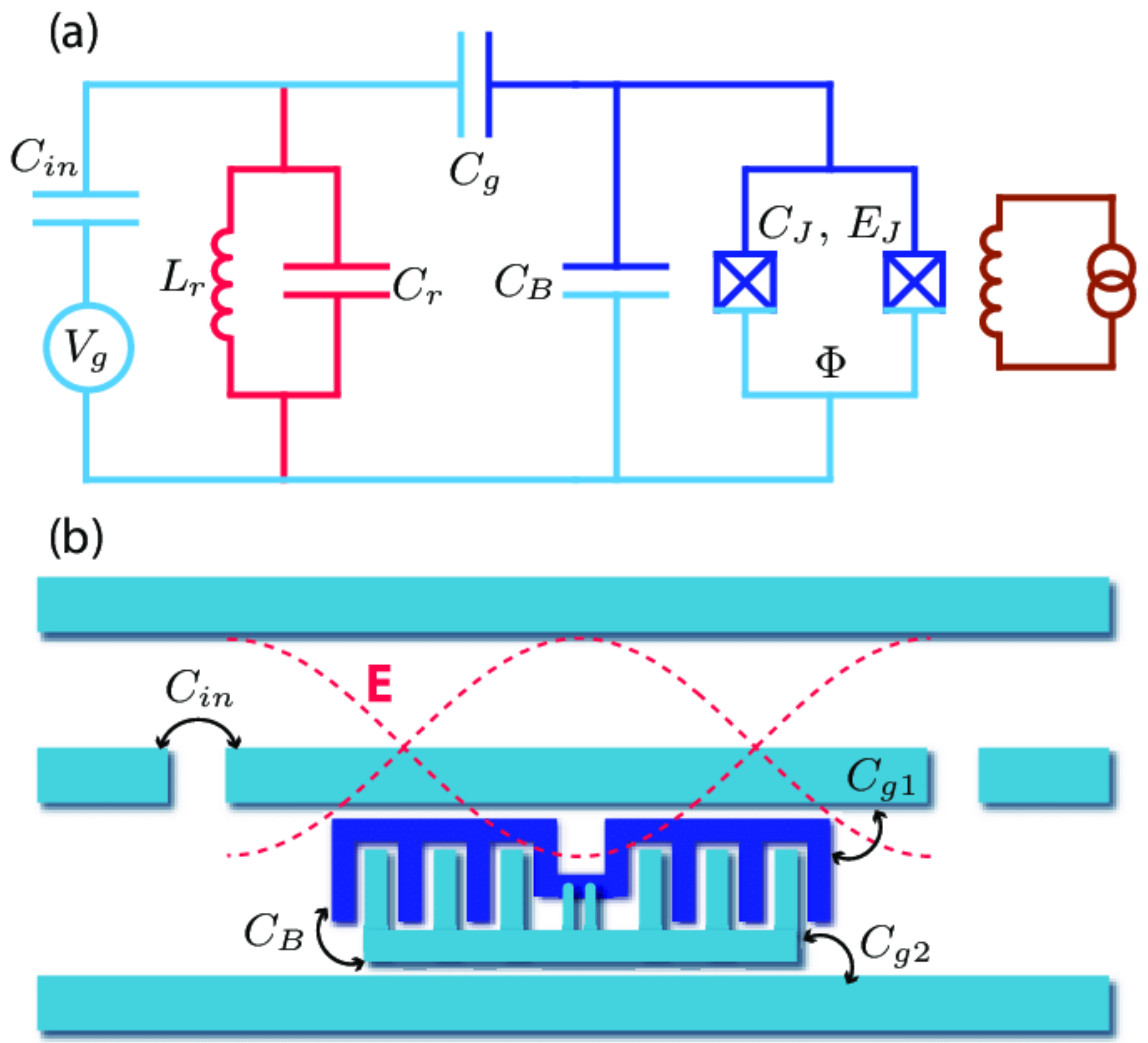
$$H = \underbrace{E_C(n - n_g)^2}_{\text{perturbation}} - E_J \cos\phi$$

Transmon Qubit

Anharmonic Oscillator

$$\frac{E_J}{E_C} \simeq 100$$

Houck et al. (Nature, 2007)
Koch *et al.* (PRA, 2002)



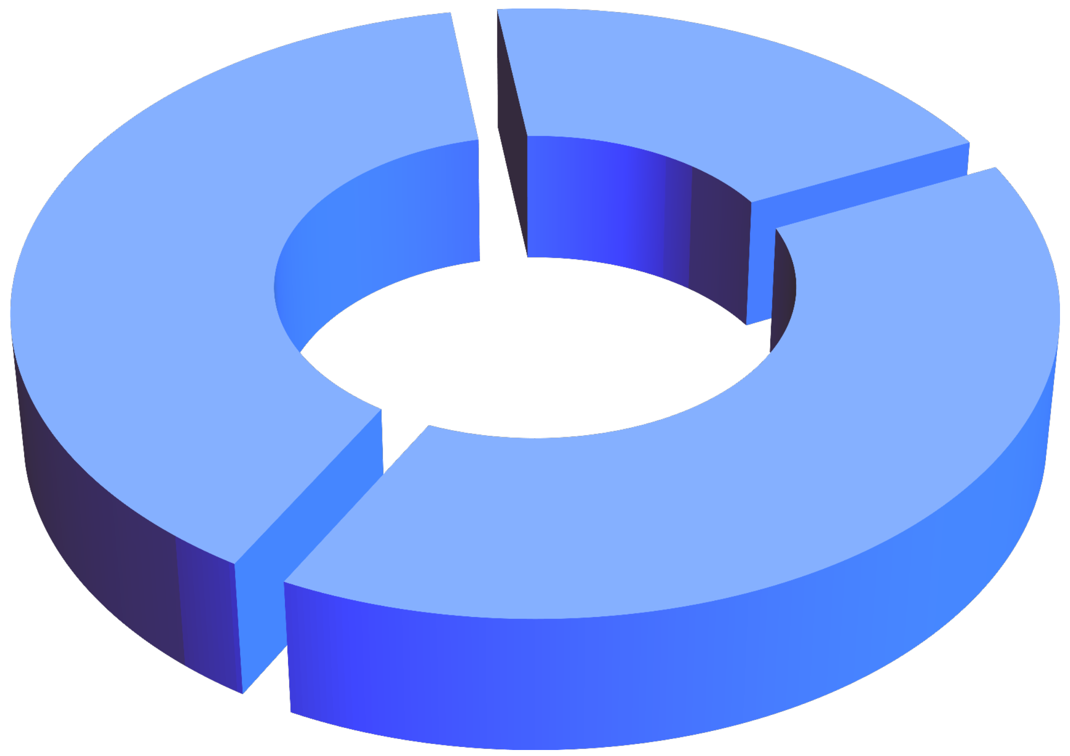
$$H_{\text{qubit}} = \frac{1}{2}\Omega\sigma^z$$

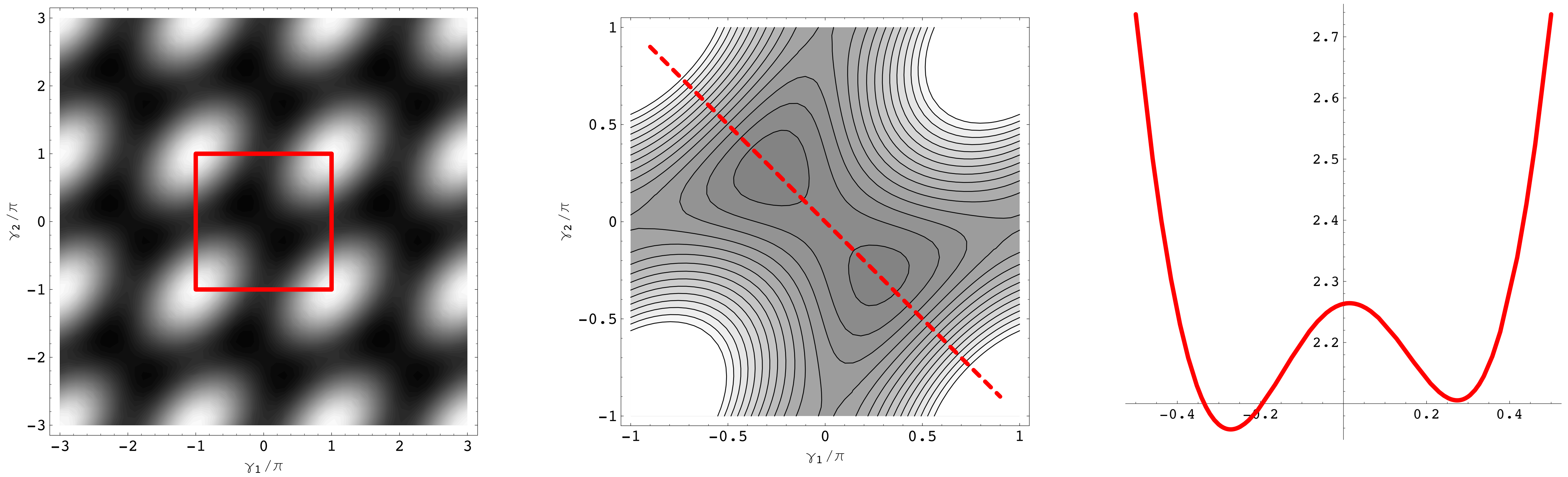
$$\Omega/2\pi \sim 5\,\text{GHz}$$

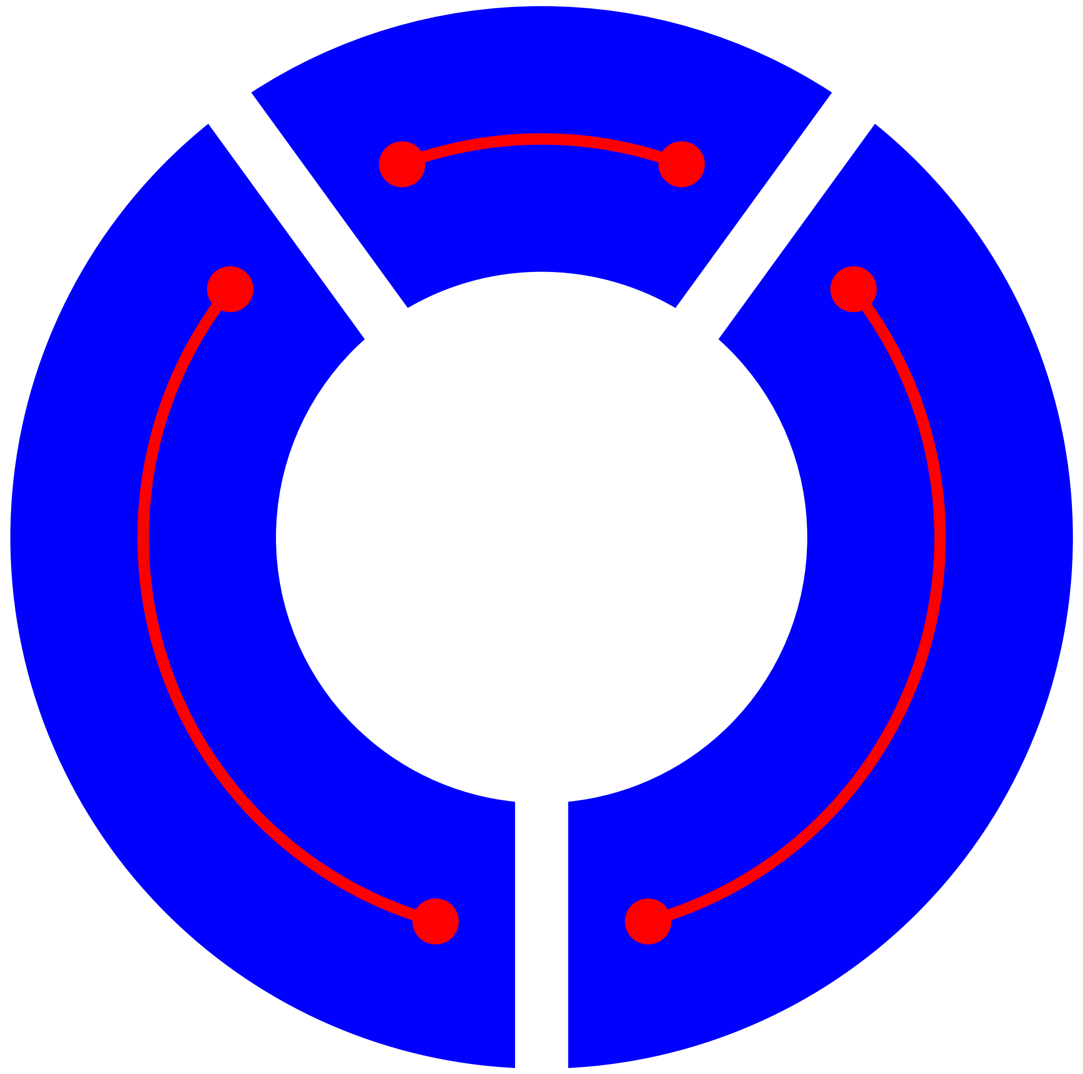
Superconducting Flux Qubits

Tri-Junction Flux Qubit

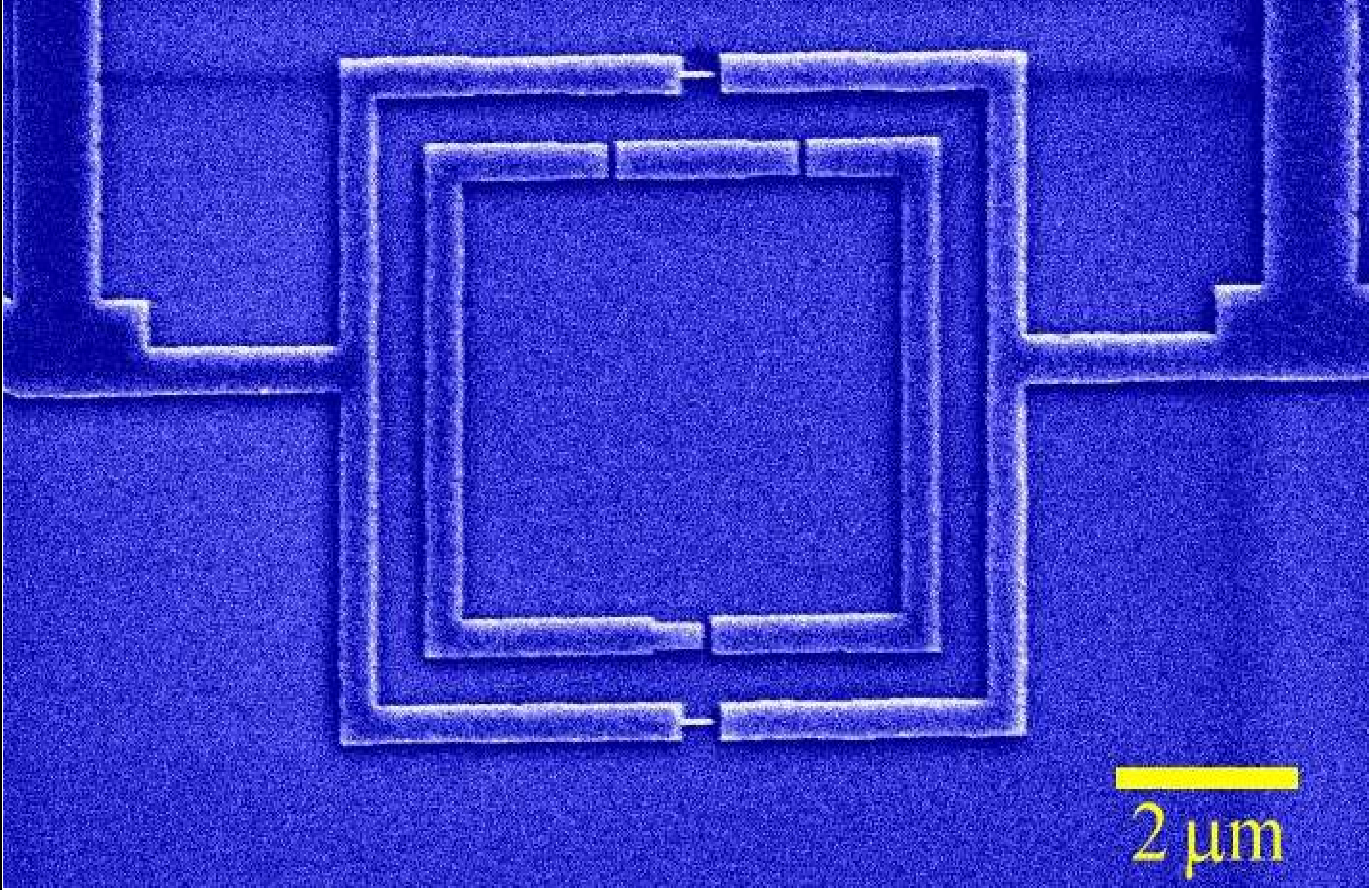
van der Wal et al., Science (2000)







$$\sum_j \varphi_j + 2\pi\Phi/\Phi_0 = 0$$


2 μm

$$\text{Lateral size of the qubit} \sim 1\mu\text{m}$$

$$I_p \approx 484 \pm 2\,\text{nA}\,, \quad \Phi_p \approx 10^{-3}\Phi_0$$

$$\Delta E \approx 0.33 \pm 0.03\,\text{GHz}$$

$$\tau_{\text{sw}} \sim 10 \sim 100\,\text{ns}\,, \quad \tau_\phi \sim 1\,\text{ms}$$

Summary

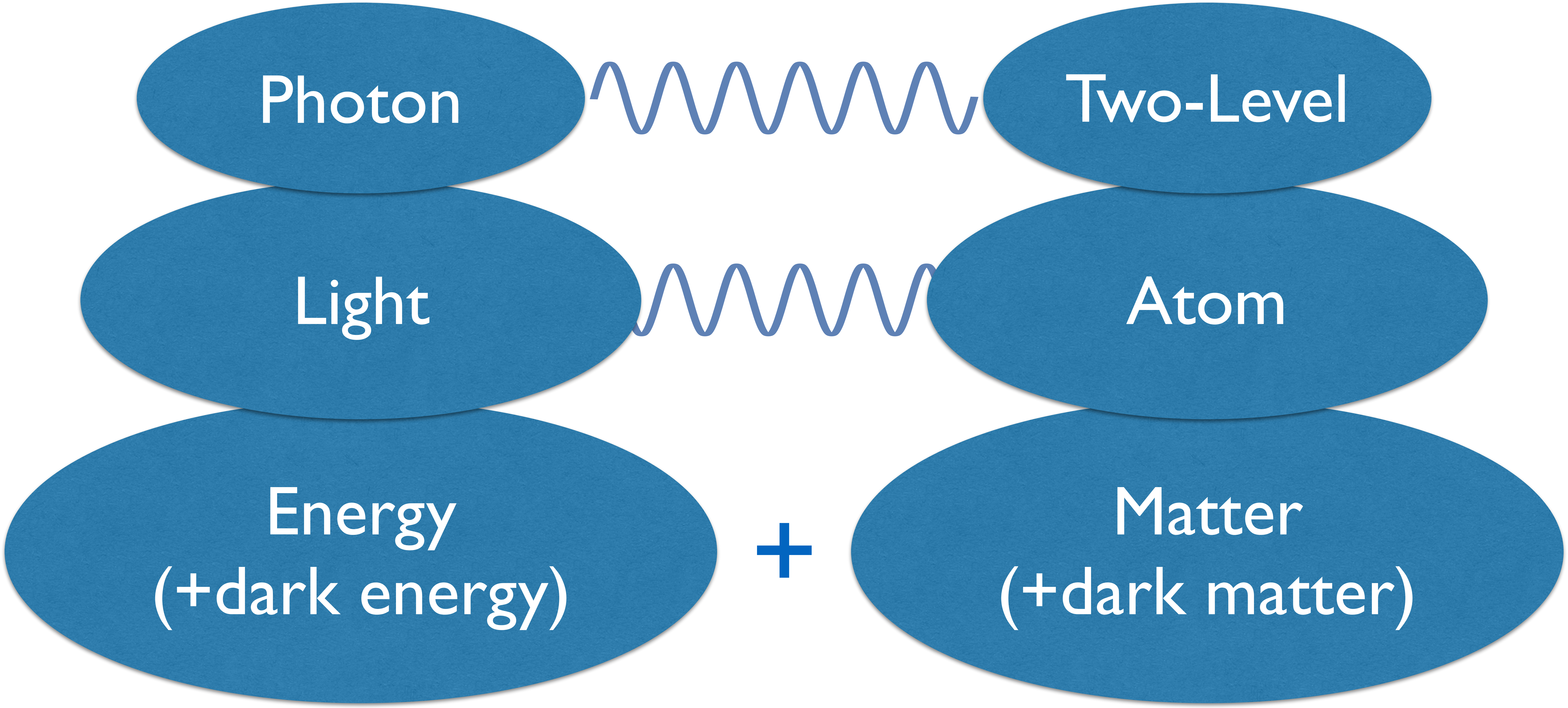
Superconducting Qubits

Summary

1. Superconductivity
2. Macroscopic Quantization
3. Superconducting Qubits

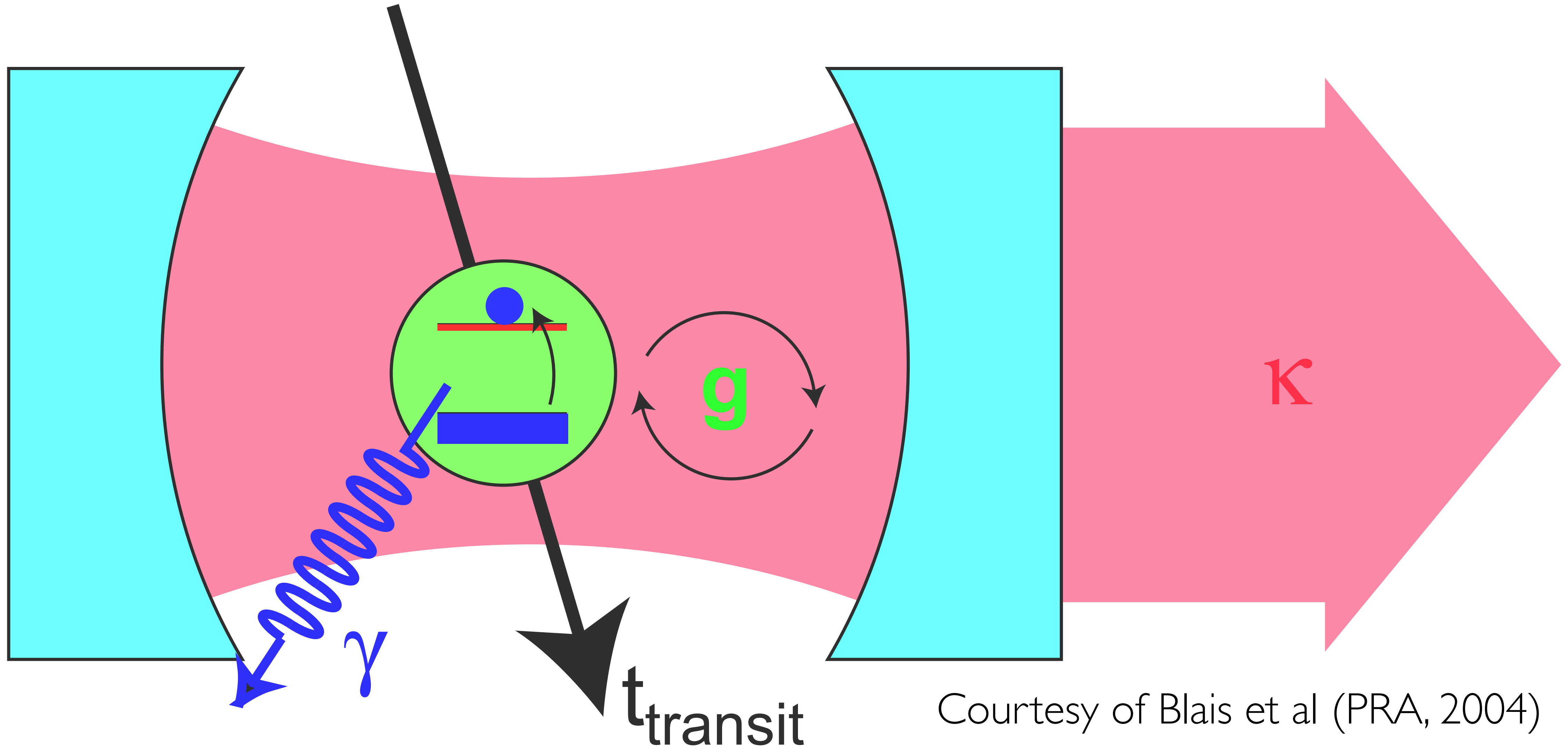
In the next tutorial …

Light-Matter Interaction



Cavity QED

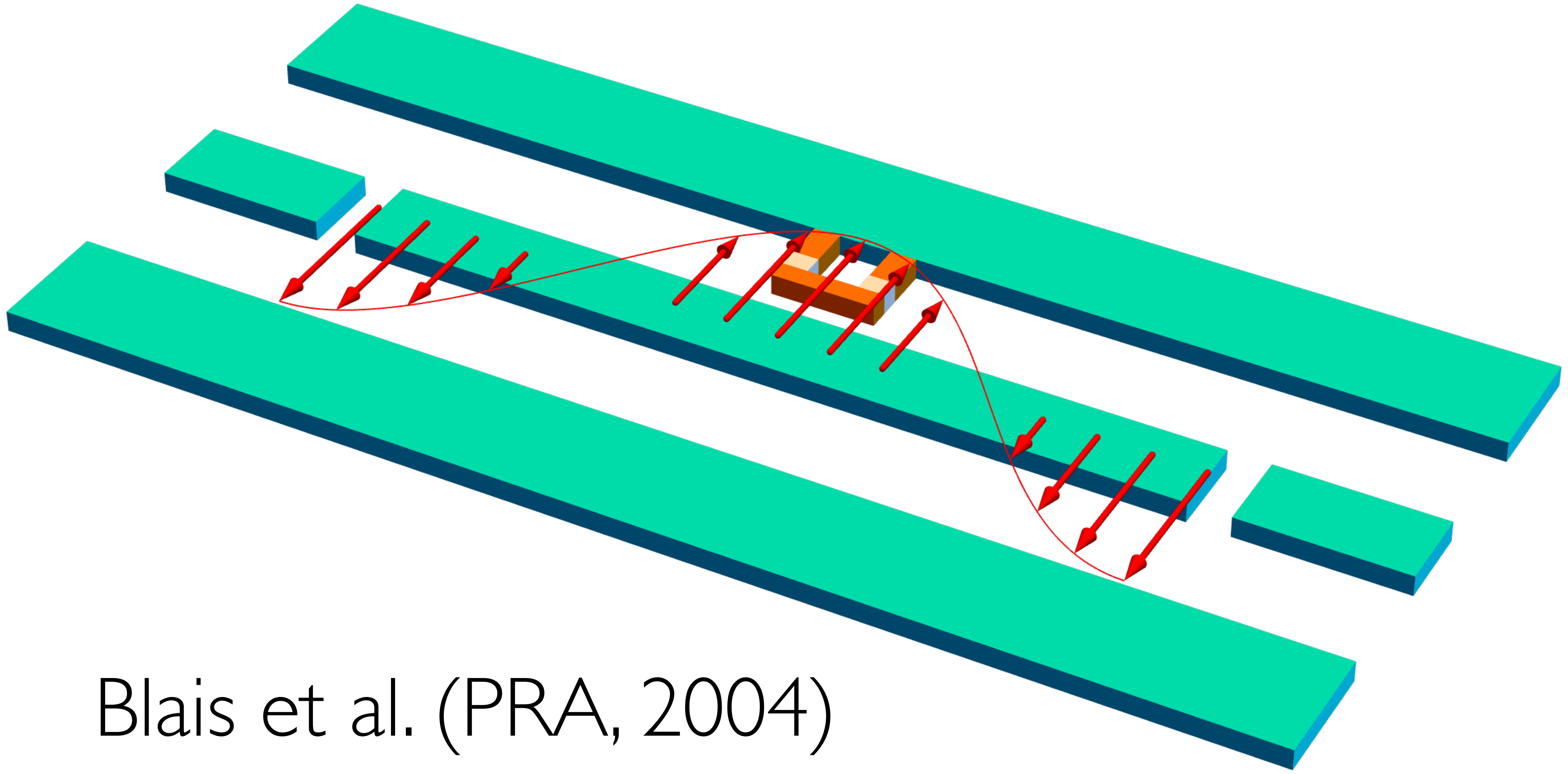
- Simple yet highly non-trivial.
- All essential features of light-matter interaction.



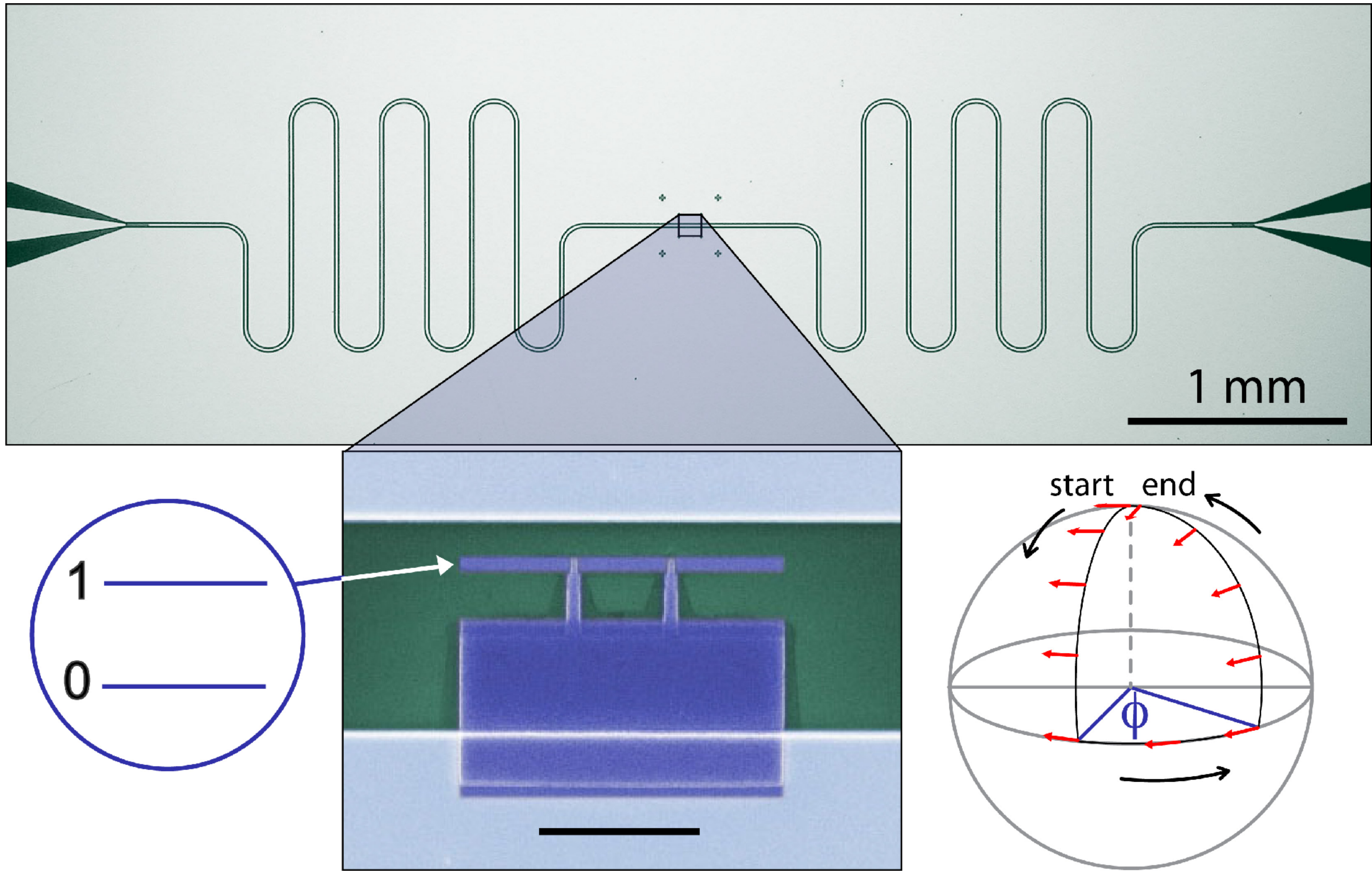
Courtesy of Blais et al (PRA, 2004)

$$H = \omega a^\dagger a + g(a + a^\dagger)\sigma^x + \frac{1}{2}\Omega\sigma^z$$

Circuit QED System



Blais et al. (PRA, 2004)



Wallraff et al. (Nature, 2004)