

Q3 SYMBOL

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Q3`

AmplitudeEmbedding

NEW IN 13.3

`AmplitudeEmbedding[{x1, x2, ..., x2n}, {s1, s2, ..., sn}]`

returns a quantum state on qubits $\{s_1, s_2, \dots, s_n\}$, the amplitudes of which encode classical input data $\{x_1, x_2, \dots, x_{2^n}\}$.

▼ Details and Options

- The *amplitude embedding* is mapping, $\{x_1, x_2, \dots, x_{2^n}\} \mapsto \sum_{k=1}^{2^n} |k-1\rangle x_k$, where $|a\rangle := |a_1\rangle \otimes |a_2\rangle \otimes \dots \otimes |a_n\rangle$ and $a := (a_1 a_2 \dots a_n)_2$ is the binary-digit representation of integer a .

▼ Examples (1)

`In[1]:= Needs["Q3`"]`

▼ Basic Examples (1)

`In[1]:= Let[Qubit, S]`

We consider a quantum register of n qubits.

```
In[2]:= $n = 4;
kk = Range[$n];
SS = S[kk, $]
```

```
Out[2]= {S1, S2, S3, S4}
```

Suppose that we have a classical input data.

```
In[3]:= Let[Real, x]
xx = x[Range@Power[2, $n]]
Out[3]= {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16}
```

The amplitude embedding constructs the following quantum state.

```
In[4]:= AmplitudeEmbedding[xx, SS]
Out[4]= |0s1 0s2 0s3 0s4⟩ x1 + |0s1 0s2 0s3 1s4⟩ x2 + |0s1 0s2 1s3 0s4⟩ x3 + |0s1 0s2 1s3 1s4⟩ x4 + |0s1 1s2 0s3 0s4⟩ x5 + |0s1 1s2 0s3 1s4⟩ x6 +
|0s1 1s2 1s3 0s4⟩ x7 + |0s1 1s2 1s3 1s4⟩ x8 + |1s1 0s2 0s3 0s4⟩ x9 + |1s1 0s2 0s3 1s4⟩ x10 + |1s1 0s2 1s3 0s4⟩ x11 +
|1s1 0s2 1s3 1s4⟩ x12 + |1s1 1s2 0s3 0s4⟩ x13 + |1s1 1s2 0s3 1s4⟩ x14 + |1s1 1s2 1s3 0s4⟩ x15 + |1s1 1s2 1s3 1s4⟩ x16
```

On an actual quantum computer, the above quantum state must be achieved through a unitary gate. This is represented by the `AmplitudeEmbedding` function.

```
In[5]:= op = AmplitudeEmbeddingGate[xx, ss]
Out[5]= AmplitudeEmbeddingGate[{x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16}, {s1, s2, s3, s4}]
```

The above unitary gate is decomposed into two uniformly controlled rotations. See [AmplitudeEmbeddingGate](#) and [UniformlyControlledRotation](#).



See Also

- [AmplitudeEmbeddingGate](#)
- [UniformlyControlledRotation](#)
- [BasisEmbedding](#)
- [BasisEmbeddingGate](#)



Tech Notes

- [Multi-Control Unitary Gates](#)
- [Quantum Information Systems with Q3](#)
- [Quick Quantum Computing with Q3](#)
- [Q3: Quick Start](#)



Related Guides

- [Quantum Information Systems](#)
- [Q3](#)

Related Links

- M. Nielsen and I. L. Chuang (2022) , *Quantum Computation and Quantum Information* (Cambridge University Press, 2011).
- Mahn-Soo Choi (2022) , *A Quantum Computation Workbook* (Springer, 2022).

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Q3`

AmplitudeEmbeddingGate

NEW IN 13.3**AmplitudeEmbeddingGate** [{ x_1, x_2, \dots, x_{2^n} }, { s_1, s_2, \dots, s_n }]represents the gate to encode classical input data $\{x_1, x_2, \dots, x_{2^n}\}$ into the amplitudes of a quantum state of qubits s_1, s_2, \dots, s_n .

▼ Details and Options

- The *amplitude embedding* is mapping, $\{x_1, x_2, \dots, x_{2^n}\} \mapsto \sum_{k=1}^{2^n} |k-1\rangle x_k$, where $|a\rangle := |a_1\rangle \otimes |a_2\rangle \otimes \dots \otimes |a_n\rangle$ and $a := (a_1 a_2 \dots a_n)_2$ is the binary-digit representation of integer a .

▼ Examples (1)

In[1]:= Needs["Q3`"]

▼ Basic Examples (1)

*In[1]:= Let[Qubit, S]*Consider a system of n qubits.

```
In[2]:= $n = 3;
$N = Power[2, $n];
kk = Range[$n];
SS = S[kk, $]
```

Out[2]= {S1, S2, S3}

We want to embed a classical input data of the form.

*In[3]:= xx = Normalize@RandomVector[\$N]**Out[3]= {-0.259904 - 0.427261 i, 0.444914 - 0.227347 i, -0.140951 + 0.0979235 i, 0.122408 + 0.459967 i,
-0.116309 + 0.123089 i, -0.020741 + 0.0522983 i, -0.314258 + 0.153372 i, -0.288961 - 0.081422 i}*

We assume that the data is normalized.

*In[4]:= Norm[xx]**Out[4]= 1.*

This is the desired quantum state embedding the classical data above.

```
In[5]:= vec = AmplitudeEmbedding[xx, SS]
Out[5]= (-0.259904 - 0.427261 i) |0_{S_1} 0_{S_2} 0_{S_3}\rangle + (0.444914 - 0.227347 i) |0_{S_1} 0_{S_2} 1_{S_3}\rangle -
(0.140951 - 0.0979235 i) |0_{S_1} 1_{S_2} 0_{S_3}\rangle + (0.122408 + 0.459967 i) |0_{S_1} 1_{S_2} 1_{S_3}\rangle - (0.116309 - 0.123089 i) |1_{S_1} 0_{S_2} 0_{S_3}\rangle -
(0.020741 - 0.0522983 i) |1_{S_1} 0_{S_2} 1_{S_3}\rangle - (0.314258 - 0.153372 i) |1_{S_1} 1_{S_2} 0_{S_3}\rangle - (0.288961 + 0.081422 i) |1_{S_1} 1_{S_2} 1_{S_3}\rangle
```

On a quantum machine, the above quantum state must be achieved through a unitary gate starting from the usual initial state $|0\rangle := |0\rangle^{\otimes n}$.

```
In[6]:= op = AmplitudeEmbeddingGate[xx, SS]
Out[6]= AmplitudeEmbeddingGate[{-0.259904 - 0.427261 i, 0.444914 - 0.227347 i,
-0.140951 + 0.0979235 i, 0.122408 + 0.459967 i, -0.116309 + 0.123089 i,
-0.020741 + 0.0522983 i, -0.314258 + 0.153372 i, -0.288961 - 0.081422 i}, {S1, S2, S3}]
```

In quantum circuit, it is depicted as follows, which consists of two uniformly controlled rotations.

```
In[7]:= qc = QuantumCircuit[op]
```

```
Out[7]=
```

Obtain the output state from the above unitary gate.

```
In[8]:= out = qc ** Ket[]
Out[8]= (-0.468873 - 0.173953 i) |0_{S_1} 0_{S_2} 0_{S_3}\rangle + (0.207993 - 0.454285 i) |0_{S_1} 0_{S_2} 1_{S_3}\rangle -
(0.04983 - 0.164235 i) |0_{S_1} 1_{S_2} 0_{S_3}\rangle + (0.381304 + 0.284887 i) |0_{S_1} 1_{S_2} 1_{S_3}\rangle - (0.0148924 - 0.168692 i) |1_{S_1} 0_{S_2} 0_{S_3}\rangle +
(0.0161672 + 0.0538881 i) |1_{S_1} 0_{S_2} 1_{S_3}\rangle - (0.151385 - 0.31522 i) |1_{S_1} 1_{S_2} 0_{S_3}\rangle - (0.277167 - 0.115353 i) |1_{S_1} 1_{S_2} 1_{S_3}\rangle
```

Apparently, it looks different from the desired state `vec` shown above. However, this is because of a physically irrelevant global phase. Indeed, the fidelity between the two states are unity.

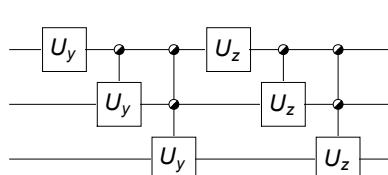
```
In[9]:= Fidelity[out, vec]
Out[9]= 1.
```

As already mentioned, the amplitude embedding gate consists of two uniformly controlled rotations. This is made explicitly using the `Expand` function.

```
In[10]:= 
Expand[op]
Out[10]= Sequence[Rotation[1.03387, {0, 1, 0}, S1],
UniformlyControlledRotation[{S1}, {1.24243, 2.40276}, {0, 1, 0}, S2],
UniformlyControlledRotation[{S1, S2}, {1.56986, 2.44945, 0.641499, 1.41884}, {0, 1, 0}, S3],
Rotation[0.710362, {0, 0, 1}, S1], UniformlyControlledRotation[{S1}, {3.21741, -2.22781}, {0, 0, 1}, S2],
UniformlyControlledRotation[{S1, S2}, {1.6449, -1.22371, -0.379524, -5.55449}, {0, 0, 1}, S3]]
```

In[11]:= qc = QuantumCircuit[Expand@op]

Out[11]=

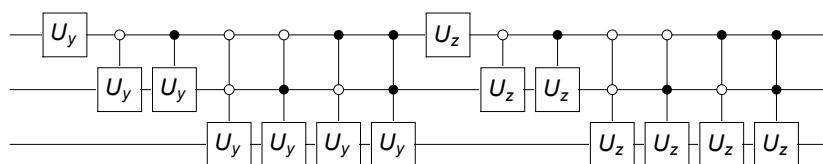


The uniformly controlled rotations themselves may be decomposed further into products of more elementary gates as follows (notice the ExpandAll command).

In[12]:=

qc = QuantumCircuit[ExpandAll@op]

Out[12]=



Check again the above statement is true by examining the output state.

In[13]:=

more = qc ** Ket[SS]

Out[13]=

$$(-0.468873 - 0.173953 i) |0_{S_1} 0_{S_2} 0_{S_3}\rangle + (0.207993 - 0.454285 i) |0_{S_1} 0_{S_2} 1_{S_3}\rangle - \\ (0.04983 - 0.164235 i) |0_{S_1} 1_{S_2} 0_{S_3}\rangle + (0.381304 + 0.284887 i) |0_{S_1} 1_{S_2} 1_{S_3}\rangle - (0.0148924 - 0.168692 i) |1_{S_1} 0_{S_2} 0_{S_3}\rangle + \\ (0.0161672 + 0.0538881 i) |1_{S_1} 0_{S_2} 1_{S_3}\rangle - (0.151385 - 0.31522 i) |1_{S_1} 1_{S_2} 0_{S_3}\rangle - (0.277167 - 0.115353 i) |1_{S_1} 1_{S_2} 1_{S_3}\rangle$$

In[14]:=

Fidelity[more, vec]

Out[14]=

1.

Note that the above quantum circuit may be simplified even further. See the examples in [UniformlyControlledRotation](#).



See Also

[AmplitudeEmbedding](#) ▪ [UniformlyControlledRotation](#) ▪ [BasisEmbedding](#) ▪ [BasisEmbeddingGate](#)



Tech Notes

- [Multi-Control Unitary Gates](#)
- [Quantum Information Systems with Q3](#)
- [Quick Quantum Computing with Q3](#)
- [Q3: Quick Start](#)



Related Guides

- Quantum Information Systems
- Q3

Related Links

- M. Möttönen *et al.*, Quantum Information and Computation 5, 467 (2005), "Transformation of quantum states using uniformly controlled rotations."
- M. Nielsen and I. L. Chuang (2022), Quantum Computation and Quantum Information (Cambridge University Press, 2011).
- Mahn-Soo Choi (2022), A Quantum Computation Workbook (Springer, 2022).

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UniformlyControlledRotation

NEW IN 13.3**UniformlyControlledRotation** [{ c_1, c_2, \dots, c_n }, { $\theta_1, \theta_2, \dots, \theta_{2^n}$ }, { x, y, z }, s]represents the uniformly controlled rotation on qubit s around axis { x, y, z } by angles $\theta_1, \theta_2, \dots, \theta_{2^n}$ depending on all possible bit sequences of control qubits c_1, c_2, \dots, c_n .**UniformlyControlledRotation** [{ c_1, c_2, \dots, c_n }, { $\theta_1, \theta_2, \dots, \theta_n$ }, $s[\dots, k]$]uses the k -axis as the rotation axis.

▼ Details and Options

- In general, **UniformlyControlledRotation** [{ c_1, c_2, \dots, c_n }, { $\theta_1, \theta_2, \dots, \theta_{2^n}$ }, { x, y, z }, s] is a product of 2^n two-level matrices.
- Note also that **UniformlyControlledRotation** [{ c_1, c_2, \dots, c_n }, { $\theta_1, \theta_2, \dots, \theta_{2^n}$ }, { x, y, z }, s] equals to $R_v(\theta_1) \oplus R_v(\theta_2) \oplus \dots \oplus R_v(\theta_{2^n})$, where $v \equiv \{x, y, z\}$, and hence is block diagonal.
- An arbitrary multi-qubit unitary matrix U can be decomposed into a product of uniformly controlled rotations; see Möttönen et al. (2004).

▼ Examples (4)

In[1]:= Needs["Q3`"]

▼ Basic Examples (2)

In[1]:= Let[Qubit, S, T]

In[2]:= \$n = 3;
\$N = Power[2, \$n];
kk = Range[\$n];
SS = S[kk, \$]

Out[2]= {S1, S2, S3}

Define a series of rotations on qubit T[1,\$].

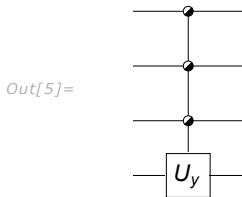
In[3]:= aa = RandomReal[{0, 1}, \$N] * Pi

Out[3]= {1.81603, 3.04719, 2.86853, 0.160244, 1.0859, 1.78237, 2.22677, 0.516285}

In[4]:= op = UniformlyControlledRotation[SS, aa, T[2]]

Out[4]= UniformlyControlledRotation[{S1, S2, S3},
{1.81603, 3.04719, 2.86853, 0.160244, 1.0859, 1.78237, 2.22677, 0.516285}, {0, 1, 0}, T]

In[5]:= qc = QuantumCircuit[op]



Out[5]=

In[6]:= op ** Ket[SS]

$$\text{Out}[6] = 0.615313 \left| 0_{S_1} 0_{S_2} 0_{S_3} 0_T \right\rangle + (0.788283 + 0. i) \left| 0_{S_1} 0_{S_2} 0_{S_3} 1_T \right\rangle$$

In[7]:= op ** S[1, 3] // Chop

$$\begin{aligned} \text{Out}[7] = & -0.137222 - 0.049277 S_1^Z S_2^Z - 0.0737621 S_1^Z S_3^Z - (0. + 0.663108 i) S_1^Z T^Y + \\ & 0.0844846 S_2^Z S_3^Z - (0. + 0.071805 i) S_2^Z T^Y - (0. + 0.0399121 i) S_3^Z T^Y + 0.272719 S_1^Z S_2^Z S_3^Z - \\ & (0. + 0.107305 i) S_1^Z S_2^Z T^Y - (0. + 0.135102 i) S_1^Z S_3^Z T^Y + (0. + 0.0272742 i) S_2^Z S_3^Z T^Y + \\ & (0. + 0.253041 i) S_1^Z S_2^Z S_3^Z T^Y + 0.586071 S_1^Z - 0.068323 S_2^Z + 0.000622466 S_3^Z - (0. + 0.0513669 i) T^Y \end{aligned}$$

In[8]:= Elaborate[op] // Chop

$$\begin{aligned} \text{Out}[8] = & 0.586071 - 0.068323 S_1^Z S_2^Z + 0.000622466 S_1^Z S_3^Z - (0. + 0.0513669 i) S_1^Z T^Y + \\ & 0.272719 S_2^Z S_3^Z - (0. + 0.107305 i) S_2^Z T^Y - (0. + 0.135102 i) S_3^Z T^Y + 0.0844846 S_1^Z S_2^Z S_3^Z - \\ & (0. + 0.071805 i) S_1^Z S_2^Z T^Y - (0. + 0.0399121 i) S_1^Z S_3^Z T^Y + (0. + 0.253041 i) S_2^Z S_3^Z T^Y + \\ & (0. + 0.0272742 i) S_1^Z S_2^Z S_3^Z T^Y - 0.137222 S_1^Z - 0.049277 S_2^Z - 0.0737621 S_3^Z - (0. + 0.663108 i) T^Y \end{aligned}$$

In[9]:= Matrix[op] // Chop // MatrixForm

$$\text{Out}[9]//\text{MatrixForm} = \left(\begin{array}{cccccccccc} 0.615313 & -0.788283 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.788283 & 0.615313 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0471855 & -0.998886 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.998886 & 0.0471855 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.136106 & -0.990694 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.990694 & 0.136106 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.996792 & -0.0800364 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0800364 & 0.996792 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.856189 & -0.516664 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.516664 & 0.856189 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

In[10]:=

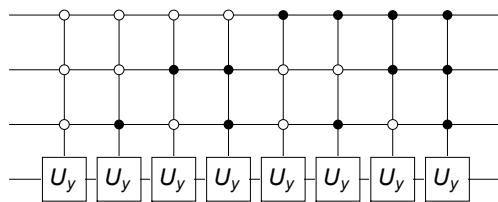
Expand[op]

Out[10]=

```
Sequence[ControlledGate[{S_1, S_2, S_3} → {0, 0, 0}, Rotation[1.81603, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {0, 0, 1}, Rotation[3.04719, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {0, 1, 0}, Rotation[2.86853, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {0, 1, 1}, Rotation[0.160244, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {1, 0, 0}, Rotation[1.0859, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {1, 0, 1}, Rotation[1.78237, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {1, 1, 0}, Rotation[2.22677, {0, 1, 0}, T]], 
ControlledGate[{S_1, S_2, S_3} → {1, 1, 1}, Rotation[0.516285, {0, 1, 0}, T]]]
```

In[11]:= qc = QuantumCircuit[Expand@op]

Out[11]=



For more examples, see the Scope section below.

▼ Scope (2)

▼ Dagger (1)

In[1]:= Let[Qubit, S, T]

In[2]:= \$n = 3;
\$N = Power[2, \$n];
kk = Range[\$n];
SS = S[kk, \$]

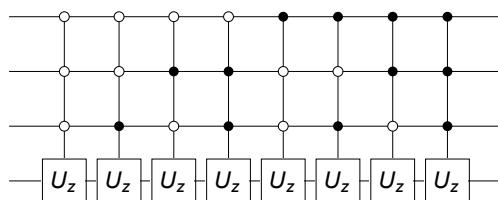
Out[2]= {S₁, S₂, S₃}

In[3]:= aa = RandomReal[{0, 1}, \$N]*Pi

Out[3]= {2.36348, 2.92257, 1.13025, 2.12854, 1.40961, 1.17361, 2.45454, 1.58282}

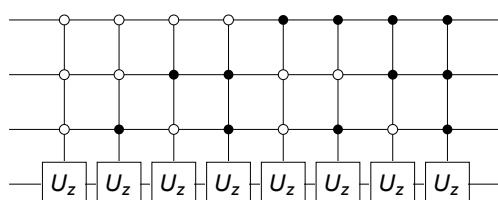
In[4]:= op = UniformlyControlledRotation[SS, aa, T[3]];
qc = QuantumCircuit[Expand@op]

Out[4]=



In[5]:= new = QuantumCircuit[Expand@Dagger@op]

Out[5]=



In[6]:= `Matrix[qc].Matrix[new] // Chop // MatrixForm`

Out[6]//MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. \end{pmatrix}$$

▼ Simplification (1)

As long as the rotation axis is *not* parallel to the x-axis, uniformly controlled rotation may be reduced to a simpler and more efficient gate sequence.

In[1]:= `Let[Qubit, S, T]`

In[2]:= `$n = 3;`
`$N = Power[2, $n];`
`kk = Range[$n];`
`SS = S[kk, $]`

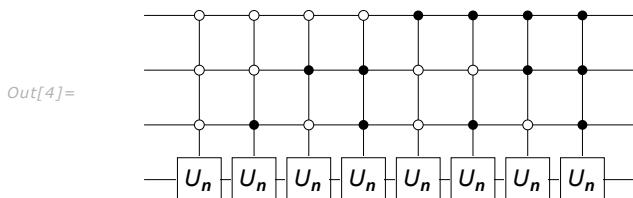
Out[2]= {S₁, S₂, S₃}

In[3]:= `aa = RandomReal[{0, 2}, $N] * Pi`

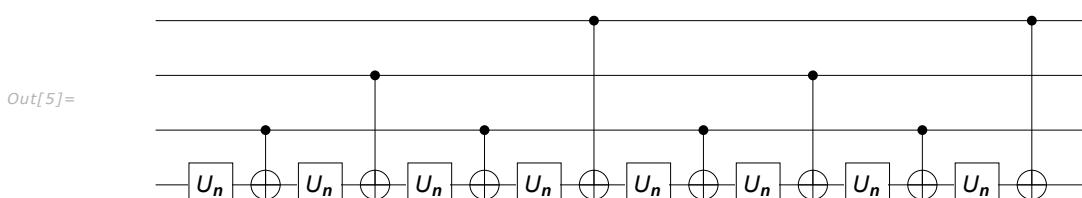
Out[3]= {5.77497, 0.816812, 3.64532, 1.99685, 1.32353, 5.2546, 5.48638, 3.91312}

Note that the rotation axis is not parallel to the x-axis.

In[4]:= `op = UniformlyControlledRotation[SS, aa, {0, 1, 1}, T];`
`qc = QuantumCircuit[Expand@op]`



In[5]:= `new = QuantumCircuit[GateFactor@op]`



In[6]:= `Matrix[new] - Matrix[qc] // Chop // MatrixForm`

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

See Also



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Related Links

- M. Möttönen *et al.*, Physical Review Letters 93, 130502 (2004), "Quantum Circuits for General Multiqubit Gates."
- M. Nielsen and I. L. Chuang (2022) , Quantum Computation and Quantum Information (Cambridge University Press, 2011).
- Mahn-Soo Choi (2022) , A Quantum Computation Workbook (Springer, 2022).