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Double quantum dot between two superconducting leads

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Abstract

A double quantum dot coupled to two superconducting leads is studied in the Coulomb blockade regime. The double dot mediates an effective Josephson coupling between the two superconducting leads, which in turn induce an exchange coupling between the spin states of the dots. The Josephson current depends on the spins on the dots, while the spin exchange coupling can be tuned by the superconducting phase difference. This spin-dependent Josephson current can be used to probe directly the correlated spin states (singlet or triplets).

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In recent years, the studies of electronic transport through strongly interacting mesoscopic systems [1] have been extended to an Anderson impurity or a quantum dot coupled to superconductors [2–6]. In particular, an effective dc Josephson effect through strongly interacting regions between superconducting leads has been analyzed [2,3,7–12]. More recently, on the other hand, it has been found that the direct coupling of two quantum dots by a tunnel junction can be used to create entanglement between spins [13], and that such spin correlations can be observed in charge transport experiments [14].

We propose a new scenario for inducing and detecting spin correlations: we study a double quantum dot (DD) each dot of which is tunnel-coupled to superconducting leads (but not directly with each other), see Fig. 1. In the Coulomb blockade regime, a spin-dependent Josephson coupling between two superconductors is induced, as well as an antiferromagnetic Heisenberg exchange coupling between the spins on the double dot which can be tuned by the superconducting phase difference. We show that correlated spin states – singlet or triplets – on the double dot can be probed via the Josephson current in a dc-SQUID setup.

The two electrodes are assumed to be conventional singlet superconductors which are characterized by the gap parameters $\Delta e^{-i\phi_L}$ and $\Delta e^{+i\phi_R}$. We consider two quantum dots, each of which has odd number of electrons. Taking into account the Coulomb interaction on each dot, each dot can be considered effectively as a single-level Anderson impurity with single-particle energy level $-\epsilon$ ($\epsilon > 0$) and on-site interaction U. (Within the self-capacitance model, $U \sim E_C = e^2/2C$ with C being the effective self-capacitance of each dot.) The tunnel coupling between each dot and the superconducting leads is specified by the level width $\Gamma = \pi t^2 N(0)$ where t is the tunneling amplitude and $N(0)$ is the normal-state density of states per

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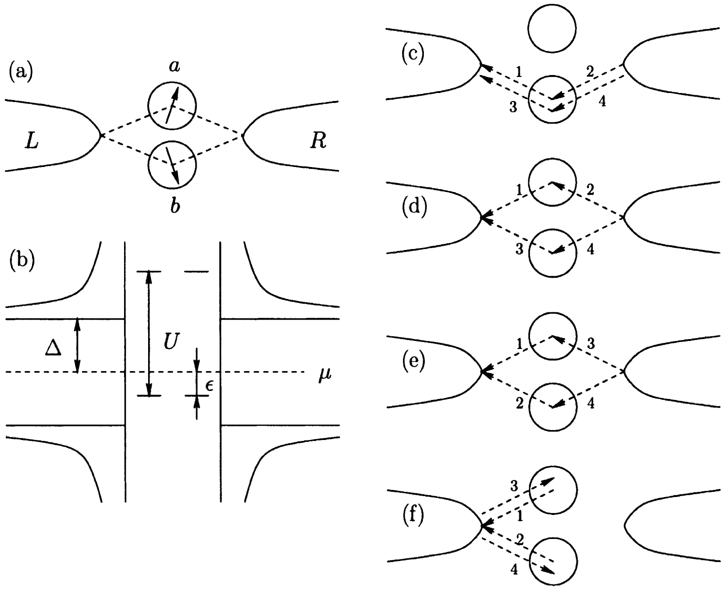

Fig. 1. Sketch of (a) the superconductor–double quantum dot–superconductor (S–DD–S) nanostructure and (b) the quasiparticle energy spectrum in the superconductors and the single-electron levels of the two quantum dots. (c)–(f): Partial listing of virtual tunneling processes contributing to $H_{\rm eff}$. The numbered arrows indicate the direction and the order of occurrence of the charge transfers.

spin of the leads at the Fermi energy. In the strong Coulomb blockade regime ($\Gamma \ll \epsilon, \Delta \ll U - \epsilon$), the whole system can be described by the effective Hamiltonian (to lowest order in Γ)

$$H_{\rm eff} = PH_T\left[(E_0 - H_0)^{-1}(1 - P)H_T\right]^3 P. \qquad (1)$$

Here P is the projection onto the reduced Hilbert space consisting of singly occupied levels of the dots and the BCS ground states on the leads, which are well separated from higher energy levels by Δ and U.

In general, there can be many virtual processes contributing to the effective Hamiltonian (1). However, in the region of our consideration ($\Gamma \ll \epsilon$, $\Delta \ll U - \epsilon$), contributions from virtual processes including doubly occupied levels are negligibly small (of order Γ^2/U). Significant are the processes including only singly occupied or empty levels in its virtual states. A partial list of such virtual processes are depicted in Fig. 1(c)–(f).

In the regime $\epsilon \ll \Delta$ (for other limits, see Ref. [15]), such processes as in Fig. 1(e) and (f) dominate over others and the effective Hamiltonian can be approximated by a single term,

$$H_{\rm eff} \approx J(1 + \cos\varphi)\left[\mathbf{S}_{\rm a}\cdot\mathbf{S}_{\rm b} - \tfrac{1}{4}\right] \quad (\varphi = \phi_{\rm L} - \phi_{\rm R}) \qquad (2)$$

with $J \approx 2\Gamma^2/\epsilon$. A remarkable feature of Eq. (2) is that a Heisenberg exchange coupling between the spins on dot a and b is induced by the superconductors. This coupling is antiferromagnetic ($J > 0$) and thus favors a singlet ground state. Furthermore, the effective Josephson current, $I_{\rm S} \approx -I_J \sin\varphi[\mathbf{S}_{\rm a}\cdot\mathbf{S}_{\rm b} - (1/4)]$ ($I_J = 2eJ/\hbar$), between the superconducting electrodes depends on the spin states of the double dot. We estimate that I_J is of order pA; in the single-dot case [2,3], where the processes of type Fig. 1(c) dominate, $I_J \sim e\Gamma^2/\hbar\Delta$ is usually too small for an experimental detection.

The result in Eq. (2) allow us to probe directly the correlated spin states on the double dot by measuring the Josephson current. But it would be useful if one could prepare the system in states other than the singlet ground state and compare their different behaviors in a single setup. We propose a dc-SQUID-like structure Fig. 2(a). One arm of the SQUID ring contains our double-dot

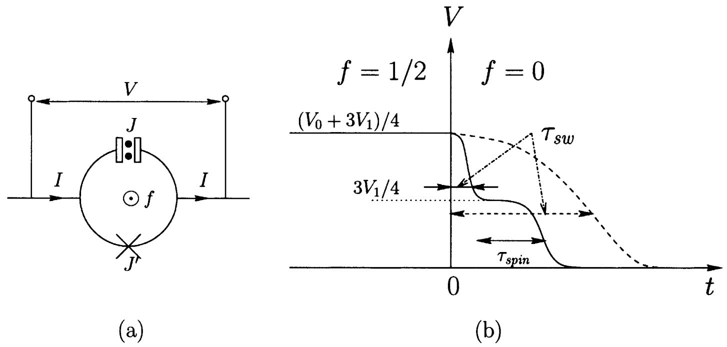

Fig. 2. (a) dc-SQUID-like geometry consisting of the S–DD–S structure (filled dots at the top) connected in parallel with another ordinary Josephson junction (cross at the bottom). (b) Schematic representation of dc voltage V vs. time when probing the spin correlations of the DD. The flux through the SQUID loop is switched from $f = 1/2$ to $f = 0$ at $t = 0$. (—) $\tau_{sw} < \tau_{spin}$, (- - -) $\tau_{sw} > \tau_{spin}$.

system (S–DD–S) and the other a usual Josephson junction (with $J' = \alpha J$). When the double dot is in the triplet state, no Josephson current can flow through S–DD–S; the critical current across the SQUID is entirely determined by the junction J' and given by $I_c = \alpha I_J$ regardless of the flux $\Phi = f\Phi_0$ ($\Phi_0 = hc/2e$) through the SQUID loop. In the singlet state, on the other hand, the critical current depends on the flux and, e.g., $I_c = (\alpha - 1)I_J$ for $f = 1/2$ and $I_c = (\alpha + 1)I_J$ for $f = 0$. Moreover, the singlet and triplets are degenerate for $f = 1/2$ while the singlet state is favored for $f = 0$. Therefore, applying a dc bias current $\alpha I_J < I < (\alpha + 1)I_J$ across the SQUID and switching the flux from $f = 1/2$ to $f = 0$ at time $t = 0$, one can observe a double-step structure in the profile of the voltage across the SQUID loop as illustrated in Fig. 2(b). In real experimental situation, it takes finite switching time τ_{sw} to turn off the flux. In order to see the effects described above, it is required that τ_{sw} should be sufficiently small compared with the spin relaxation time τ_{spin} on the double dot.

To our knowledge, there are no experimental reports on quantum dots coupled to superconductors. However, hybrid systems consisting of superconductors (e.g., Al or Nb) and 2DES (InAs) have been investigated by a number of group [16–18].

In conclusion, we have investigated a double quantum dot each dot of which is coupled to two superconductor leads. The superconductors induce an effective exchange coupling between spins on the double dot whereas the Josephson current from one lead to the other depends on the spin state. This leads to the possibility to probe the spin states of the dot electrons by measuring a Josephson current.

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