Kondo effect of quantum dots in the quantum Hall regime

Mahn-Soo Choi, N. Y. Hwang, and S.-R. Eric Yang
Department of Physics, Korea University, 1 5-ka Anam-dong, Seoul 136-701, Korea

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We report on the Kondo effect of quantum dots involving the precursor of the Landau-level filling factor $\nu=1$ state in the quantum Hall regime. We argue that pairs of degenerate single-Slater determinant states may give rise to a Kondo effect, which can be mapped into an ordinary Kondo effect in a fictitious magnetic field. We report on several properties of this Kondo effect using scaling and numerical renormalization group analysis. We suggest an experiment to investigate this Kondo effect.

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The Kondo effect, one of the most extensively studied phenomena in condensed-matter physics,[1] has recently enjoyed a revival in mesoscopic systems. Examples include quantum dots,[2–6] quantum point contacts,[7] and carbon nanotube coupled to superconductors.[8] The main attraction of the Kondo effect in such systems is its tunability, which makes it possible to test various aspects of the Kondo effect that cannot be directly investigated in bulk solids. For example, a scattering phase shift at the Kondo resonance in a quantum dot has been measured using a two-path interferometer.[6] In mesoscopic systems, the role of magnetic impurities embedded in bulk solids is played by nanoscale quantum dots or carbon nanotube (as a whole) coupled to electrodes. The Kondo effect arises essentially from the spin-degenerate energy levels associated with a single unpaired electron in the artificial atom, and is accompanied by the Kondo resonance at the Fermi level. In quantum dots, a Kondo effect may arise when an external magnetic field induces energy degeneracy,[9–12] while in bulk systems a magnetic field lifts the spin degeneracy and leads to splitting of the Kondo resonance peak.[13,14]

In this paper, we report on the Kondo effect of quantum dots involving the precursor of the Landau-level filling factor $\nu=1$ state in the quantum Hall regime. An attractive feature of these quantum dots is that the single-Slater-determinant states are exact ground states of the many-body Hamiltonian under certain circumstances.[15–17] The Slater-determinant states, denoted by $|N_\uparrow,N_\downarrow\rangle$, have $N_\uparrow$ spin-up and $N_\downarrow$ spin-down electrons, see Eq. (2). In this work, we focus on the parameter region where $|N,0\rangle$ and $|N-1,1\rangle$ become nearly degenerate ground states[15,16] (see Fig. 1). The degeneracy of $|N,0\rangle$ and $|N-1,1\rangle$ results from the interplay of many-body interaction, parabolic confinement energy, and (real) Zeeman energy. These states are stable ground states in wide regions of the parameter space[15,16] and are easily probed in experiments.[18,19] We show that this pair of degenerate many-body ground states gives rise to a Kondo effect that can be mapped into an ordinary Kondo problem with a *fictitious* magnetic field. Also, our investigation suggests that the effects of the fictitious field may be removed by moving off the phase boundary. Our analysis indicates that a similar effect can arise at each phase boundary between $|N_\uparrow,N_\downarrow\rangle$ and $|N_\uparrow-1,N_\downarrow+1\rangle$ ground states. We report on several properties of this Kondo effect using scaling and numerical renormalization group (NRG) analysis. We also suggest an experiment to investigate this Kondo effect.

We first describe briefly our model for a quantum dot in a strong magnetic field. It consists of two-dimensional electrons confined to a finite area that is coupled to two leads. The confining potential is regarded parabolic, $V(x,y)=\frac{1}{2}m^*\Omega^2(x^2+y^2)$, where m^* is the effective electron mass and Ω is the frequency. A magnetic field B is applied perpendicular to the quantum dot, i.e., in the z direction. The field is assumed to be so strong that the Landau-level spacing $\hbar\omega_c$, where $\omega_c=eB/m^*c$, is sufficiently large compared to the confinement energy $\hbar\Omega$ and the electron-electron interaction energy $e^2/\epsilon\ell$ ($\ell\equiv\sqrt{\hbar c/eB}$ and ϵ is the dielectric constant). In this limit, only the lowest Landau level is relevant. In the symmetric gauge, the single-electron orbits of an isolated quantum dot are labeled by the orbital angular momentum m ($m=0,1,2,\dots,$) and spin index $\sigma=\uparrow,\downarrow$. An electron

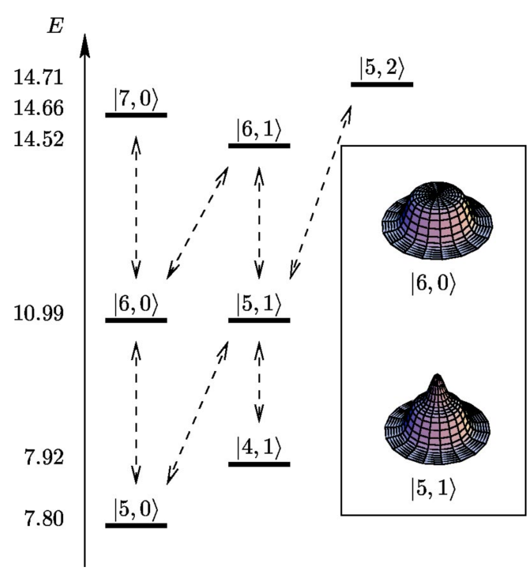

FIG. 1. Energy level structure of single-Slater-determinant states $|N_\uparrow,N_\downarrow\rangle$ for $N=6$. The energies are in units of $e^2/\epsilon\ell$. States $|6,0\rangle$ and $|5,1\rangle$ are degenerate ground states for certain values of the parameters (see the text).[15] The virtual states that are relevant to the Kondo effect are also Slater determinants since ground states are given by Slater determinants. The arrows connect degenerate ground states to Slater determinant states with one more or less total electron number. Inset: Density profiles of the maximum-density-droplet state $|N,0\rangle$ and the state $|N-1,1\rangle$ with one spin flipped.

in the angular momentum state m is located in a ring of the width ℓ and a radius $R_m=\sqrt{2(m+1)}\ell$. The single-particle energy of an orbit with m and σ is given by $\epsilon_{m,\sigma}=\gamma(1+m)+\frac{1}{2}g\mu_B B\sigma$, where $\gamma\equiv\hbar\omega_c(\Omega/\omega_c)^2$ and $g\mu_B B$ is the Zeeman splitting (we ignore the zero-point energy, which does not affect the Kondo effect).

The many-body Hamiltonian of an isolated quantum dot in a strong magnetic field is then given by

$$H_0=\sum_{m,\sigma}\varepsilon_{m,\sigma}d^\dagger_{m,\sigma}d_{m,\sigma}+\sum_{m_1,m_1',m_2,m_2'}\sum_{\sigma_1,\sigma_2}U_{m_1',m_2';m_1,m_2}d^\dagger_{m_2',\sigma_2}d^\dagger_{m_1',\sigma_1}d_{m_1,\sigma_1}d_{m_2,\sigma_2}, \tag{1}$$

where $d^\dagger_{m,\sigma}$ ($d_{m,\sigma}$) creates (annihilates) an electron in the state m,σ, and $U_{m_1',m_2';m_1,m_2}$ are the matrix elements of the electron-electron Coulomb interaction. As mentioned above, in quantum dots in a strong magnetic field, the single-Slater-determinant state of the form

$$|N_\uparrow,N_\downarrow\rangle=d^\dagger_{N_\downarrow-1,\downarrow}\cdots d^\dagger_{0\downarrow}d^\dagger_{N_\uparrow-1,\uparrow}\cdots d^\dagger_{0\uparrow}|0\rangle, \tag{2}$$

so-called *maximum-density-droplet states*, can be an exact eigenstate of the many-body Hamiltonian, Eq. (1), in a wide range of parameters $\tilde{\gamma}\equiv\gamma/(e^2/\epsilon\ell)$ and $\tilde{g}=g\mu_B B/(e^2/\epsilon\ell)$ (see Figs. 1 and 2 in Ref. 15).

In the presence of the coupling to the leads, the many-body states in the quantum dot are hybridized with the conduction bands of the leads. The coupling can be considered within the tunneling model

$$H_T=\sum_{k,m,\sigma}V(d^\dagger_{m,\sigma}c_{k,\sigma}+c^\dagger_{k,\sigma}d_{m,\sigma}), \tag{3}$$

where $c^\dagger_{k,\sigma}$ and $c_{k,\sigma}$ are operators for conduction electrons, and V is the tunneling amplitude (we ignore the m dependence of V for simplicity, see below). We assume that there are N electrons in the quantum dot in equilibrium, and that the Fermi level E_F of the leads lies between the successive electrochemical potentials μ_{N-1} and μ_N of the quantum dot:[20] $\mu_{N-1}<E_F<\mu_N$. Here, note that, the electrochemical potential of the quantum dot includes the contribution from the gate voltage V_g applied to the quantum dot, which is given by $\mu_N\equiv E^0_{N+1}-E^0_N+eV_g$ where E^0_N is the N-electron ground-state energy. Below, we consider the Kondo effect in the limit $\Gamma\ll E_F-\mu_{N-1}$ and $\Gamma\ll\mu_N-E_F$ ($\Gamma\equiv 2\pi\rho_0|V|^2$). We emphasize, however, that the spins involved here are not real spins but pseudospins, corresponding to the degenerate ground states $|N,0\rangle$ and $|N-1,1\rangle$.

Given the setup prescribed above, transport of the conduction electrons through the quantum dot via sequential tunneling is highly suppressed. The hybridization of the dot levels is possible only through the virtual tunneling processes. To simplify the discussion, we will further assume that $\mu_N-E_F\gg E_F-\mu_{N-1}$. The dominant contributions then come from the virtual processes involving the $(N-1)$-electron states $|N-1,0\rangle$ and $|N-2,1\rangle$, while the processes involving the virtual states with $(N+1)$ electrons in the dot can be ignored. [In the NRG study to be discussed below, however, we have taken into account all the virtual processes with $(N-1)$- and $(N+1)$-electron states.] Effectively, the tunneling Hamiltonian takes the form (see also Fig. 1)

$$H_T=\sum_k V(c^\dagger_{k\uparrow}|N-1,0\rangle\langle N,0|+c^\dagger_{k\downarrow}|N-1,0\rangle \times\langle N-1,1|c^\dagger_{k\uparrow}|N-2,1\rangle\langle N-1,1|)+\text{H.c.}. \tag{4}$$

Each term in Eq. (4) leads to one of the virtual processes listed in Fig. 2. Within the spirit of the Schrieffer-Wolf transformation taking into account these virtual processes, one can show that for small Γ and at low temperatures, the impurity model $H=H_D+H_c+H_T$ is equivalent to a Kondo-like model:

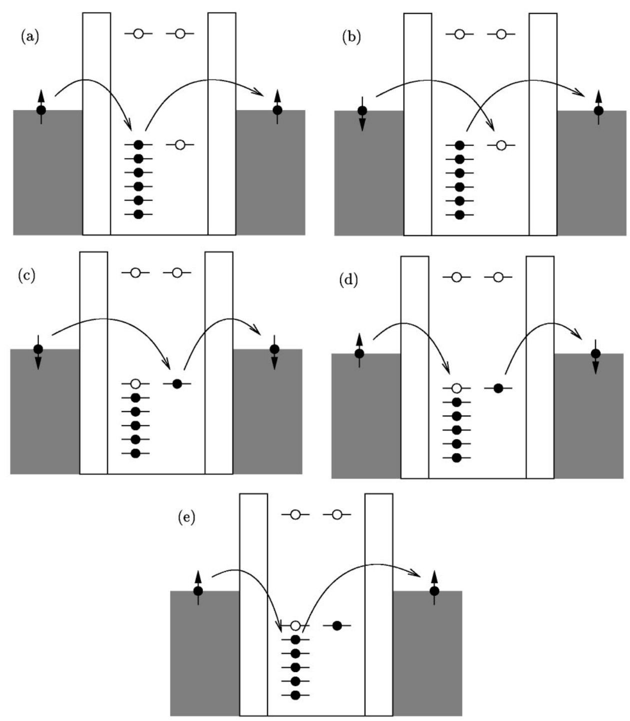

FIG. 2. Virtual processes mediating the transitions (a) from $|N,0\rangle$ to $|N,0\rangle$, (b) from $|N,0\rangle$ to $|N-1,1\rangle$, (c) from $|N-1,1\rangle$ to $|N-1,1\rangle$, (d) from $|N-1,1\rangle$ to $|N,0\rangle$, and (e) from $|N-1,1\rangle$ to $|N-1,1\rangle$. The virtual process (e) does not have a counterpart in the usual Anderson model and gives an extra kinetic energy gain for the electrons in the state $|N-1,1\rangle$.

$$H = -\sum_{k,q} J_{\perp} t (S^{+} c_{q,\uparrow} c^{\dagger}_{k,\downarrow} + S^{-} c_{q,\downarrow} c^{\dagger}_{k,\uparrow}) - \sum_{k,q} S^{z} (J_{\uparrow} c_{q,\uparrow} c^{\dagger}_{k,\uparrow} - J_{\downarrow} c_{q,\downarrow} c^{\dagger}_{k,\downarrow}), \tag{5}$$

where we have adopted the notation $S^{+} = |N,0\rangle\langle N-1,1|$, $S^{-} = |N-1,1\rangle\langle N,0|$, and $S^{z} = (|N,0\rangle\langle N,0| - |N-1,1\rangle\langle N-1,1|)/2$ to emphasize the roles of the states $|\Uparrow\rangle \equiv |N,0\rangle$ and $|\Downarrow\rangle \equiv |N-1,1\rangle$ as pseudospin components. The coupling constants $J_{\perp}$, $J_{\downarrow}$, and $J_{\uparrow}$ in Eq. (5) are given by

$$J_{\perp} = \frac{|V|^2}{E_F + E_{N-1,0} - E_{N,0} - eV_g}, \tag{6}$$

$$J_{\uparrow} = \frac{|V|^2}{E_F + E_{N-1,0} - E_{N,0} - eV_g} - \frac{|V|^2}{E_F + E_{N-2,1} - E_{N-1,1} - eV_g}, \tag{7}$$

and

$$J_{\downarrow} = \frac{|V|^2}{E_F + E_{N-1,0} - E_{N-1,1} - eV_g}. \tag{8}$$

Before going further, it will be useful to discuss here an important difference between our impurity problem, the model studied in Ref. 9, and the usual Anderson model. In the Anderson model, the degenerate states are spin states associated with a single *orbital* in the impurity. The impurity spin is isotropic. In particular, the spin components $\uparrow$ and $\downarrow$ are equal in the contributions to the transport. In our case, the pseudo-spin component $|\Downarrow\rangle$ ($|N-1,1\rangle$) allows for one additional virtual-process channel compared with $|\Uparrow\rangle$, namely, the one depicted in Fig. 2(e), coming from the third term in the parenthesis in Eq. (4). This additional channel is responsible for the difference between the coefficients $J_{\downarrow}$ and $J_{\uparrow}$ in Eq. (5). Physically, this additional channel allows the electrons in the $|\Downarrow\rangle$ states for more kinetic energy gain than those in the $|\Uparrow\rangle$ states. The difference $\Delta_z \equiv J_{\uparrow} - J_{\downarrow}$ leads to an *effective* Zeeman splitting between $|\Uparrow\rangle$ and $|\Downarrow\rangle$ states. It is reminiscent of the original Kondo impurity in an external magnetic field.[13,14,25] It should be emphasized, however, that in our case, the fictitious field Δ_z arises *intrinsically*. Moreover, as we see below, the effects of Δ_z can be removed by detuning the levels of $|N,0\rangle$ and $|N-1,1\rangle$ (i.e., going off the phase boundary in the parameter space). The two degenerate states involved in the Kondo effect studied in Ref. 9 become degenerate due to competition between the single-particle orbital-level spacing and the Zeeman energy. In the case of the Kondo effect near the singlet-triplet transition studied in Refs. 10,11, the degeneracy purely comes from the many-body exchange interaction, and is fourfold (ignoring very small Zeeman energy). In our case, it is a result of interplay among many-body interaction, parabolic confinement energy, and (real) Zeeman effect. We also remark that the tunneling amplitude V in Eq. 3 should, in general, depend on the orbital m. Its effect introduces another contribution to the fictitious Zeeman splitting, and does not affect the results qualitatively.

To investigate the low-energy physical properties of the Kondo-like model, Eq. (5), we first follow the poor man's scaling approach[1] and trace out the conduction electrons in the range $D - \delta D < |\varepsilon_k| < D$. The resultant renormalization group (RG) equations are given by

$$\frac{d}{d\ell} J_{\perp} = 2\rho_0 J_z J_{\perp}, \quad \frac{d}{d\ell} J_z = 2\rho_0 J_{\perp}^2, \quad \frac{d}{d\ell} \Delta_z = 0, \tag{9}$$

where $J_z \equiv (J_{\downarrow} + J_{\uparrow})/2$ and $\ell \equiv -\ln(\rho_0 D)$. The first two RG equations in Eq. (9) are exactly the same as those of the usual Anderson model, which exhibit a Kosterlitz-Thouless-type RG flow diagram.[1] This implies that the effective impurity model in question exhibits a Kondo effect. The additional coupling constant Δ_z in Eq. (9) plays a role of a fictitious magnetic field on the pseudospin. Therefore, for Δ_z larger than the Kondo temperature T_K, the Kondo resonance peak at the Fermi level will be split into two peaks with separation given by Δ_z. The spectral weights of the split peaks will be diminished.[13,14] Further, since Δ_z arises intrinsically in our model, the effect of this fictitious field may be removed by detuning the levels of $|N,0\rangle$ and $|N-1,1\rangle$. All these arguments are confirmed by the NRG calculations, see below.

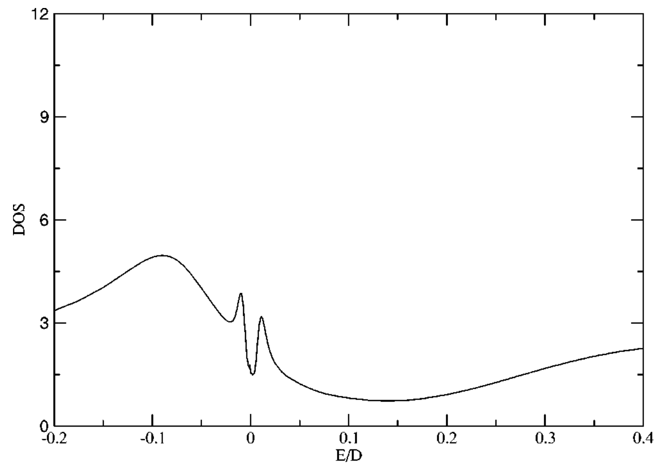

FIG. 3. Density of states on the quantum dot in a strong magnetic field with degenerate energy levels $E_{6,0} = E_{5,1}$.

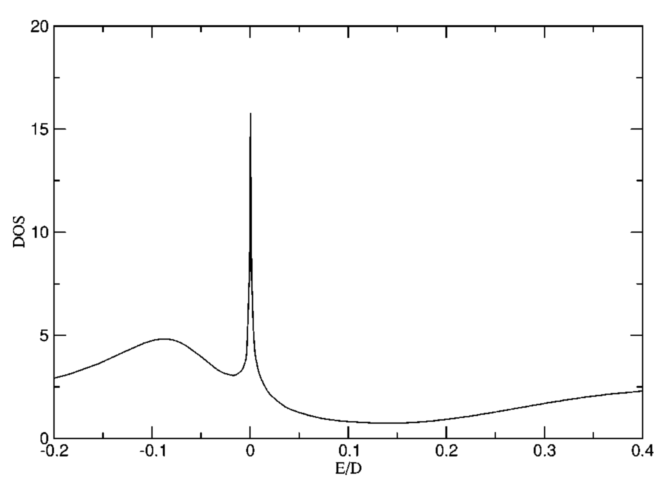

FIG. 4. Density of states on the quantum dot in a strong magnetic field with small detuning of energy levels $E_{6,0}$ and $E_{5,1}$.

The RG equations, Eqs. (9), and the corresponding arguments above are based on a second-order perturbation in $J_\perp$, $J_\downarrow$, and $J_\uparrow$. One may question the validity of the perturbative RG analysis, in particular, the marginal behavior of Δ_z in Eq. (9). We perform a NRG calculation,[21–24] and justify that the above conclusions are qualitatively correct. Figure 3 shows the local density of states of the quantum dot in a strong magnetic field. For the calculations, we have chosen $\Gamma=0.025D$ and $E_F-(E_{N,0}-E_{N-1,0}+eV_g)=0.085D$. Near the Fermi level, there are two Kondo peaks separated by an $\Delta_z\approx 0.01D$. Since this fictitious Zeeman splitting comes from the differences in the kinetic energy gains for $|\Uparrow\rangle$ and $|\Downarrow\rangle$ states, it can be removed by detuning the degenerate levels $E_{N,0}$ and $E_{N-1,1}$. In Fig. 4, we have detuned the energy levels of $E_{N,0}$ and $E_{N-1,1}$ by an amount Δ_z, i.e., $E_F-(E_{N,0}-E_{N-1,0}+eV_g)=(0.085-0.005)D$ and $E_F-(E_{N-1,1}-E_{N-1,0}+eV_g)=(0.085+0.005)D$. One can clearly see that the Kondo resonance peak at the Fermi level has been recovered.

Before concluding, we briefly discuss possible experiments. The maximum-density-droplet states have already been probed in transport measurements through vertical[18] and lateral[19] quantum dots. The ordinary Kondo effects have also been observed in quantum dots.[2,4,5] To observe a Kondo effect involving the maximum-density-droplet states, vertical quantum dots are particularly convenient. One advantage is that these vertical dots provide a relatively strong confinement potential. Another important advantage of vertical dots is that the leads can be made of bulk electrodes. In bulk electrodes, the densities of states for both spins near the Fermi level are insensitive to the energy (we assume that the Zeeman energy is small compared with the Fermi energy) and the spin-polarization effects can be safely neglected.[5] We estimate the typical values of the relevant parameters to be $N\sim 15$, $B\sim 5$ T ($g\mu_B B\sim 0.15$ meV), $\hbar\Omega\sim 3$ meV, $T_K\sim 0.05$–0.1 K, and $\Delta_z\sim 0.1$ meV.

We have reported on the Kondo effect of quantum dots in the quantum Hall regime, involving the precursor of the Landau-level filling factor $\nu=1$ state. Unlike the ordinary Anderson impurity model, we find that the Kondo resonance splits due to an internally generated fictitious magnetic field. We showed that by detuning the energies of states involved in the Kondo effect, the resonance peak at the Fermi level may be restored. We have argued that these effects may be observed in vertical dots.

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[25] There is one-to-one correspondence between our impurity problem and the common Anderson model (in the $U\to\infty$ limit) : $\epsilon_d\leftrightarrow E_F-(E_{N,0}-E_{N-10}+eV_g)$, $|0\leftrightarrow\rangle|N-10\rangle$, $|\uparrow\rangle\leftrightarrow|\Uparrow\rangle$, and $|\downarrow\rangle\leftrightarrow|\Downarrow\rangle$.