QCLab Workshop

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Physics
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교학상장 (敎學相長)

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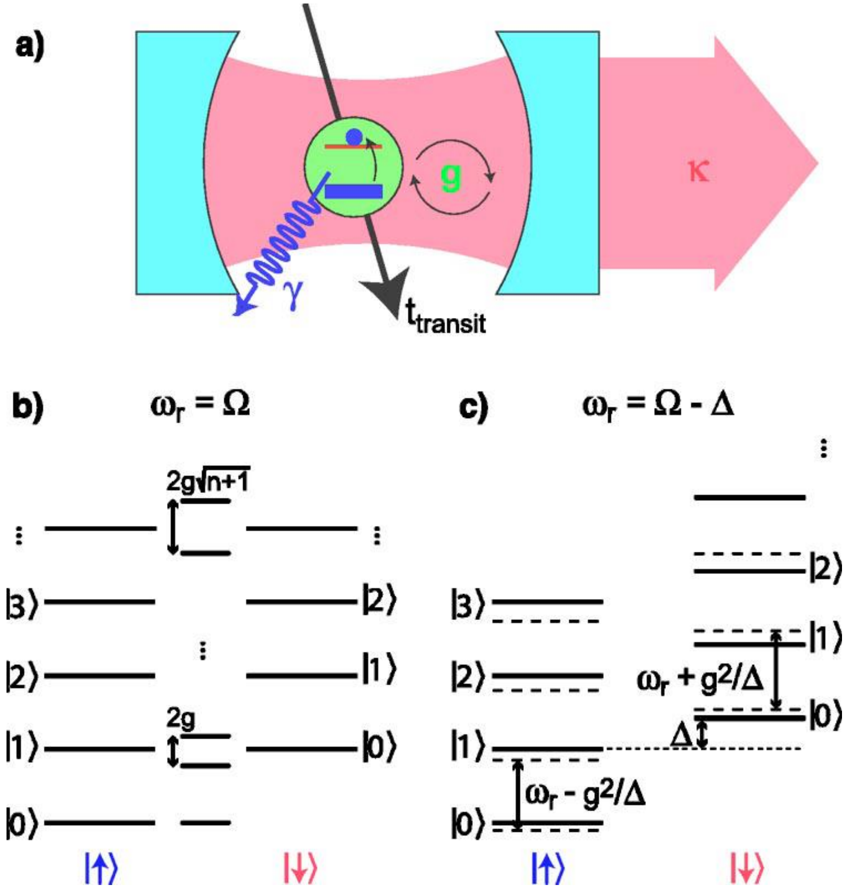
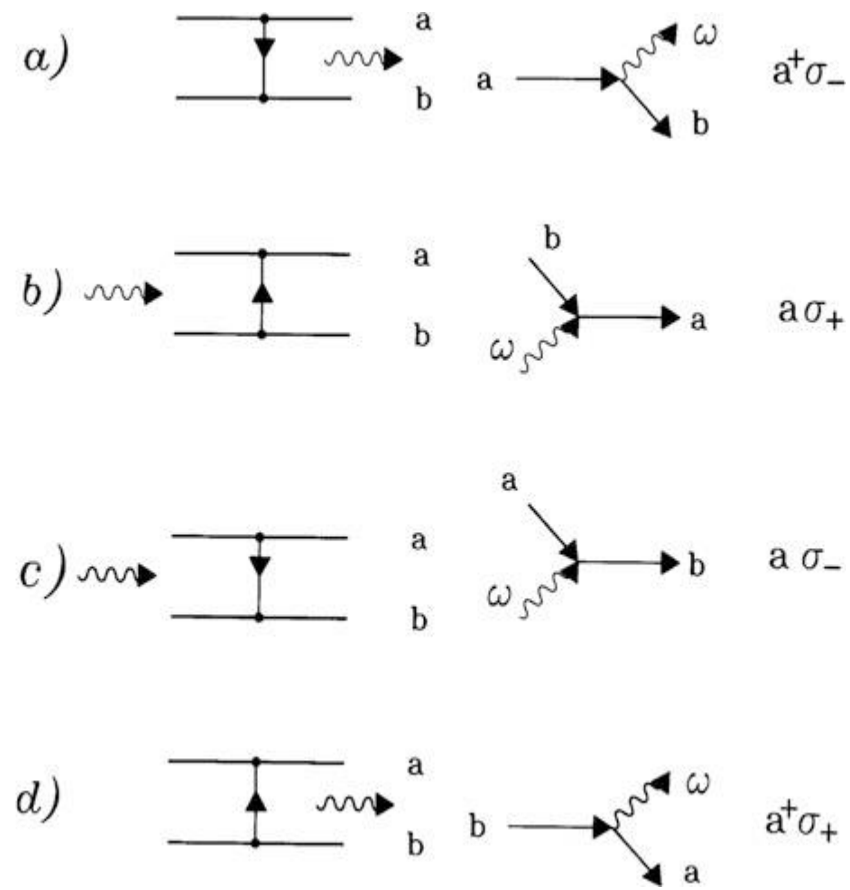
2-1 Introduction





Atom coupled to a photon inside of an optical cavity

$$H = \hbar\omega_r(a^{\dagger}a + 1/2) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^{\dagger} + a)\sigma^x$$

2-2 Matrix Representation

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \quad |2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \quad |3\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}.$$

$$\mathcal{H} = \hbar\omega \begin{pmatrix} 1/2 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 3/2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 5/2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 7/2 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 9/2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

$$a = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & 0 & \cdots \\ 0 & 0 & 0 & 0 & \sqrt{4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2-3 Simple Harmonic Oscillator

- Simple Case

$$\hat{H} = \hbar\omega(\hat{a}^\dagger \hat{a} + 1/2)$$

- With External term

$$\hat{H} = (\hat{a}^\dagger + \alpha)(\hat{a} + \alpha) \; = \; \hat{a}^\dagger \hat{a} + \alpha(\hat{a}^\dagger + \hat{a}) + \alpha^2$$

$$\hat{b} = \hat{a} + \alpha$$

$$\hat{H} = \hat{b}^\dagger \hat{b}$$

$$[\hat{b}, \hat{b}^\dagger] = 1$$

Campbell-Baker-Hausdorff Formula & Displacement Operator & Coherent State

- Campbell-Baker-Hausdorff Formula

$$e^{\lambda}\mu e^{-\lambda} = \mu + [\lambda, \mu] + \frac{1}{2!}[\lambda, [\lambda, \mu]] + \frac{1}{3!}[\lambda, [\lambda, [\lambda, \mu]]]...$$

- Displacement Operator

$$D(\alpha) = e^{\alpha \hat{a}^{\dagger} - \alpha^{*} \hat{a}}$$

$$D^{\dagger}(\alpha) = D^{-1}(\alpha) = D(-\alpha)$$

- Using Formula

$$D^{\dagger}(\alpha)\hat{a}D(\alpha) = \hat{a} + \alpha$$

$$D^{\dagger}(\alpha)\hat{a}^{\dagger}D(\alpha) = \hat{a}^{\dagger} + \alpha^{*}$$

Continue..

- Unitary Transform of SHO

$$D^\dagger(\alpha)\hat{H}D(\alpha) = D^\dagger(\alpha)\hat{a}^\dagger D(\alpha)D^\dagger(\alpha)\hat{a}D(\alpha) = (\hat{a}^\dagger + \alpha)(\hat{a} + \alpha)$$

- Coherent state

$$\hat{a} \mid \alpha \rangle = \alpha \mid \alpha \rangle$$

- Coherent state generating

$$\mid \alpha \rangle = D(\alpha) \mid 0 \rangle$$

Continue…

- Fock space representation

$$| \alpha \rangle = e^{\frac{1}{2}\alpha^2} \sum_n \frac{(\alpha \hat{a}^\dagger)^n}{n!} | 0 \rangle$$

- Expectation Value

$$\begin{aligned} \langle x \rangle &= \frac{\lambda}{\sqrt{2}}(\langle \alpha \mid \hat{a} \mid \alpha \rangle + \langle \alpha \mid \hat{a}^\dagger \mid \alpha \rangle) \\ &= \frac{\lambda}{\sqrt{2}}(\alpha + \alpha^*) \\ &= \frac{\lambda}{\sqrt{2}} 2\ Re(\alpha) \end{aligned}$$

$$\begin{aligned} \langle p \rangle &= \frac{\sqrt{2}\hbar}{2i\lambda}(\langle \alpha \mid \hat{a} \mid \alpha \rangle - \langle \alpha \mid \hat{a}^\dagger \mid \alpha \rangle) \\ &= \frac{\sqrt{2}\hbar}{2i\lambda}(\alpha - \alpha^*) \\ &= \frac{\sqrt{2}\hbar}{\lambda} 2\ Im(\alpha) \end{aligned}$$

- Uncertainty

$$\Delta x \Delta p = \frac{\hbar}{2}$$

Mathematica

```
In[2]:= nn = Range[0, 10];
              범위

        Egg = RotateLeft@DiagonalMatrix@Sqrt@nn;
               왼쪽으로 회전   대각 행렬           제곱근

        Dgg = Transpose@Egg;
               전치

In[5]:= {Egg // MatrixForm, Dgg // MatrixForm}
                  행렬 형식              행렬 형식
```

Out[5]=

$$\left\{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{10} & 0 \end{pmatrix}\right\}$$

Mathematica

```
In[6]:= H[g_] := DiagonalMatrix[nn] + g (Dgg + Egg) + g^2 IdentityMatrix[11]
```

대각 행렬 · 항등 행렬

```
In[7]:= H[g] // MatrixForm
```

행렬 형식

Out[7]//MatrixForm=

$$
\begin{pmatrix}
g^2 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
g & 1+g^2 & \sqrt{2}\, g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{2}\, g & 2+g^2 & \sqrt{3}\, g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3}\, g & 3+g^2 & 2\, g & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2\, g & 4+g^2 & \sqrt{5}\, g & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sqrt{5}\, g & 5+g^2 & \sqrt{6}\, g & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{6}\, g & 6+g^2 & \sqrt{7}\, g & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sqrt{7}\, g & 7+g^2 & 2\sqrt{2}\, g & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2}\, g & 8+g^2 & 3\, g & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\, g & 9+g^2 & \sqrt{10}\, g \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{10}\, g & 10+g^2
\end{pmatrix}
$$

Mathematica

- Eigenvector of Hamiltonian vs Coherent state

```
In[8]:= {Reverse@Part[Transpose@Eigenvectors[H[0.5]], 1],
        반전         부분    전치          고유 벡터
  Exp[-1/2*1/2*1/2] Sum[(0.5^i*(Dot@@ConstantArray[Dgg, i]).{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0})/Factorial[i],
  지수 함수             총합          내적  복제 배열                                              계승
    {i, 0, 10}]} // Transpose // MatrixForm
                     전치          행렬 형식
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0.882497 & 0.882497 \\ 0.441248 & 0.441248 \\ -0.156005 & 0.156005 \\ 0.0450347 & 0.0450347 \\ 0.0112587 & 0.0112587 \\ 0.0025175 & 0.00251752 \\ 0.000512379 & 0.000513886 \\ 0.0000925562 & 0.0000971154 \\ 0.0000126522 & 0.0000171677 \\ 9.77723\times10^{-7} & 2.86129\times10^{-6} \\ 2.67685\times10^{-8} & 4.5241\times10^{-7} \end{pmatrix}$$

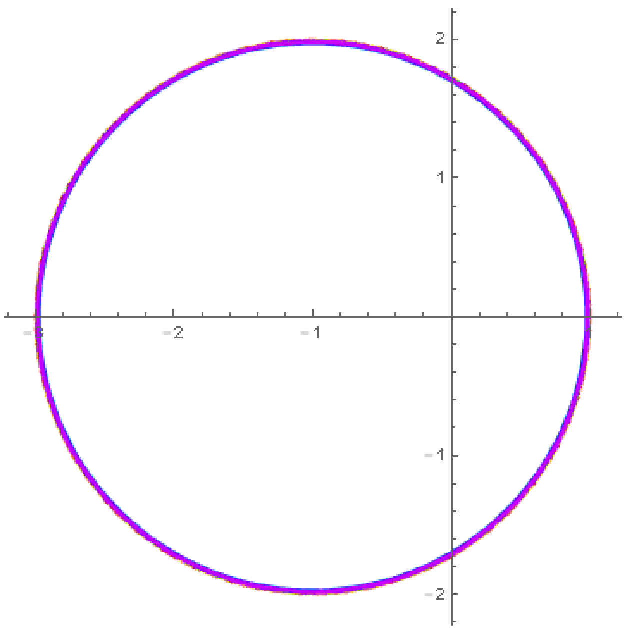
Mathematica

```
In[11]:= {ee, ev} = Eigensystem[H[0.5]];
                    고유치와 고유 벡터의 목록

         psi[t_] := Transpose@Conjugate[ev].((Exp[-I ee t] ev).(CS))
                    전치        켤레 복소수            지수…  허수 단위

         XP[t_] := With[{Psi = psi[t]}
                   인수 대체

           , {(Conjugate[psi[t]].(Egg + Dgg).psi[t]),
               켤레 복소수

            (Conjugate[psi[t]].(((I Egg - I Dgg)).psi[t]))}]
             켤레 복소수                 허수…   허수 단위

         fig4 := ParametricPlot[XP[t], {t, 0, 800}, ColorFunction → Function[{x, y, t}, Hue[0.8 t]]]
                 파라 메트릭 플롯                      색상 함수        함수                  색조
```

$$\Psi(t) = \sum_{0}^{n} \psi_n \langle \psi_n \mid \psi_i \rangle e^{-\frac{iE_n t}{\hbar}}$$

```
In[137]:= Show[fig4]
          전시
```

Out[137]=



- X vs P Plot

Center is (-1.0,0) which is Expectation value of X, P

2-4 Jaynes Cummings Hamiltonian

- Approximation

$$(\hat{a} + \hat{a}^\dagger) \otimes (\sigma^+ + \sigma^-) \simeq \hat{a} \otimes \sigma^+ + \hat{a}^\dagger \otimes \sigma^-$$

$$\hat{H}_{JC} \;=\; \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger \sigma^- + \hat{a}\sigma^+) + \frac{1}{2}\sigma^z$$

- Eigen state

$$|\,\psi_\pm\,\rangle \;=\; \frac{1}{\sqrt{2}}(|\,n,\uparrow\,\rangle \pm \,|\,n+1,\downarrow\,\rangle)$$

$$\hat{H}_{JC}\,|\,\psi_+\,\rangle \;=\; \frac{1}{\sqrt{2}}[(n + g\sqrt{n+1} + 1/2)\,|\,n,\uparrow\,\rangle \;+\; (n + 1 + g\sqrt{n+1} - 1/2)\,|\,n,\downarrow\,\rangle$$

$$=\; \frac{1}{\sqrt{2}}(n + 1/2 + g\sqrt{n+1})(|\,n,\uparrow\,\rangle + \,|\,n+1,\downarrow\,\rangle)$$

$$\hat{H}_{JC}\,|\,\psi_-\,\rangle \;=\; \frac{1}{\sqrt{2}}[(n + g\sqrt{n+1} + 1/2)\,|\,n,\uparrow\,\rangle \;-\; (n + 1 + g\sqrt{n+1} - 1/2)\,|\,n,\downarrow\,\rangle$$

$$=\; \frac{1}{\sqrt{2}}(n + 1/2 - g\sqrt{n+1})(|\,n,\uparrow\,\rangle - \,|\,n+1,\downarrow\,\rangle)$$

Mathematica

```
In[29]:= nn = Range[0, 5];
             범위

         Egg = RotateLeft@DiagonalMatrix@Sqrt@nn;
               왼쪽으로 회전 대각 행렬      제곱근

         Dgg = Transpose@Egg;
               전치

In[32]:= JC[g_] := Pauli[0]⊗DiagonalMatrix[nn] + g * (Pauli[4]⊗Egg + Pauli[5]⊗Dgg) + 1/2 Pauli[3]⊗IdentityMatrix[6]
                             대각 행렬                                                                    항등 행렬

In[33]:= JC[g] // MatrixForm
                  행렬 형식
```

Out[33]//MatrixForm=

$$
\begin{pmatrix}
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 \\
0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}\, g & 0 & 0 & 0 \\
0 & 0 & \frac{5}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3}\, g & 0 & 0 \\
0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 2\, g & 0 \\
0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}\, g \\
0 & 0 & 0 & 0 & 0 & \frac{11}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & \sqrt{2}\, g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\
0 & 0 & \sqrt{3}\, g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 \\
0 & 0 & 0 & 2\, g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 \\
0 & 0 & 0 & 0 & \sqrt{5}\, g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2}
\end{pmatrix}
$$

Mathematica

```
In[34]:= {ee1, ev1} = Eigensystem[JC[0.1]];
```

고유치와 고유 벡터의 목록

```
In[36]:= ev1 // Transpose // MatrixForm
```

전치 행렬 형식

Out[36]//MatrixForm=

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.707107	0.	-0.707107
0.	0.	0.	0.	0.	0.	0.	0.707107	0.707107	0.	0.	0.
0.	0.	0.	0.	0.	0.707107	-0.707107	0.	0.	0.	0.	0.
0.	0.	0.	0.707107	0.707107	0.	0.	0.	0.	0.	0.	0.
0.	-0.707107	-0.707107	0.	0.	0.	0.	0.	0.	0.	0.	0.
1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.707107	0.	0.707107
0.	0.	0.	0.	0.	0.	0.	0.707107	-0.707107	0.	0.	0.
0.	0.	0.	0.	0.	0.707107	0.707107	0.	0.	0.	0.	0.
0.	0.	0.	0.707107	-0.707107	0.	0.	0.	0.	0.	0.	0.
0.	-0.707107	0.707107	0.	0.	0.	0.	0.	0.	0.	0.	0.

```
{ee1, Reverse@Flatten[Table[{i + 1 / 2 - 0.1 Sqrt[i + 1], i + 1 / 2 + 0.1 Sqrt[i + 1]}, {i, 0, 5}]]} // Transpose // MatrixForm
```

반전 평활화 표 제곱근 제곱근 전치 행렬 형식

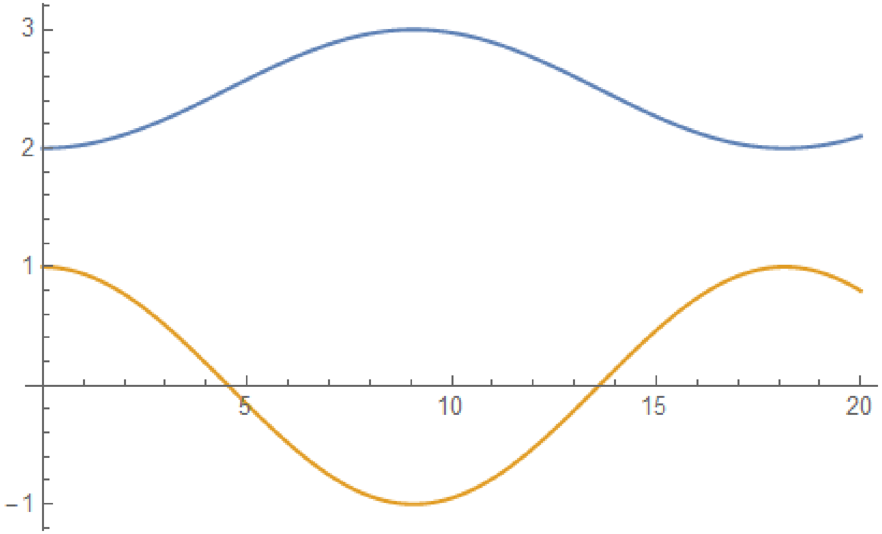
Out[71]//MatrixForm=

5.5	5.74495
4.72361	5.25505
4.27639	4.72361
3.7	4.27639
3.3	3.7
2.67321	3.3
2.32679	2.67321
1.64142	2.32679
1.35858	1.64142
0.6	1.35858
-0.5	0.6
0.4	0.4

Mathematica

In[80]:=
```
psi1[t_] := Transpose@Conjugate[ev1].((Exp[-I ee1 t] ev1).({0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}))
            전치              켤레 복소수                  지수…    허수 단위
Plot[{Conjugate[psi1[t]].(Pauli[0]⊗DiagonalMatrix[nn]).psi1[t],
플롯      켤레 복소수                        대각 행렬
  Conjugate[psi1[t]].(Pauli[3]⊗IdentityMatrix[6]).psi1[t]}, {t, 0, 20}]
  켤레 복소수                        항등 행렬
```

Out[81]=



2-5 Rabi Hamiltonian

$$\hat{H}_{Rabi} = \hat{a}^\dagger \hat{a} - g(\hat{a}^\dagger + \hat{a})\sigma^x + \sigma^z$$

- Rabi Hamiltonian can't solve exactly
- So we observe property using mathematica

Mathematica

```
In[110]:= nn = Range[0, 5];
              범위
          Egg = RotateLeft@DiagonalMatrix@Sqrt@nn;
                왼쪽으로 회전  대각 행렬           제곱근
          Dgg = Transpose@Egg;
                전치

In[135]:= Rabi[g_] := Pauli[0]⊗DiagonalMatrix[nn] - g*Pauli[1]⊗(Egg + Dgg) + 1/2 Pauli[3]⊗IdentityMatrix[6]
                                 대각 행렬                                                  항등 행렬

In[122]:= Rabi[g] // MatrixForm
                     행렬 형식
```

Out[122]//MatrixForm=

$$
\begin{pmatrix}
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 & 0 \\
0 & \frac{3}{2} & 0 & 0 & 0 & 0 & -g & 0 & -\sqrt{2}\,g & 0 & 0 & 0 \\
0 & 0 & \frac{5}{2} & 0 & 0 & 0 & 0 & -\sqrt{2}\,g & 0 & -\sqrt{3}\,g & 0 & 0 \\
0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & -\sqrt{3}\,g & 0 & -2\,g & 0 \\
0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & -2\,g & 0 & -\sqrt{5}\,g \\
0 & 0 & 0 & 0 & 0 & \frac{11}{2} & 0 & 0 & 0 & 0 & -\sqrt{5}\,g & 0 \\
0 & -g & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
-g & 0 & -\sqrt{2}\,g & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & -\sqrt{2}\,g & 0 & -\sqrt{3}\,g & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\
0 & 0 & -\sqrt{3}\,g & 0 & -2\,g & 0 & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 \\
0 & 0 & 0 & -2\,g & 0 & -\sqrt{5}\,g & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 \\
0 & 0 & 0 & 0 & -\sqrt{5}\,g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2}
\end{pmatrix}
$$

Mathematica

- With small g~0.1
- We can observe that property is similar to JC model

```
In[46]:= {ee2, ev2} = Eigensystem[Rabi[0.1]];
                      고유치와 고유 벡터의 목록

In[47]:= ev2 // Transpose // MatrixForm
                전치          행렬 형식
```

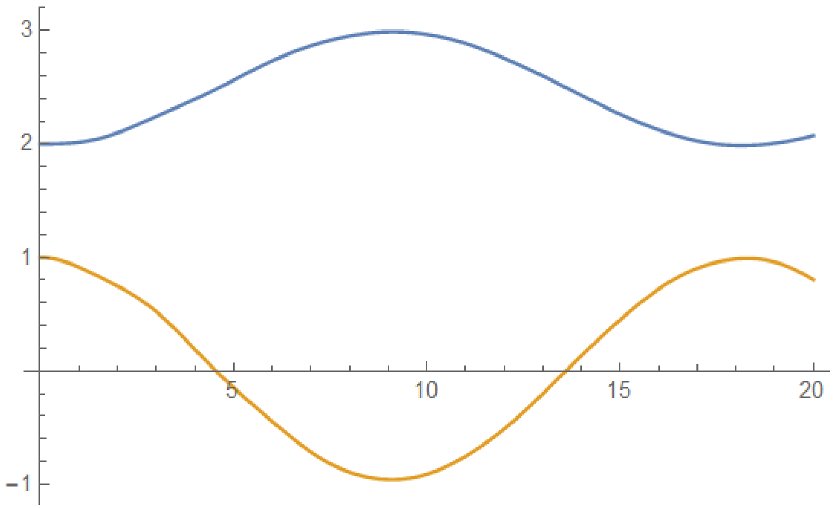
Out[47]//MatrixForm=

0.	3.98178×10^{-6}	-7.44038×10^{-6}	0.	0.	-0.00221852	-0.00287807
0.0000166392	0.	0.	0.00377905	0.00511263	0.	0.
0.	0.00504168	-0.00754154	0.	0.	-0.735402	-0.673842
0.0109865	0.	0.	0.738812	0.668162	0.	0.
0.	0.719676	-0.686941	0.	0.	0.0744214	-0.0681147
0.993779	0.	0.	-0.0824964	0.0748141	0.	0.
2.76172×10^{-7}	0.	0.	0.0000901041	0.000134682	0.	0.
0.	0.000168541	-0.000281797	0.	0.	-0.0480855	-0.0524496
0.000473366	0.	0.	0.0585666	0.0648365	0.	0.
0.	0.0648551	-0.0775955	0.	0.	-0.671748	0.733818
0.110825	0.	0.	0.666263	-0.737382	0.	0.
0.	0.691256	0.722519	0.	0.	-0.00908091	0.00699432

Mathematica

```
In[48]:= psi2[t_] := Transpose@Conjugate[ev2].((Exp[-I ee2 t] ev2).({0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}))
                    전치          켤레 복소수          지수 ···  허수 단위
         Plot[{Conjugate[psi2[t]].(Pauli[0]⊗DiagonalMatrix[nn]).psi2[t],
         플롯     켤레 복소수                  대각 행렬
           Conjugate[psi2[t]].(Pauli[3]⊗IdentityMatrix[6]).psi2[t]}, {t, 0, 20}]
           켤레 복소수                  항등 행렬
```

Out[49]=



Mathematica

- With large g~1.0

```
In[50]:= {ee2, ev2} = Eigensystem[Rabi[1.0]];
                      고유치와 고유 벡터의 목록

In[51]:= ev2 // Transpose // MatrixForm
               전치          행렬 형식
```

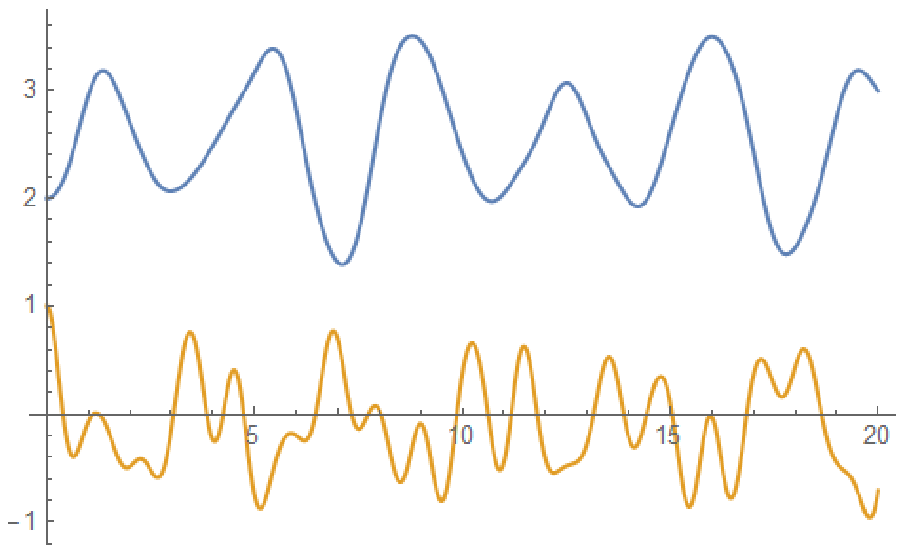
Out[51]//MatrixForm=

0.	-0.00428236	0.	-0.0595873	0.	0.170465	-0.667248
0.0275868	0.	-0.184183	0.	0.689744	0.	0.
0.	-0.135992	0.	-0.636549	0.	0.485311	0.190479
0.357426	0.	-0.693695	0.	-0.20028	0.	0.
0.	-0.721022	0.	0.00292333	0.	-0.449838	-0.340334
0.710592	0.	0.53458	0.	0.288957	0.	0.
0.00350433	0.	-0.0338348	0.	0.209205	0.	0.
0.	-0.0290161	0.	-0.239149	0.	0.38217	-0.515245
0.11207	0.	-0.424555	0.	0.484631	0.	0.
0.	-0.351275	0.	-0.54469	0.	-0.244253	0.28567
0.594956	0.	-0.133025	0.	-0.349301	0.	0.
0.	-0.580843	0.	0.487203	0.	0.572143	0.235767

Mathematica

In[52]:=
```
psi2[t_] := Transpose@Conjugate[ev2].((Exp[-I ee2 t] ev2).({0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}))
            전치       켤레 복소수          지수…  허수 단위

Plot[{Conjugate[psi2[t]].(Pauli[0]⊗DiagonalMatrix[nn]).psi2[t],
플롯  켤레 복소수                     대각 행렬
  Conjugate[psi2[t]].(Pauli[3]⊗IdentityMatrix[6]).psi2[t]}, {t, 0, 20}]
  켤레 복소수                   항등 행렬
```

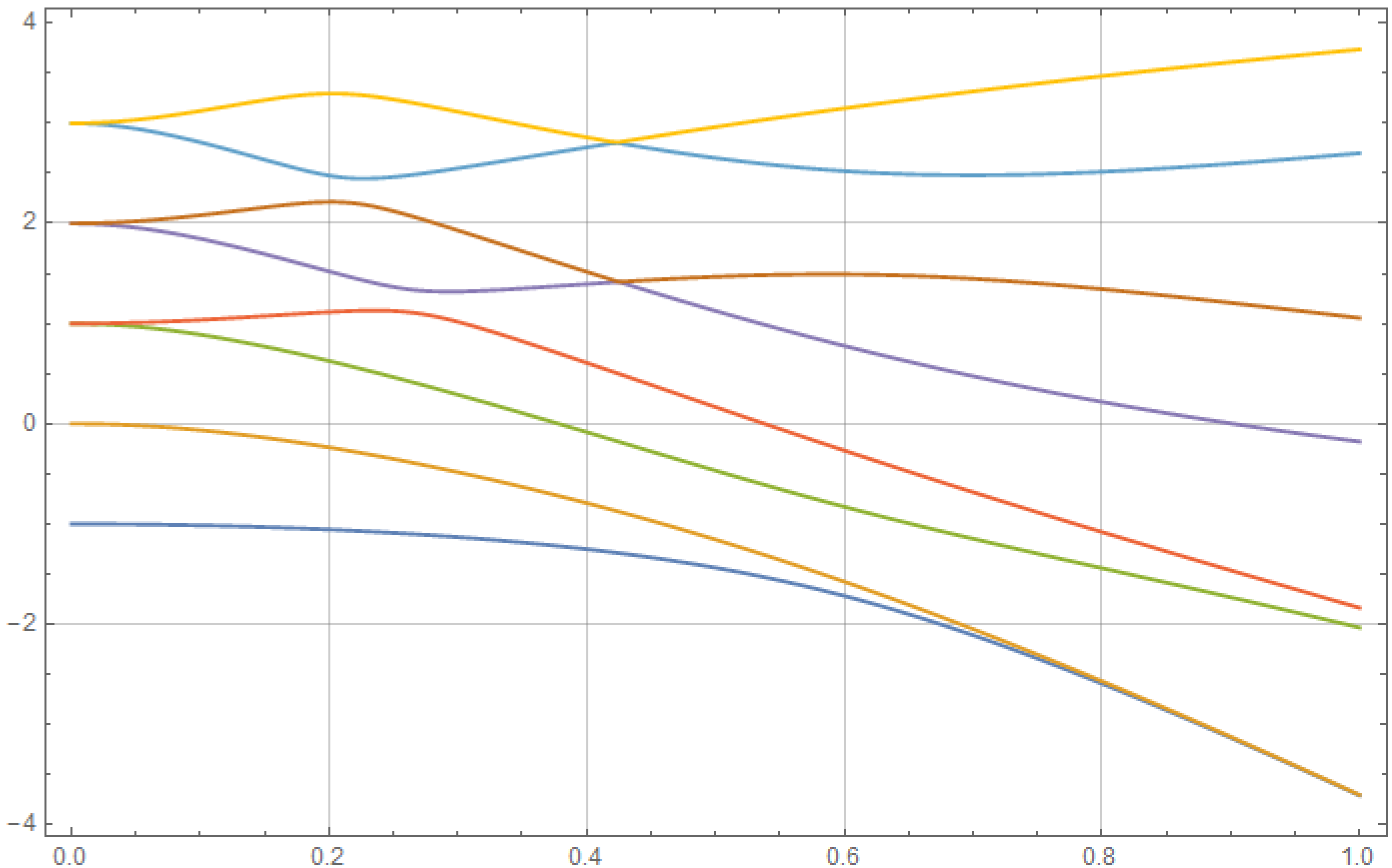
Out[53]=



Mathematica

In[184]:=
```
la = Transpose@Table[Take[Sort@Eigenvalues@Rabi[g], Ceiling[8]], {g, 0, 2, 0.01}];
     전치        표       갖기   분류   고유치                 올림
ga = ListPlot[la, Joined → True, DataRange → {0, 1}, ImageSize → Large,
     목록에 해당하는 점···  점의 결합  참된  데이터 범위          이미지 크기  큰
  GridLines → Automatic, Axes → None, Frame → True]
  격자 선        자동        축     없음    테두리   참된
```

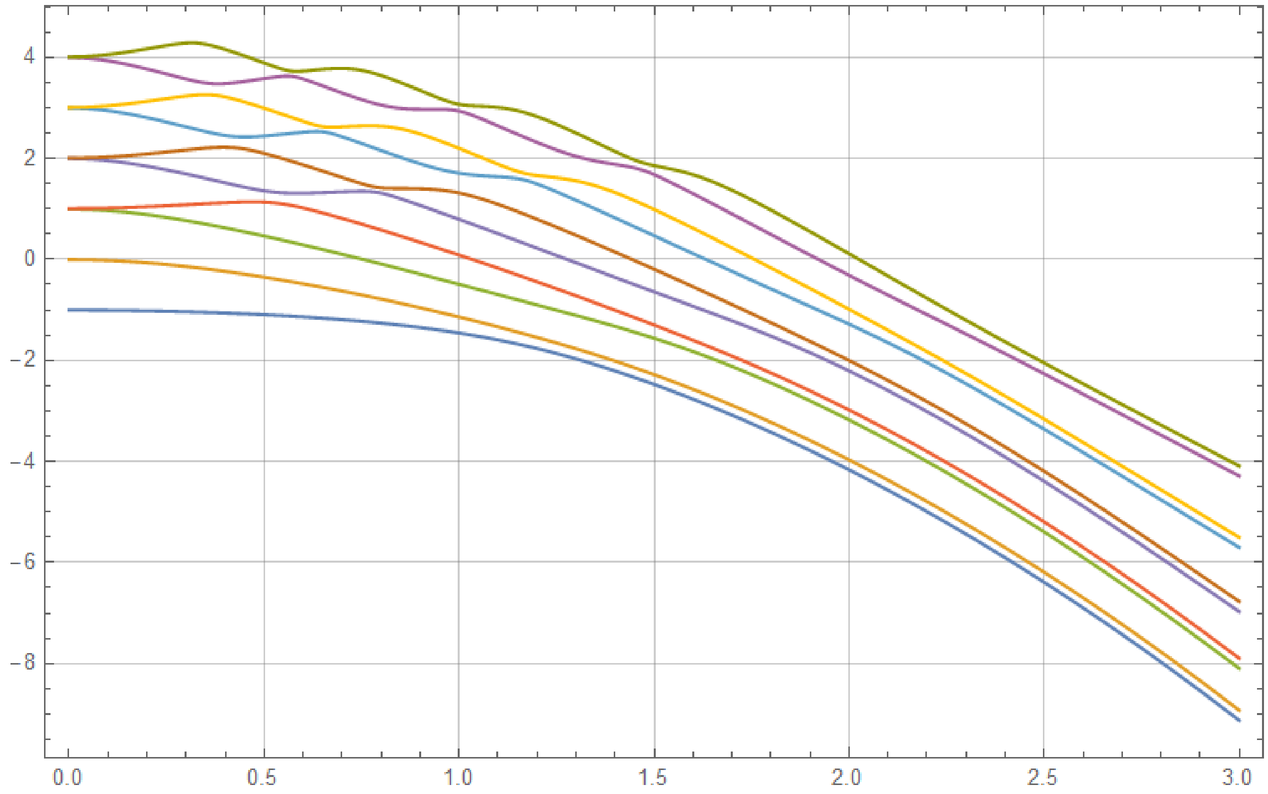
Out[185]=



Mathematica

$$\hat{H} = \hat{a}^\dagger \hat{a} - g(\hat{a}^\dagger + \hat{a})\sigma^x + \sigma^z + \sigma^x$$

```
la = Transpose@Table[Take[Sort@Eigenvalues@H[g, 0.1], Ceiling[$N / 2]], {g, 0, 3, 0.01}];
     전치          표      갖기   분류   고유치                          올림

ga = ListPlot[la, Joined → True, DataRange → {0, 3}, ImageSize → Large,
     목록에 해당하는 점…  점의 결합   참된    데이터 범위          이미지 크기    큰
  GridLines → Automatic, Axes → None, Frame → True]
  격자 선      자동           축     없음    테두리   참된
```



Conservated Quantity

- State transition in Rabi Hamiltonian

$$|\,0,\uparrow\,\rangle,\ |\,1,\downarrow\,\rangle,\ |\,2,\uparrow\,\rangle,\ \ldots$$

$$|\,0,\downarrow\,\rangle,\ |\,1,\uparrow\,\rangle,\ |\,2,\downarrow\,\rangle,\ \ldots$$

- Parity Operator

$$\tau^z = \cos(\pi \hat{a}^\dagger \hat{a}) \otimes \sigma^z$$

$$[\hat{\tau}^z, \hat{H}_{Rabi}] = 0$$

Continue…

- We can construct Hamiltonian to Block Diagonal Matrix

```
In[189]:= Rabi[g_] := Pauli[0] ⊗ DiagonalMatrix[nn] - g * Pauli[0] ⊗ (Egg + Dgg) + 1 / 2 Pauli[3] ⊗ DiagonalMatrix[Cos[Pi nn]]
                                 대각 행렬                                                                   대각 행렬            코…  원주율

In[190]:= Rabi[g] // MatrixForm
                     행렬 형식
```

Out[190]//MatrixForm=

$$\begin{pmatrix}
\frac{1}{2} & -g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-g & \frac{1}{2} & -\sqrt{2}\, g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sqrt{2}\, g & \frac{5}{2} & -\sqrt{3}\, g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sqrt{3}\, g & \frac{5}{2} & -2\, g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\, g & \frac{9}{2} & -\sqrt{5}\, g & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\sqrt{5}\, g & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -g & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -g & \frac{3}{2} & -\sqrt{2}\, g & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{2}\, g & \frac{3}{2} & -\sqrt{3}\, g & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{3}\, g & \frac{7}{2} & -2\, g & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\, g & \frac{7}{2} & -\sqrt{5}\, g \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{5}\, g & \frac{11}{2}
\end{pmatrix}$$

X-P Plot

```
$N=10;
Let[Reals,t]
nn:=Range[0,$N];
nnn:=ConstantArray[1,$N+1]
Aegg=RotateLeft@DiagonalMatrix@Sqrt[nn];
Adgg=Dag[Aegg];
CS[g_]:=Exp[-1/2(g)^2]*(g)^nn/Sqrt[nn!]*nnn
Id:=IdentityMatrix[$N+1];
Idd:=DiagonalMatrix[Cos[Pi nn]];
Rabi[g_]:=Pauli[0]⊗DiagonalMatrix[nn]-g*Pauli[0]⊗(Aegg+Adgg)+ 1/2Pauli[3]⊗DiagonalMatrix[Cos[Pi nn]]
```

```
NRabi[g_] := Rabi[g][[1 ;; 11, 1 ;; 11]]
NRabi[g] // MatrixForm
```

행렬 형식

$$
\begin{pmatrix}
\frac{1}{2} & -g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-g & \frac{1}{2} & -\sqrt{2}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sqrt{2}\,g & \frac{5}{2} & -\sqrt{3}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sqrt{3}\,g & \frac{5}{2} & -2\,g & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\,g & \frac{9}{2} & -\sqrt{5}\,g & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\sqrt{5}\,g & \frac{9}{2} & -\sqrt{6}\,g & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\sqrt{6}\,g & \frac{13}{2} & -\sqrt{7}\,g & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{7}\,g & \frac{13}{2} & -2\sqrt{2}\,g & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\sqrt{2}\,g & \frac{17}{2} & -3\,g & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3\,g & \frac{17}{2} & -\sqrt{10}\,g \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{10}\,g & \frac{21}{2}
\end{pmatrix}
$$

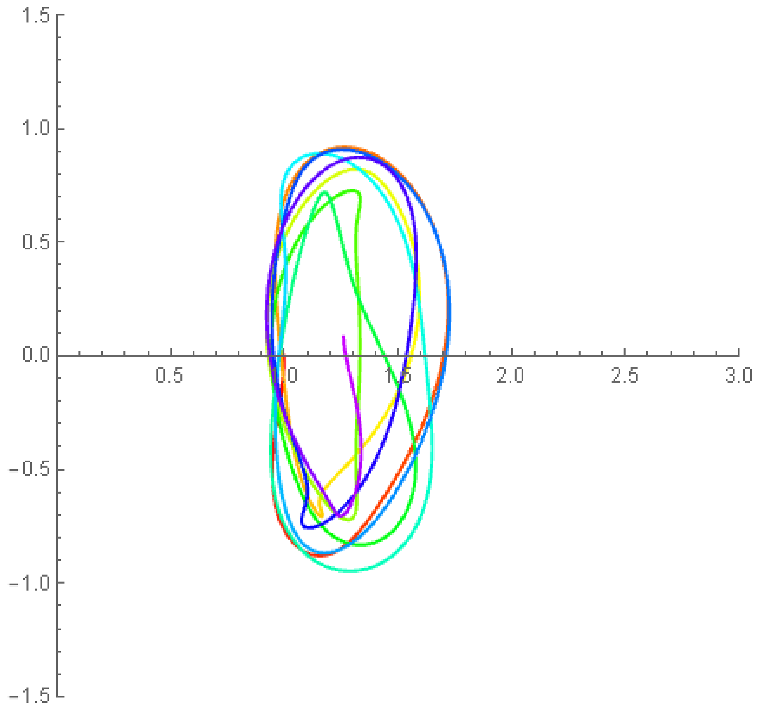
Continue…

In[28]:=
```
{ee,ev}=Eigensystem[NRabi[0.5]];
psi[t_]:=Transpose@Conjugate[ev].((Exp[-I ee t] ev).(CS[0.5]))
XP[t_]:=With[{Psi=psi[t]}
,{(Conjugate[psi[t]].(((Aegg+Adgg)).psi[t])),
(Conjugate[psi[t]].(((I Aegg-I Adgg)).psi[t]))}]
fig:=ParametricPlot[XP[t],{t,0,50},ColorFunction→Function[{x,y,t},Hue[0.8t]],PlotRange→{{0,3},{-1.5,1.5}}]
```

In[32]:=
```
Show[fig]
```

전시

Out[32]=



Continue…

In[194]:=

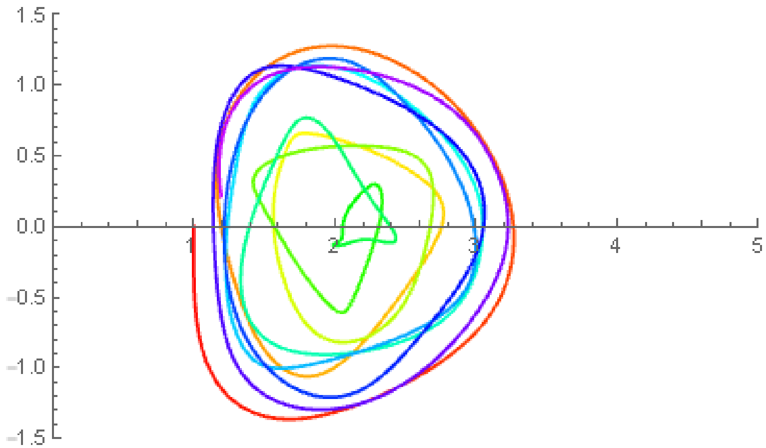
```
{ee1,ev1}=Eigensystem[NRabi[1.01]];
psi1[t_]:=Transpose@Conjugate[ev1].((Exp[-I ee1 t] ev1).(CS[0.5]))
XP1[t_]:=With[{Psi=psi1[t]}
,{(Conjugate[psi1[t]].(((Aegg+Adgg)).psi1[t])),
(Conjugate[psi1[t]].(((I Aegg-I Adgg)).psi1[t]))}]
fig1:=ParametricPlot[XP1[t],{t,0,50},ColorFunction→Function[{x,y,t},Hue[0.8t]],PlotRange→{{0,4},{-1.5,1.5}}]
```

In[193]:=

```
Show[fig1]
```

전시

Out[193]=



frame… labels… axes image size more…

3 Package

3-1 Cauchy Package

```
In[1]:= << Mathey`Cauchy`

        Mathey\Cauchy.m v3.23 (2016-01-07) Mahn-Soo Choi

In[5]:= Let[Reals, x]
              실수 영역

In[6]:= RealQ[x]

Out[6]= True

In[2]:= Let[Complexes, y]
              복소수 영역

In[3]:= ComplexQ[y]

Out[3]= True

In[10]:= {z * Conjugate[z], x * Conjugate[x]}
             켤레 복소수              켤레 복소수

Out[10]= {z z*, x^2}
```

3-2 Pauli Package

In[1]:=
```
<< Quaphy`Pauli`
```

```
Mathey\Cauchy.m v3.23 (2016-01-07) Mahn-Soo Choi
Quaphy\Pauli.m v2.33 (2016-02-22) Mahn-Soo Choi
```

In[3]:=
```
{MatrixForm@Ket[0], MatrixForm@Ket[1], MatrixForm@Pauli[3]}
```
행렬 형식 행렬 형식 행렬 형식

Out[3]= $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\right\}$

In[5]:=
```
{MatrixForm@Wigner[1, 1], MatrixForm@Wigner[1, 2], MatrixForm@Wigner[1, 3]}
```
행렬 형식 행렬 형식 행렬 형식

Out[5]= $\left\{\begin{pmatrix}0 & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ 0 & \frac{1}{\sqrt{2}} & 0\end{pmatrix}, \begin{pmatrix}0 & -\frac{i}{\sqrt{2}} & 0\\ \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}}\\ 0 & \frac{i}{\sqrt{2}} & 0\end{pmatrix}, \begin{pmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & -1\end{pmatrix}\right\}$

In[6]:=
```
ClebschGordanTable[1, 1]
```

	$\|2,2\rangle$	$\|2,1\rangle$	$\|2,0\rangle$	$\|2,-1\rangle$	$\|2,-2\rangle$	$\|1,1\rangle$	$\|1,0\rangle$	$\|1,-1\rangle$	$\|0,0\rangle$
$\langle 1,1\|$	1	0	0	0	0	0	0	0	0
$\langle 1,0\|$	0	$\frac{1}{\sqrt{2}}$	0	0	0	$\frac{1}{\sqrt{2}}$	0	0	0
$\langle 1,-1\|$	0	0	$\frac{1}{\sqrt{6}}$	0	0	0	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{3}}$
$\langle 0,1\|$	0	$\frac{1}{\sqrt{2}}$	0	0	0	$-\frac{1}{\sqrt{2}}$	0	0	0
$\langle 0,0\|$	0	0	$\sqrt{\frac{2}{3}}$	0	0	0	0	0	$-\frac{1}{\sqrt{3}}$
$\langle 0,-1\|$	0	0	0	$\frac{1}{\sqrt{2}}$	0	0	0	$\frac{1}{\sqrt{2}}$	0
$\langle -1,1\|$	0	0	$\frac{1}{\sqrt{6}}$	0	0	0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{3}}$
$\langle -1,0\|$	0	0	0	$\frac{1}{\sqrt{2}}$	0	0	0	$-\frac{1}{\sqrt{2}}$	0
$\langle -1,-1\|$	0	0	0	0	1	0	0	0	0

4

Question

Reference

1. Mahn-Soo Choi, Myung-Joong Hwang 10.1103/PhysRevA.82.025802
2. Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, S. M. Girvin, and R. J. Schoelkopf 10.1103/PhysRevA.69.062320

Thank you for listening