Invited talk: Korea University, Seoul.

Quantum Electrodynamic Aharonov-Bohm Effect

of a Superconducting Charge Qubit in Electromagnetic-Field-Free Region

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Dec. 17. 2015

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Dept. of Physics, Chonnam Natl. Univ., Korea


전남대학교 캠퍼스 안내도
CHONNAM NATIONAL UNIVERSITY CAMPUS MAP
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물리학과

Outlook

- Backgrounds
  - Aharonov-Bohm effect
  - Circuit quantum electrodynamics
- Quantum electrodynamic Aharonov-Bohm effect
- Summary

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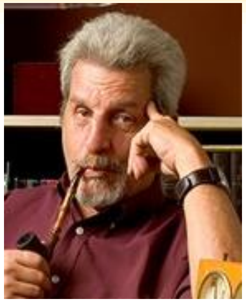
Aharonov-Bohm Effect



Photographs are taken from “Wikimedia”

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 115, NO. 3 AUGUST 1, 1959

Significance of Electromagnetic Potentials in the Quantum Theory

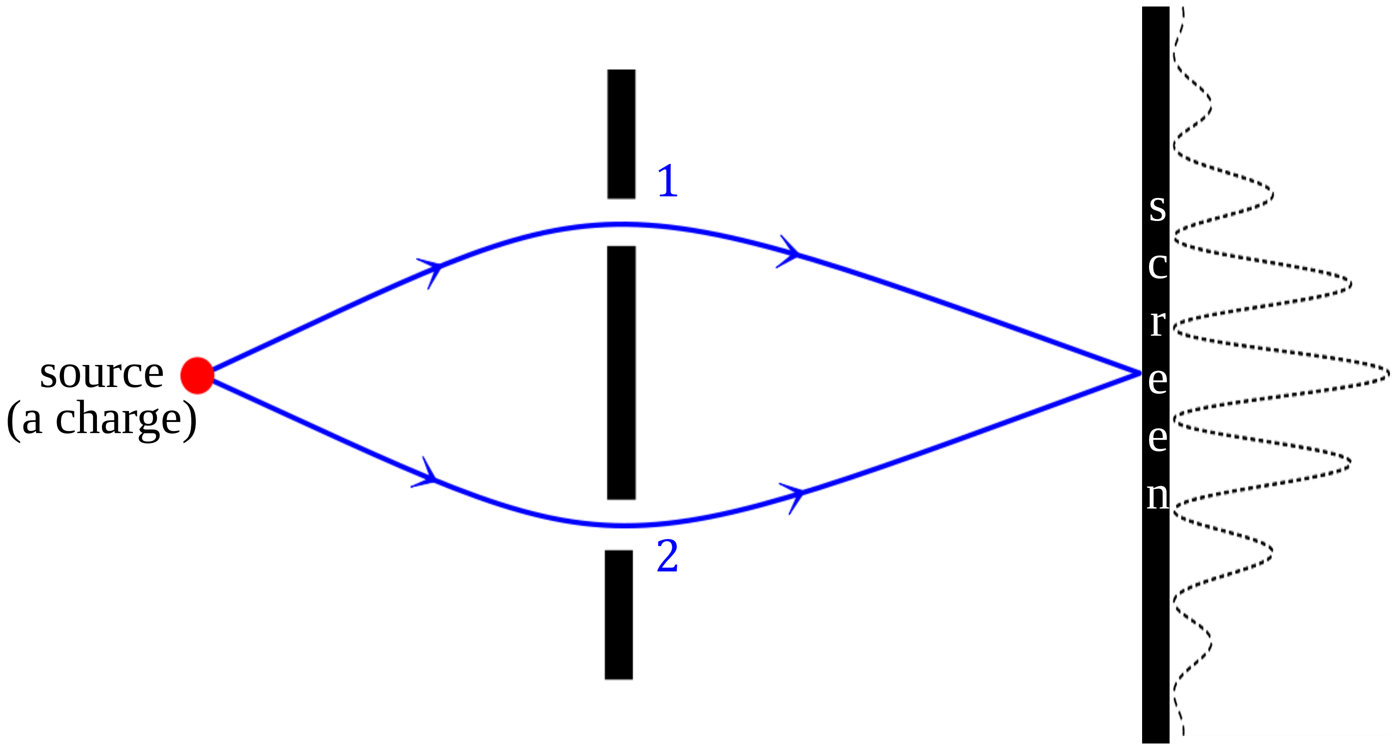
Y. AHARONOV AND D. BOHM
H. H. Wills Physics Laboratory, University of Bristol, Bristol, England

(Received May 28, 1959; revised manuscript received June 16, 1959)

In this paper, we discuss some interesting properties of the electromagnetic potentials in the quantum domain. We shall show that, contrary to the conclusions of classical mechanics, there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish. We shall then discuss possible experiments to test these conclusions; and, finally, we shall suggest further possible developments in the interpretation of the potentials.

Magneto**static** Aharonov-Bohm Effect

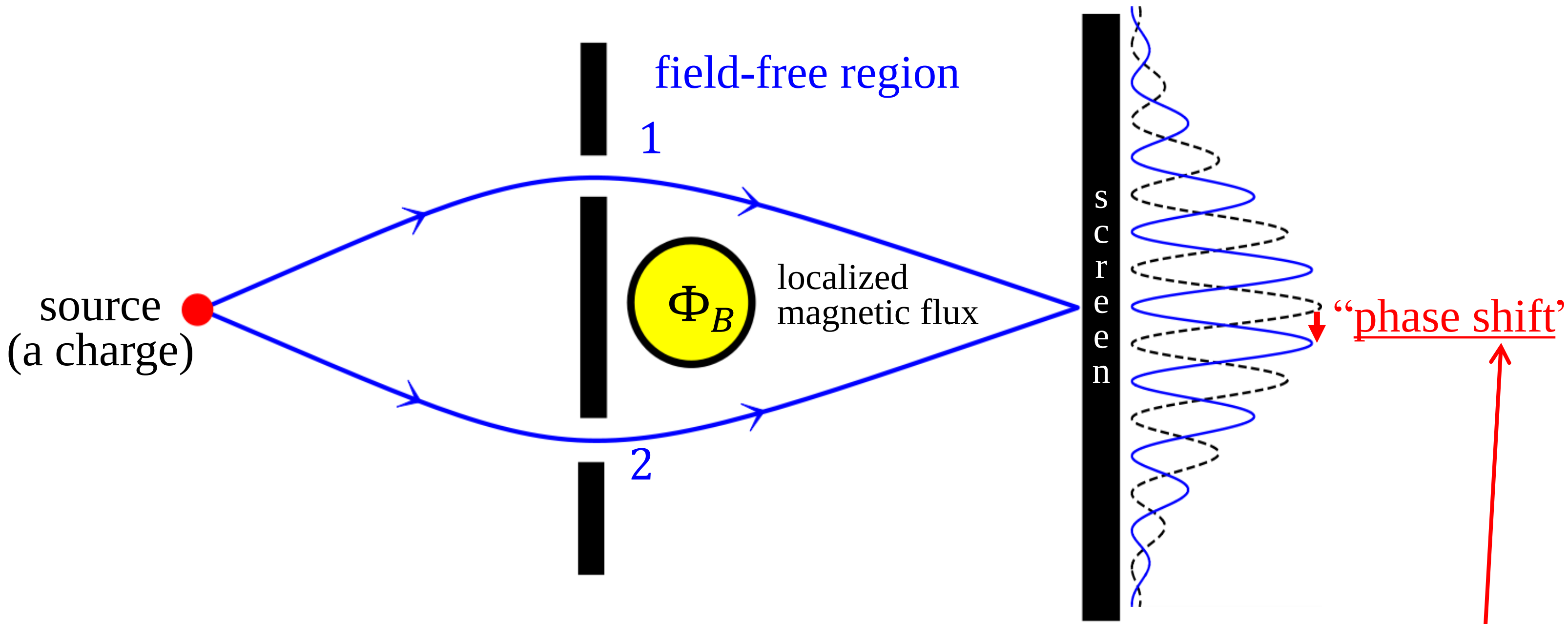
- Double slit experiment -



Superposition of wave function at the screen : $\psi = \psi_1 + \psi_2$

Magneto**static** Aharonov-Bohm Effect

- Double slit experiment with a solenoid -



Superposition of wave function at the screen : $\psi(\mathbf{x}) = \psi_1 e^{-\frac{ie}{\hbar c}\int_1^x \mathbf{A}\cdot d\mathbf{x}} + \psi_2 e^{-\frac{ie}{\hbar c}\int_2^x \mathbf{A}\cdot d\mathbf{x}}$

Relative phase : $\Delta\theta = \frac{e}{\hbar c}\left(\int_2^x \mathbf{A}\cdot d\mathbf{x} - \int_1^x \mathbf{A}\cdot d\mathbf{x}\right) \Rightarrow \Delta\theta = \frac{e}{\hbar c}\oint \mathbf{A}\cdot d\mathbf{x} = \frac{e}{\hbar c}\int \mathbf{B}\cdot d\mathbf{a} = \frac{e\Phi_B}{\hbar c}$

"**Static**" vector potential

Φ_B : magnetic flux in the solenoid.

Magneto**static** Aharonov-Bohm Effect

An experiment : A. Tonomura et al., PRL 56, 24 (1986).

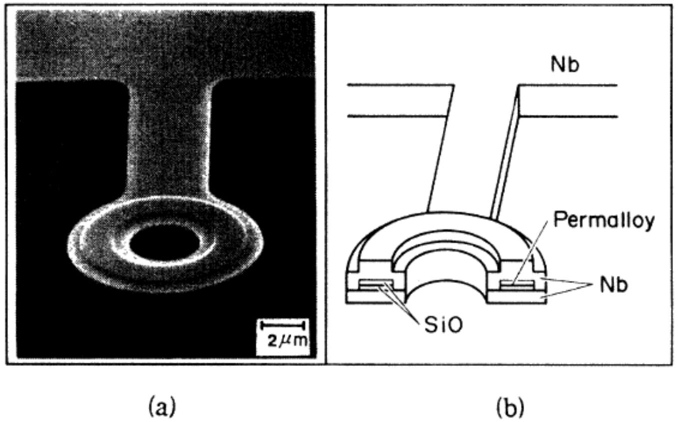


FIG. 2. Toroidal magnet. (a) Scanning electron micrograph; (b) diagram. The toroid is connected to a Nb plate by a tiny bridge for high thermal conductivity.

A photograph is taken from “www.dongascience.com”.

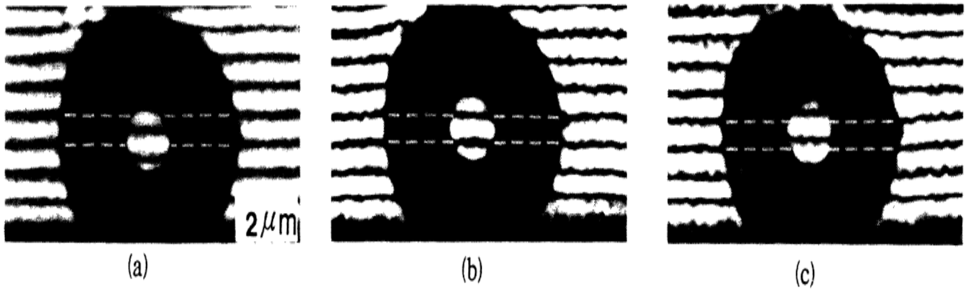


FIG. 6. Interference micrographs of a toroidal magnet at low temperatures. (a) $T = 4.5$ K (phase amplification, 1×); (b) $T = 4.5$ K (phase amplification, 2×); (c) $T = 15$ K (phase amplification, 2×). The enclosed flux is quantized in units of $h/2e$ when $T < T_c$ ($= 9.2$ K). The number of fluxons is odd.

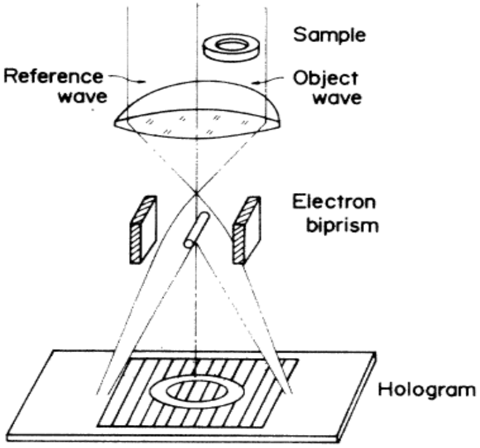


FIG. 3. Electron-optical system for hologram formation.

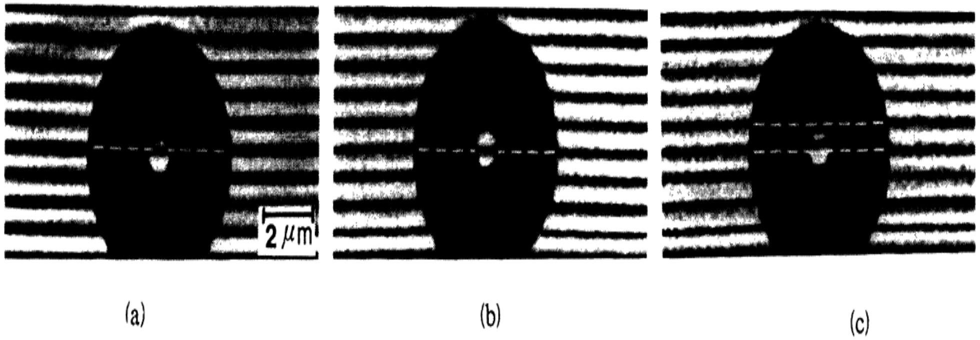
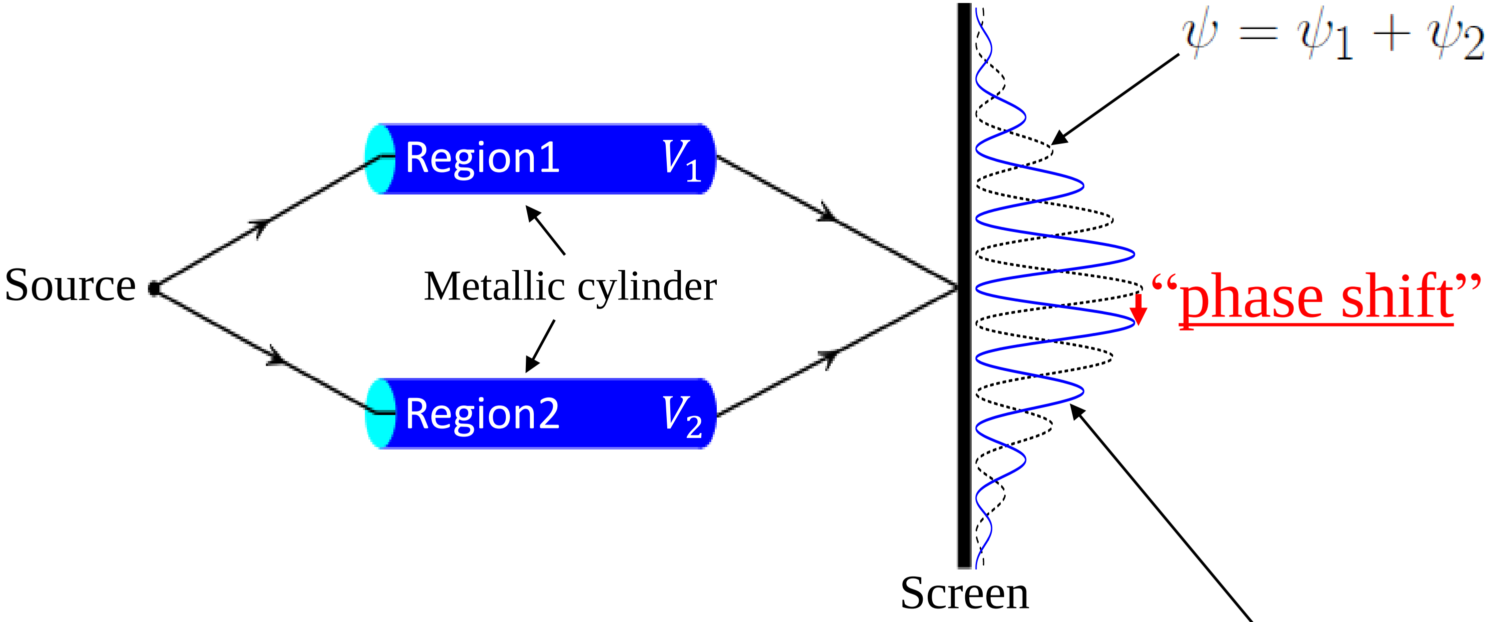


FIG. 7. Interference micrographs of a toroidal magnet at low temperatures. (a) $T = 4.5$ K (phase amplification, 1×); (b) $T = 4.5$ K (phase amplification, 2×); (c) $T = 15$ K (phase amplification, 2×). The number of fluxons is even.

Electro**static** Aharonov-Bohm Effect

Superposition of the wave function without the scalar potential :

$$\psi = \psi_1 + \psi_2.$$



Superposition of the wave function with the scalar potential :

$$\psi = \psi_1 e^{-\frac{ie}{\hbar}\int_1 V_1 dt} + \psi_2 e^{-\frac{ie}{\hbar}\int_2 V_2 dt}.$$

Relative phase : $\Delta\theta = \frac{e}{\hbar}\left(\int_2 V_2 dt - \int_1 V_1 dt\right) = \frac{e}{\hbar}\oint V dt$

Electro**static** scalar potential

Aharonov-Bohm Effect

Static potentials affect an electron (or charged particle) in electromagnetic-field-free region

Observable intriguing quantum phenomena

Outlook

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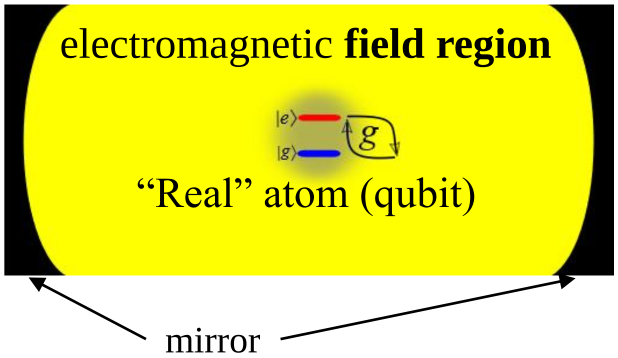
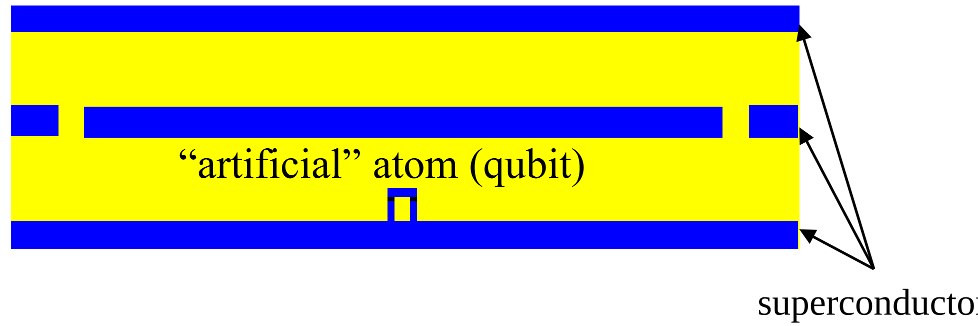
Cavity/Circuit Quantum Electrodynamics (QED)

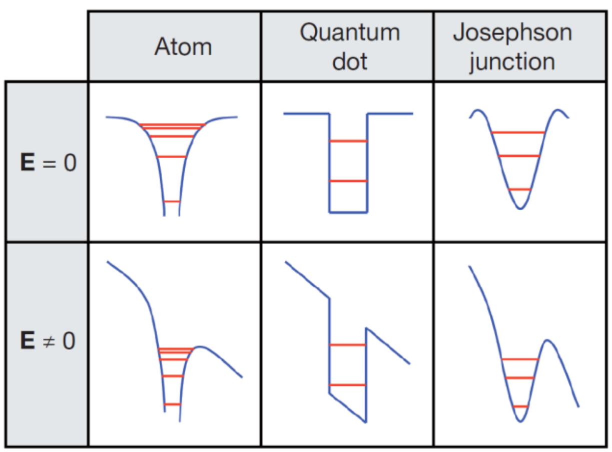
➡ Study of the **interaction** between **light** and **matter**!

Cavity QED

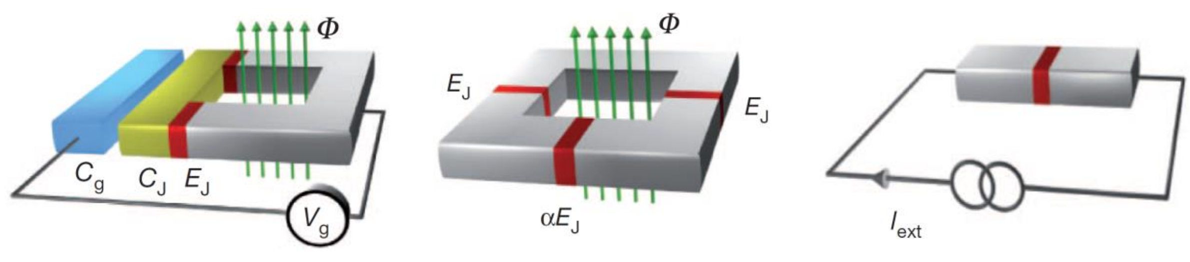


Circuit QED



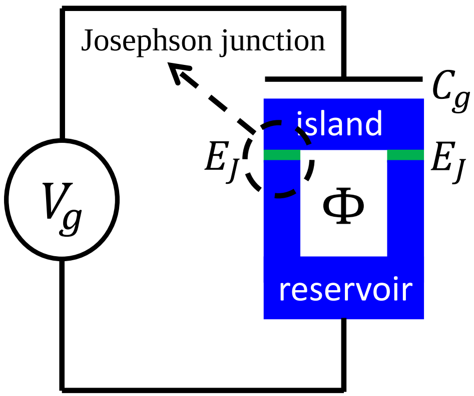


Charge qubit | Flux qubit | Phase qubit



Figures are taken from You & Nori, Nature2011.

Superconducting charge qubit



Cooper pair box (an artificial atom)

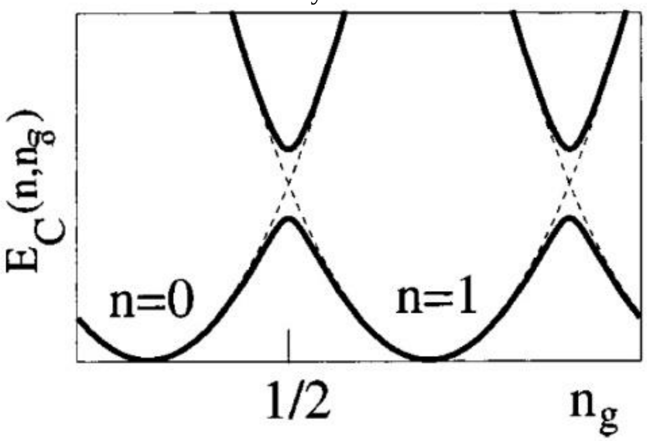
$$H_A = \sum_{n=0} E_c\left(n - n_g\right)^2 \ |n\rangle\langle n| - \frac{E_J}{2}\left(|n+1\rangle\langle n| + |n\rangle\langle n+1|\right)$$

E_C : single Cooper pair charging energy

Φ : magnetic flux (control Josephson energy)

n_g : gate charge number

Makhlin-Rev.Mod.Phys. 2001.



Qubit

$$H_q = \frac{\hbar\omega_a}{2}\hat{\sigma}_z$$

Transition frequency of the qubit

$$\omega_a = \sqrt{\left[E_c(1-2n_g)\right]^2 + E_J^2}$$

Eigenvector of the qubit

$$|g\rangle = \cos\frac{\gamma}{2}\ |0\rangle + \sin\frac{\gamma}{2}\ |1\rangle$$

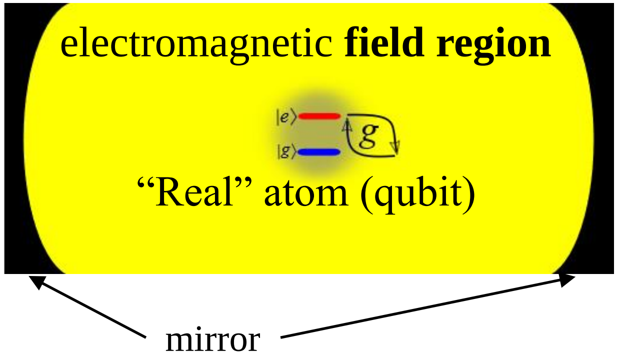
$$|e\rangle = -\sin\frac{\gamma}{2}\ |0\rangle + \cos\frac{\gamma}{2}\ |1\rangle$$

$$\gamma = \arctan\left(\frac{E_J}{E_c(1-2n_g)}\right)$$

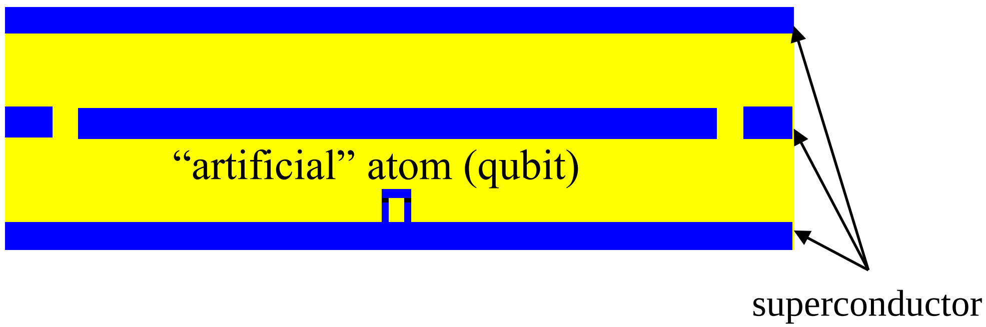
Cavity/Circuit Quantum Electrodynamics (QED)

➡ Study of the **interaction** between **light** and **matter**!

Cavity QED



Circuit QED



Total Hamiltonian :

$$H = H_{atom} + H_{cavity} + H_{int}$$

Interaction Hamiltonian :

$$H_{int} = -\vec{d} \cdot \vec{E}$$

"Local" interaction !!

$\vec{E}$: electric field in the cavity.
$\vec{d}$: electric dipole moment of the atom.

Jaynes-Cummings model

Total Hamiltonian :

atom / cavity / interaction

$$H = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \hbar g(\sigma_+ + \sigma_-)\left(a + a^\dagger\right)$$

RWA

$a^\dagger\sigma_+ \,,\ \sigma_- a$ counter-rotating

$a^\dagger\sigma_- \,,\ \sigma_+ a$ rotating

Jaynes-Cummings Hamiltonian :

$$H_{JC} = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \hbar g\left(\sigma_+ a + a^\dagger\sigma_-\right)$$

Eigenvalue

$$E_{0g} = -\frac{\hbar\Delta}{2}$$

$$E_{n\pm} = \pm\frac{\hbar}{2}\sqrt{4g^2(n+1) + \Delta^2}$$

$\Delta = \omega_a - \omega$ (detuning)

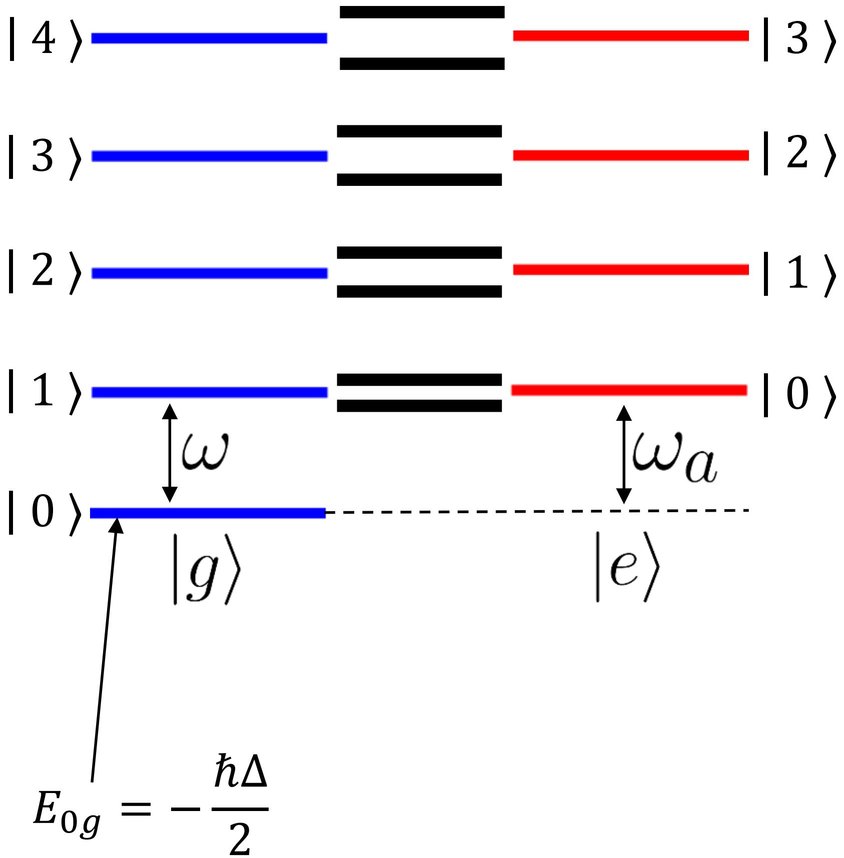
Eigenvector

$$|n+\rangle = \cos\beta\,|e,n\rangle + \sin\beta\,|g,n+1\rangle$$

$$|n-\rangle = -\sin\beta\,|e,n\rangle + \cos\beta\,|g,n+1\rangle$$

$$\tan 2\beta = \frac{2g\sqrt{n+1}}{\Delta}$$

Jaynes-Cummings Model: Resonant Case ($\Delta = 0$)



$$E_{n\pm} = \pm\frac{\hbar}{2}\sqrt{4g^2(n+1) + \Delta^2}$$

$$|n+\rangle = \cos\beta\,|e,n\rangle + \sin\beta\,|g,n+1\rangle$$

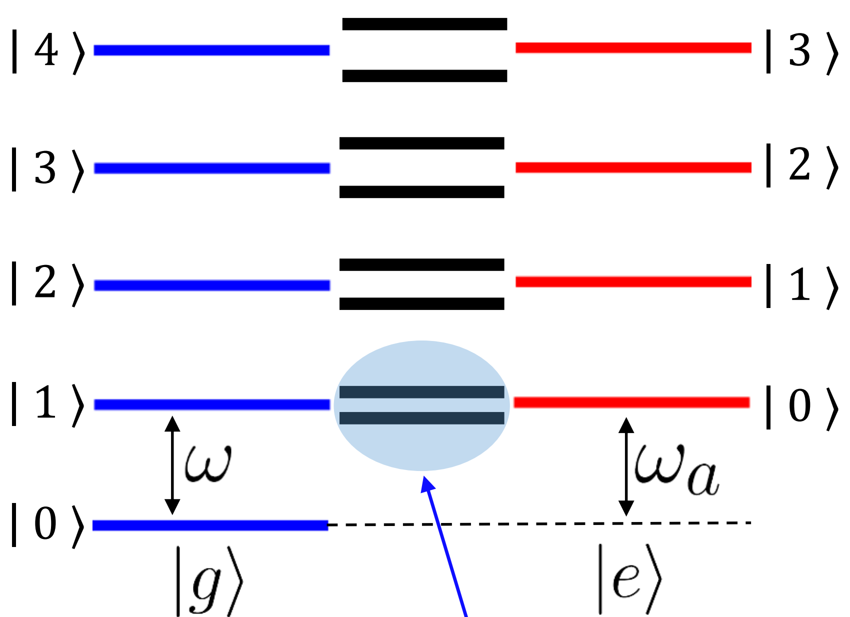
$$|n-\rangle = -\sin\beta\,|e,n\rangle + \cos\beta\,|g,n+1\rangle$$

Jaynes-Cummings Model: Resonant Case ($\Delta = 0$)

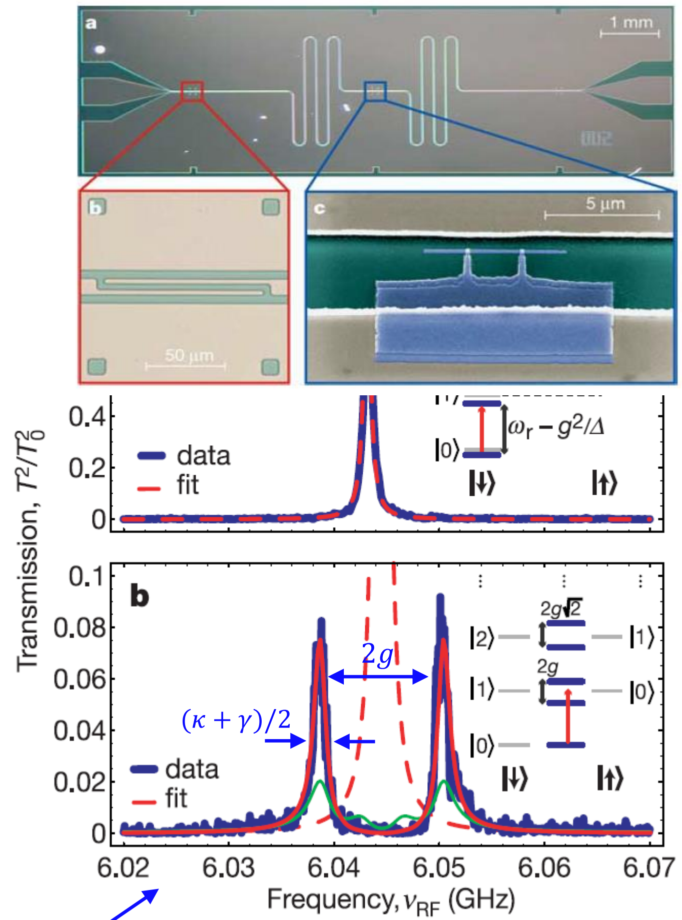
Circuit QED is in strong coupling ($g \gg \kappa, \gamma$) regime.

κ: cavity-decay rate
γ: qubit-decay rate

$$E_{n\pm} = \pm \frac{\hbar}{2}\sqrt{4g^2(n+1) + \Delta^2}$$



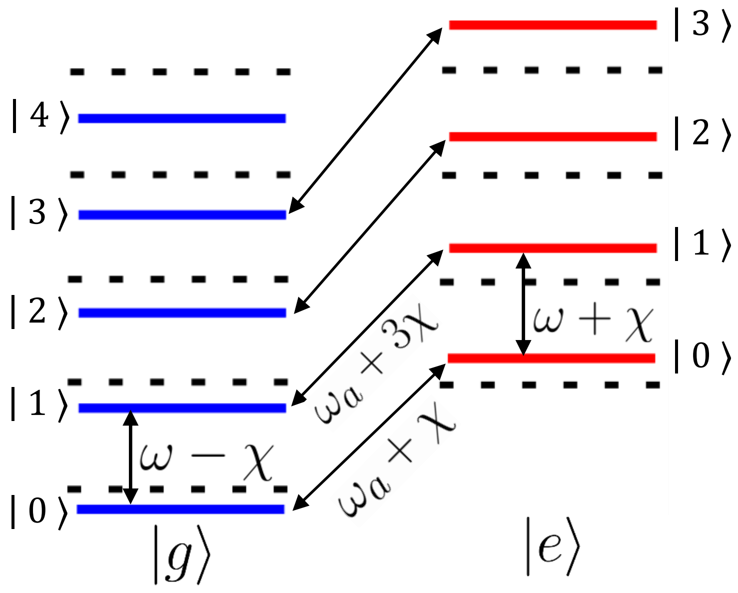
Wallraff,Nature 2004



Vacuum Rabi mode splitting was observed!!

Jaynes-Cummings Model: Dispersive Limit ($\Delta \gg g$)

$$H_d = \hbar\omega\left(a^\dagger a + 1/2\right) + \hbar\omega_a\sigma_z/2 + \hbar\chi\left(a^\dagger a + 1/2\right)\sigma_z$$

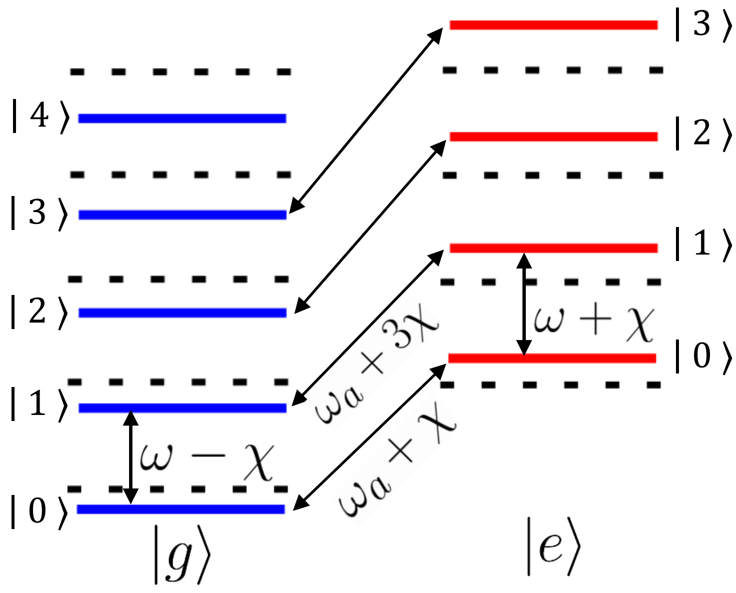


- Qubit-state-dependent cavity frequency shift
  - ➡ Quantum non demolition measurement for qubit state
- Lamb & AC stark shift
  - ➡ Resolving photon number states

$\chi = g^2/\Delta$ is the dispersive shift.

Jaynes-Cummings Model: Dispersive Limit ($\Delta \gg g$)

$$H_d = \hbar\omega\left(a^\dagger a + 1/2\right) + \hbar\omega_a\sigma_z/2 + \hbar\chi\left(a^\dagger a + 1/2\right)\sigma_z$$



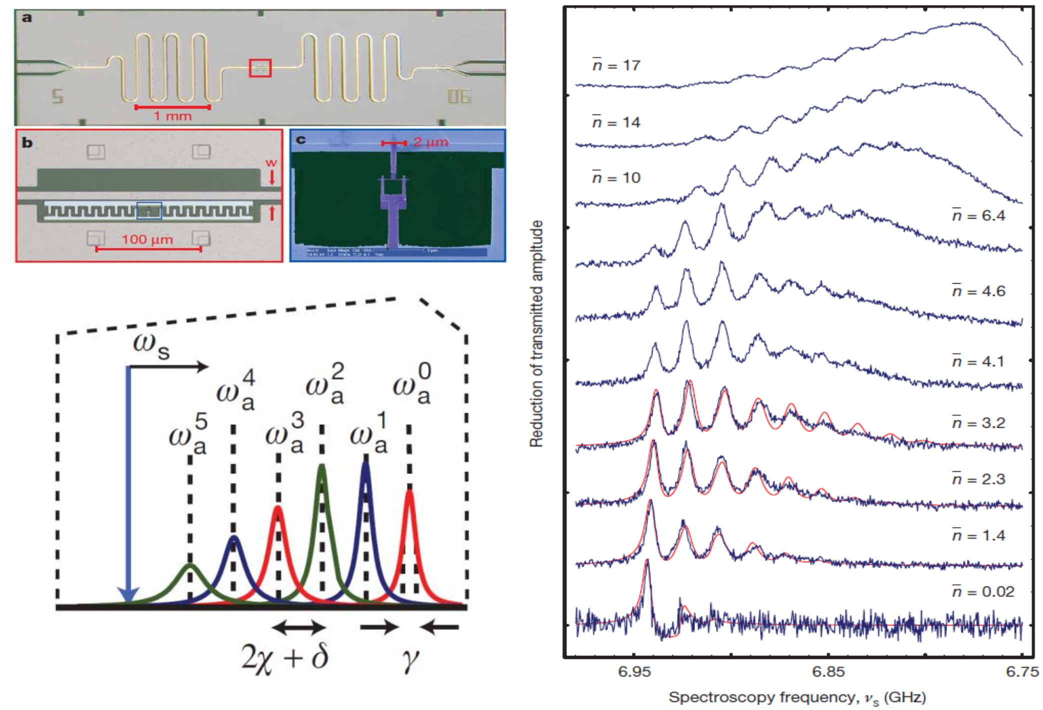
$\chi = g^2/\Delta$ is the dispersive shift.

Vol 445|1 February 2007|doi:10.1038/nature05461 nature

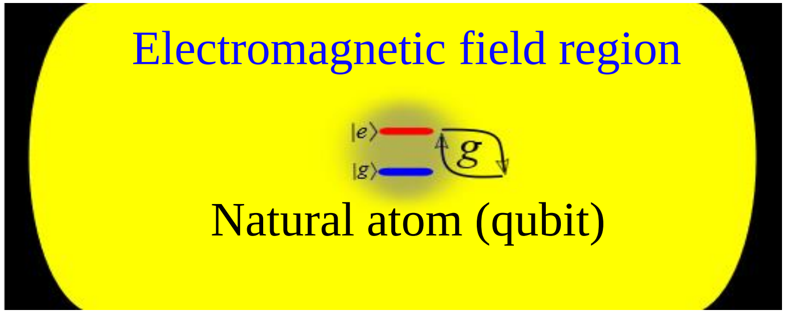
LETTERS

Resolving photon number states in a superconducting circuit

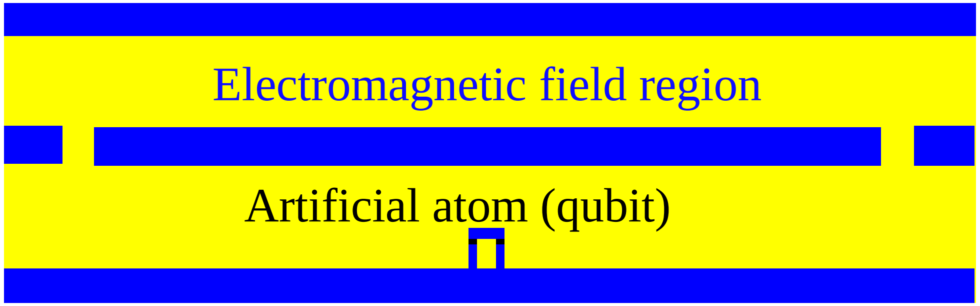
D. I. Schuster[1]*, A. A. Houck[1]*, J. A. Schreier[1], A. Wallraff[1]†, J. M. Gambetta[1], A. Blais[1]†, L. Frunzio[1], J. Majer[1], B. Johnson[1], M. H. Devoret[1], S. M. Girvin[1] & R. J. Schoelkopf[1]



Cavity/Circiut Quatum Electrodynamics



Cavity quantum electrodynamics



Circuit quantum electrodynamics

- An atom(qubit) is located in electromagnetic field region!!
- Local interaction between the atom and the electromagnetic field in cavity

Outlook

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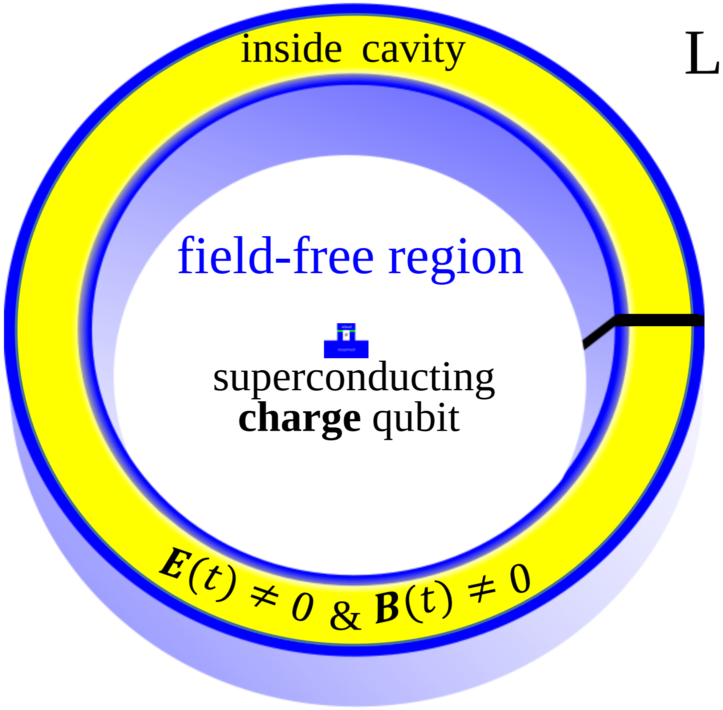
Quantum Electrodynamic Aharonov-Bohm effect

A charge qubit is located in the electromagnetic-field [***E(t)*** & ***B(t)***]-free region.



schematic setup

Lagrangian:

$$L = L_{charge} + L_{cavity} + L_{int}.$$

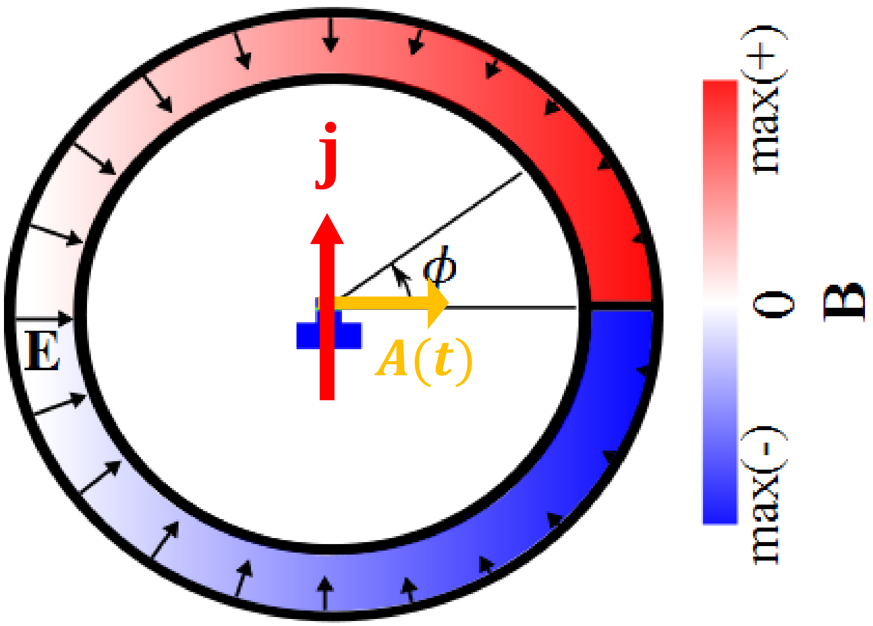
Lorentz-covariant interaction :

$$L_{int} = \frac{1}{c}\int \mathbf{j}\cdot\mathbf{A}\,d\tau - \int \rho V\,d\tau\,.$$

ρ : charge density of the qubit.
$\mathbf{j}$: current density of the qubit.

The **time-dependent potential** affects a charge qubit

Potentials are generated by the cavity



the lowest mode

electromagnetic field [**E** & **B**]

⇓ boundary condition

surface charge (σ_s) & current ($\boldsymbol{K}_s$) density

$$V(\boldsymbol{r},t) = \int \frac{\sigma_s(\boldsymbol{r}',t-R/c)}{R}da$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{1}{c}\int \frac{\boldsymbol{K}_s(\boldsymbol{r}',t-R/c)}{R}da$$

Potentials at the qubit location

scalar potential

$$\hat{V}(t) = -V_0\left[\hat{a}e^{-i(\omega t-\theta)} + \hat{a}^\dagger e^{i(\omega t-\theta)}\right],$$

$$V_0 \propto \frac{\hbar\omega}{|e|}\sqrt{k\delta z}\,.$$

vector potential

$$\boldsymbol{A}(t) = A(t)\hat{x}\,.$$

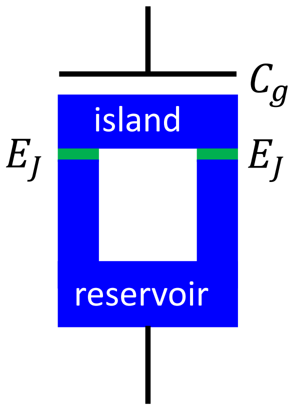
$$\mathbf{j}\cdot\boldsymbol{A}(t) = 0$$

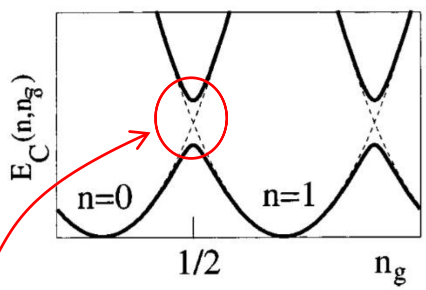
We can ignore the interaction by the vector potential.

Interaction

$$H = H_{qubit} + H_{cavity} + H_{int}$$

Superconducting charge qubit





Makhlin-Rev.Mod.Phys. 2001.

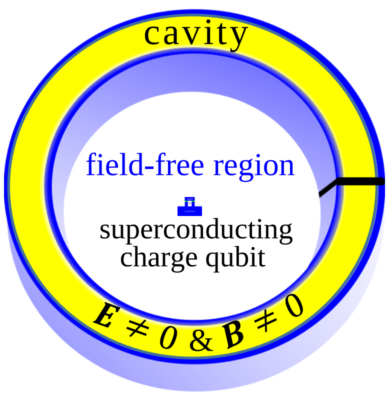
$$|e\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle),$$

$$|g\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle).$$

Charge operator

$$\hat{q} = -2|e||1\rangle\langle 1| = -|e|(\hat{\sigma}_+ + \hat{\sigma}_-),$$

where $\hat{\sigma}_+ = |e\rangle\langle g|$, $\hat{\sigma}_- = |g\rangle\langle e|$.



$$H_{int} = \hat{q}\hat{V}$$

electrodynamic potential

$$H_{int} = \hbar g\,(\hat{\sigma}_+ + \hat{\sigma}_-)\left(\hat{a}e^{i\theta} + \hat{a}^\dagger e^{-i\theta}\right),$$

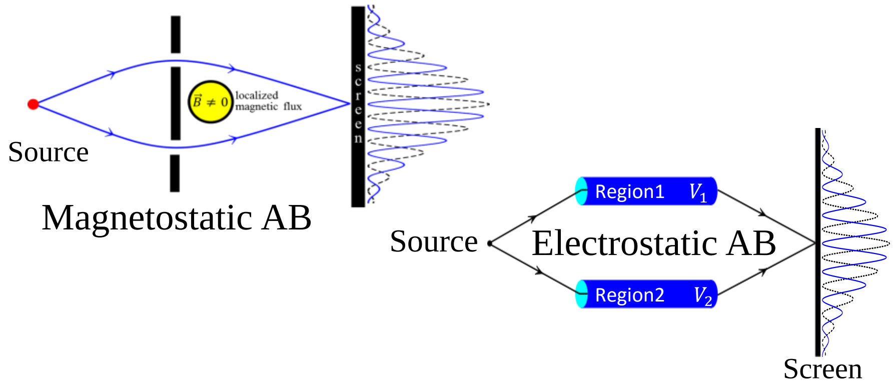
with the interaction energy : $\hbar g = -\frac{qV_0}{2}$.

Y.-W. Kim and K. Kang

Department of Physics, Chonnam Nat. Univ.

Electrodynamic Aharonov-Bohm effect

"static" Aharonov-Bohm effect



The paths of a charged particle are in electro-magnetic-field-free region.

The "static" potentials affect the behavior of the charged particle.

⇓

"Aharonov-Bohm phase"

$$\theta_{AB} = \frac{e}{\hbar c} \oint \boldsymbol{A} \cdot d\boldsymbol{x}$$

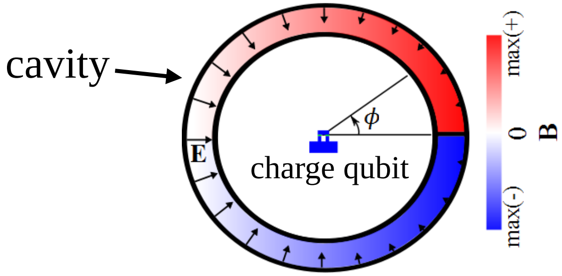
Magnetostatic AB

$$\theta_{AB} = \frac{e}{\hbar} \oint V dt$$

Electrostatic AB

"There is a shift in interference pattern"

"electrodynamic" Aharonov-Bohm effect (our work)



A charge qubit is located in the electro-magnetic-field-free region.

The "electrodynamic" potential (V(t)) affects the charge qubit.

⇓

"Nonlocal exchange of the charge and the photon"

$$H_{int} = \hbar g\left(\hat{\sigma}_+ \hat{a} e^{i\theta} + \hat{a}^\dagger \hat{\sigma}\_ e^{-i\theta} \right),$$

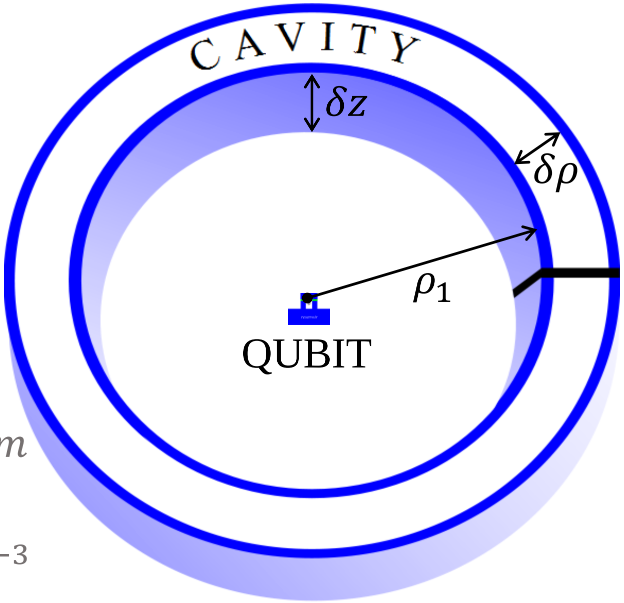
and $\hbar g = -\frac{qV_0}{2}$.

Y.-W. Kim and K. Kang

Department of Physics, Chonnam Nat. Univ.

Electrodynamic Aharonov-Bohm effect: interaction strength

Interaction energy : $\hbar g = -\frac{qV_0}{2}.$ $\Rightarrow$ $\frac{g}{\omega} = f\left(\frac{\delta\rho}{\rho_1}\right)\sqrt{\frac{\alpha\delta z}{\rho_1}}$



$\rho_1 = 2.50mm$
$\delta\rho\ /\rho_1 = 0.1$
$\delta z/\rho_1 = 10^{-3}$
$\omega/2\pi = 9.09$ GHz

Interaction strength:
$g/2\pi = 5.27$ MHz.

This effect can be strong enough to be observed.

Observable phenomena by the QED AB effect

- ✓ vacuum Rabi splitting
- ✓ vacuum Rabi oscillation
- ✓ etc.

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Summary

- ✓ We predict the electrodynamic Aharonov-Bohm effect with time-dependent potential under the condition that a charge is placed in a region free of electromagnetic field.

- ✓ This effect can be realized with a superconducting charge qubit interacting nonlocally with a cavity electromagnetic field.

- ✓ All the exotic phenomena with cavity QED can be observed in this electrodynamic Aharonov-Bohm setup.

Thank you
for your attention!!