Condensed Matter Seminar (8 May 2019)

Quantum Theory of Measurement

An Introduction

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MEASUREMENT TYPES

- Projection measurement
- Weak continuous measurement
- Weak measurement

- Quantum non-demolition (QND) measurement
- POVM (positive operator-valued measure)

- Standard quantum limit (SQL, shot noise limit)
- Precision measurement beyond SQL

Basic Picture

Von Neumann's Picture

- A "probe" is coupled to the "system".
- The probe is "read out".
- Both the system and probe are quantum.
- A *classical* device is assumed to read out the probe.

$$\hat{H}_{\text{total}} = \hat{H}_{\text{system}} + \hat{H}_{\text{probe}}(\hat{X}, \hat{P}) + g\hat{A} \otimes \hat{P}$$

$\hat{A} = \sum_a a \left|a\right\rangle \left\langle a\right|$ the quantity to measure

$\hat{X}$ the "scale" to read out

$\hat{P}$ conjugate to $\hat{X}$, $[\hat{X}, \hat{P}] \neq 0$

Ideal Case

- Measurement time is short enough; the coupling, g, is strong enough.

$$\left(\tau_m \sim \frac{1}{g}\right) \ll \tau_S\,, \tau_P$$

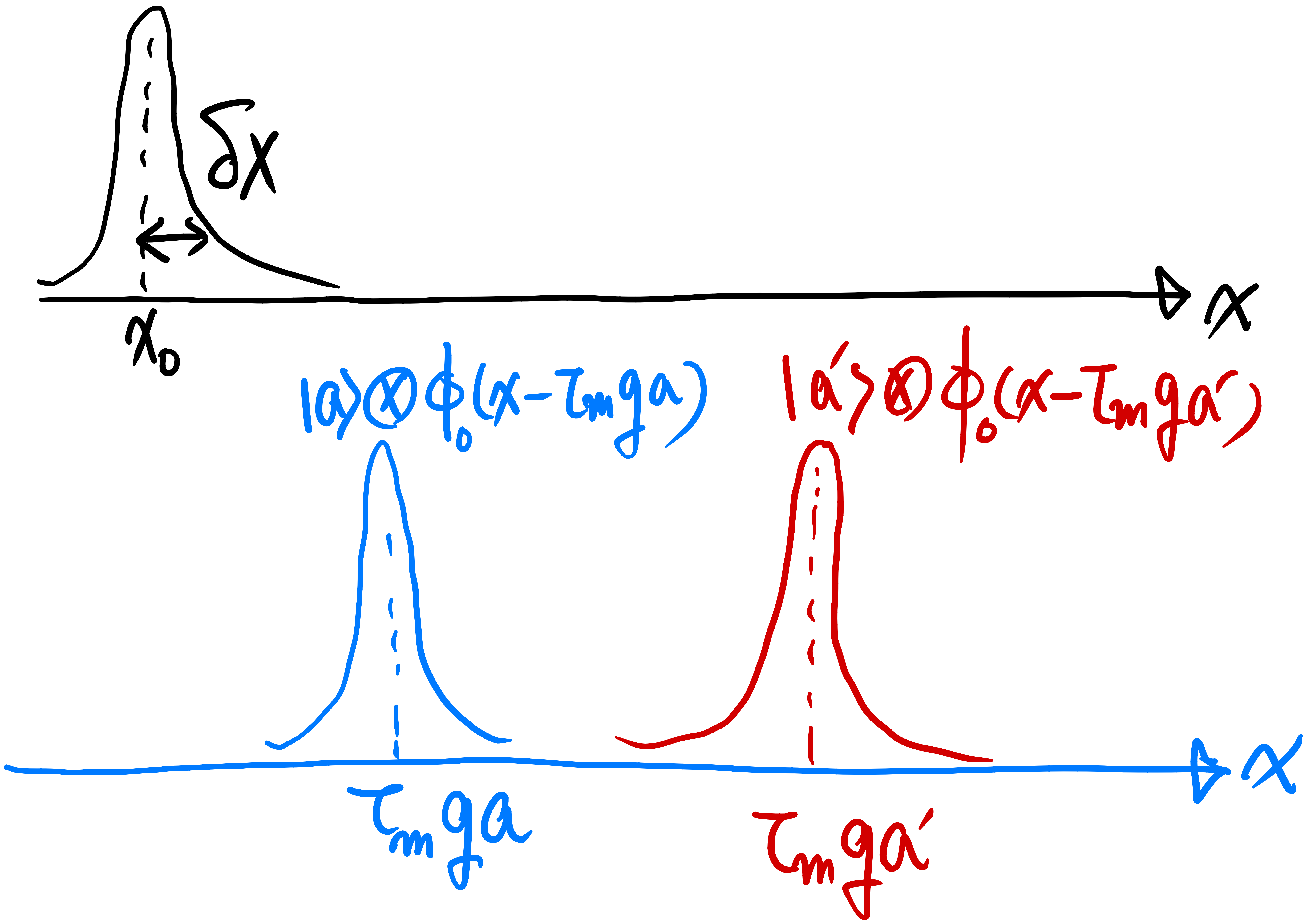
- The probe "position" discriminates well the eigenvalues of the observable in question.

$$g\tau_m\,\Delta a \gg \delta X$$

$$|\Psi_{\mathrm{total}}(0)\rangle = |\psi_{\mathrm{in}}\rangle \otimes |\phi_0\rangle$$

$$\hat{U}(t) := \exp\left[-it\hat{H}_{\mathrm{total}}\right] \approx \exp\left[-itg\hat{A} \otimes \hat{P}\right]$$

$$|\Psi_{\mathrm{total}}(\tau_m)\rangle = \hat{U}(\tau_m)\,|\psi_{\mathrm{in}}\rangle \otimes |\phi_0\rangle \approx \sum_a |a\rangle \otimes e^{-i\tau_m g a\hat{P}}\,|\phi_0\rangle$$



δx
x
x0
|a⟩⊗φ0(x−τmga)
|a′⟩⊗φ0(x−τmga′)
x
τmga
τmga′

Remaining Question

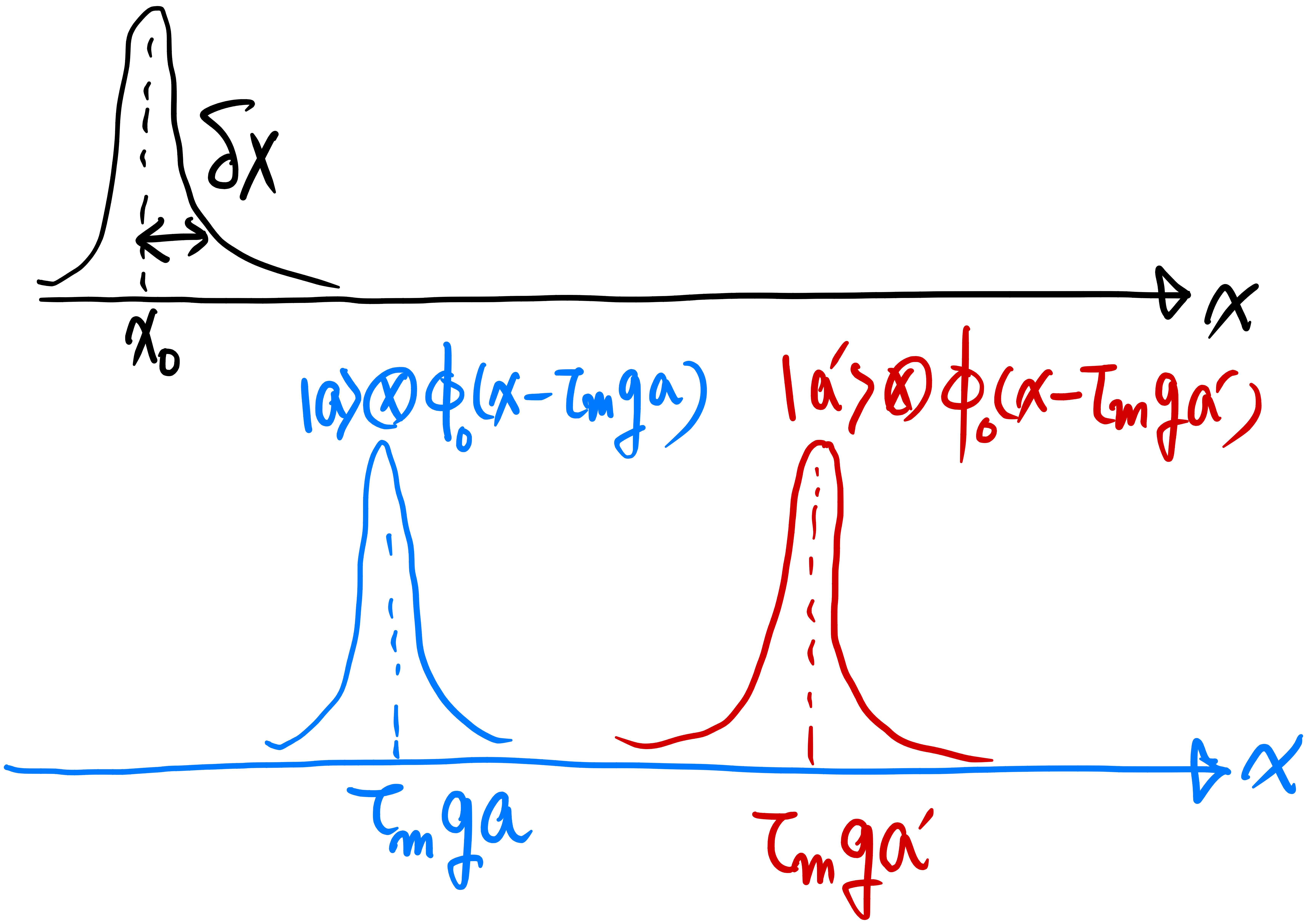
How to estimate the parameter?

For a given a,

$$\hat{U}(\tau_m) \approx \exp\left[-i\theta\hat{P}\right] , \quad \theta := \tau_m g a$$

READ-OUT TYPES

- Direct read-out
- Interferometer (e.g., Ramsey interferometer)



δx
x
x₀
|a⟩⊗φ₀(x−τ_m g a)
|a′⟩⊗φ₀(x−τ_m g a′)
x
τ_m g a
τ_m g a′

BEST STRATEGE

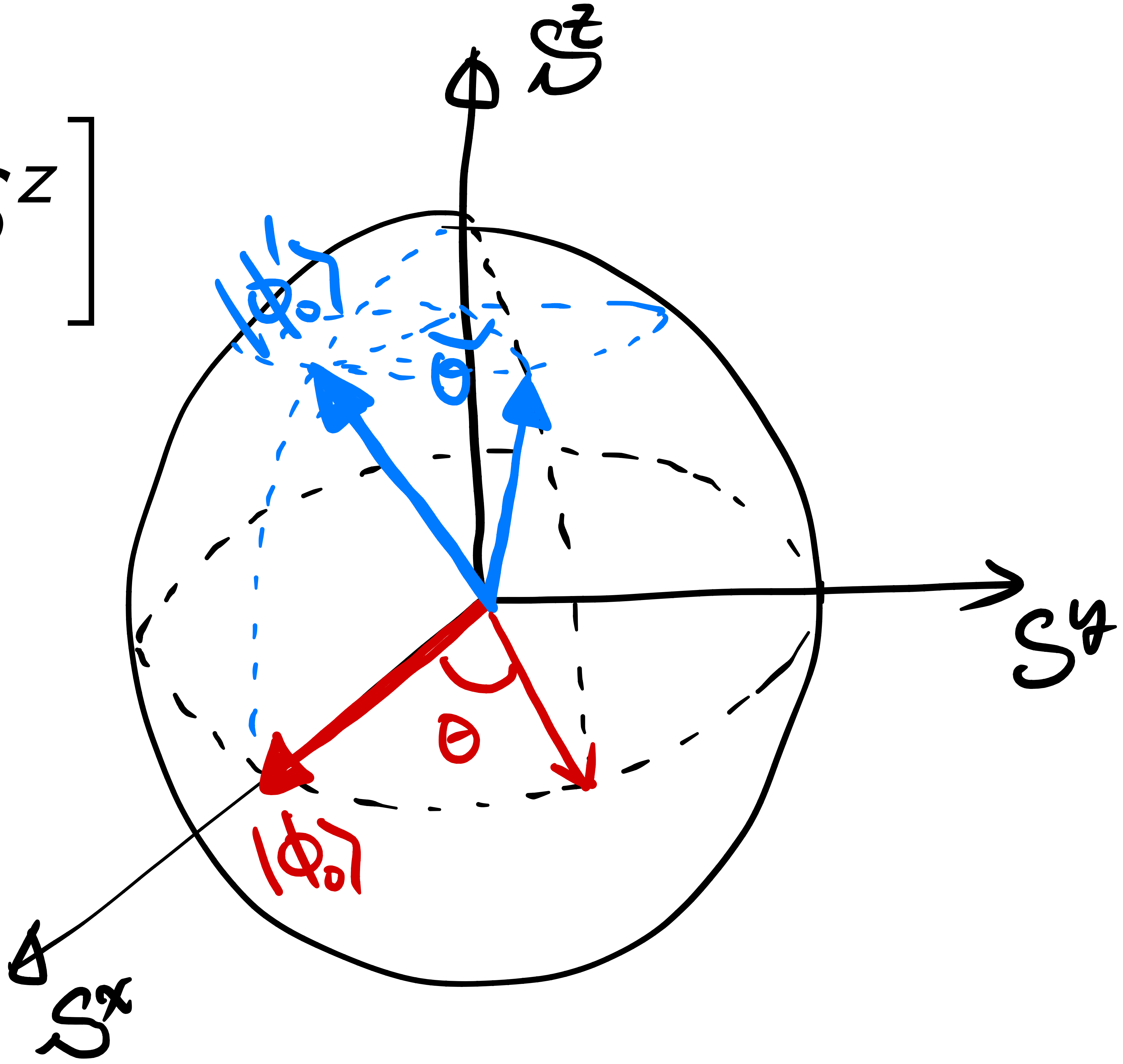
- Small variance of "position" in the probe state.
- Probe state sensitive to the "momentum" operator.

EXAMPLE

SPIN MEASUREMENT

$$H_{\text{int}} = g\hat{A} \otimes \hat{S}^z$$

$$\hat{U}(\tau_m) \approx \exp\left[-i\theta\hat{S}^z\right]$$



Standard Quantum Limit

(Shot-Noise Limit)

$$\delta\theta \sim \frac{1}{\sqrt{N}}$$

Quantum Non-Demolition Measurement

Principle and Applications

$$\hat{H}_{\text{total}} = \hat{H}_{\text{system}} + \hat{H}_{\text{probe}}(\hat{X}, \hat{P}) + g\hat{A} \otimes \hat{P}$$

$\hat{A} = \sum_a a \left|a\right\rangle \left\langle a\right|$ the quantity to measure

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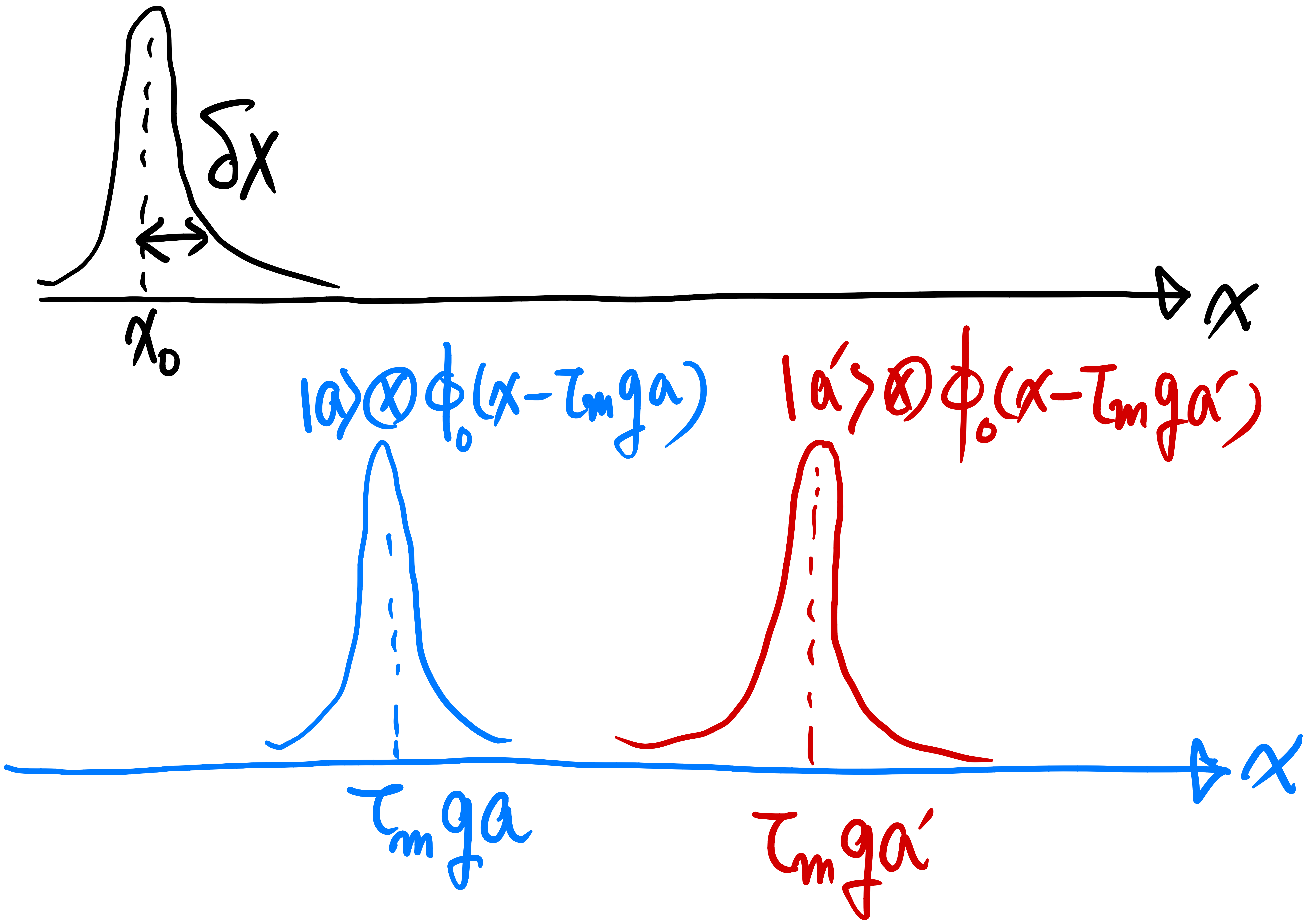
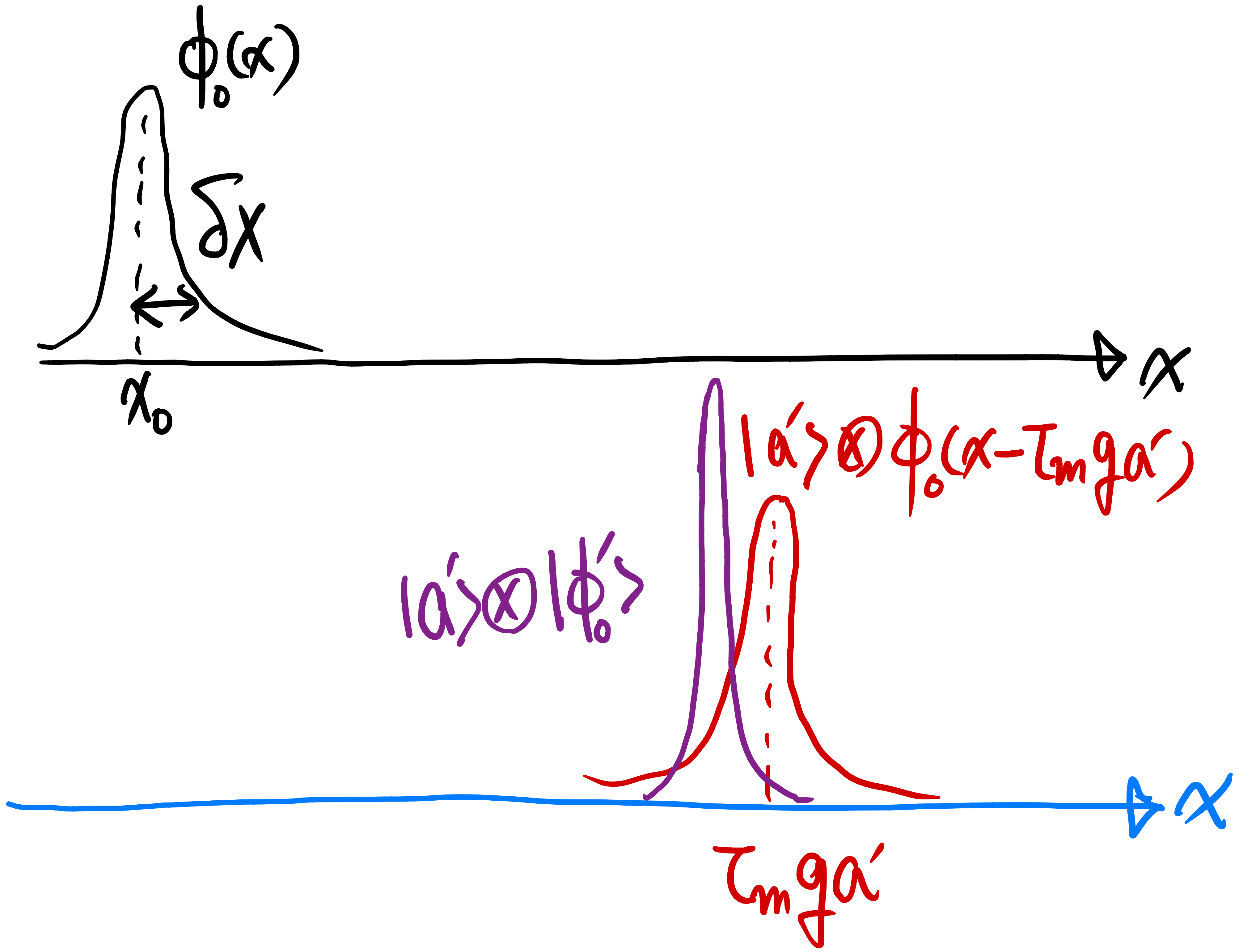


δx
x
x_0
$|a\rangle \otimes \phi_0(x - \tau_m g a)$
$|a'\rangle \otimes \phi_0(x - \tau_m g a')$
x
$\tau_m g a$
$\tau_m g a'$

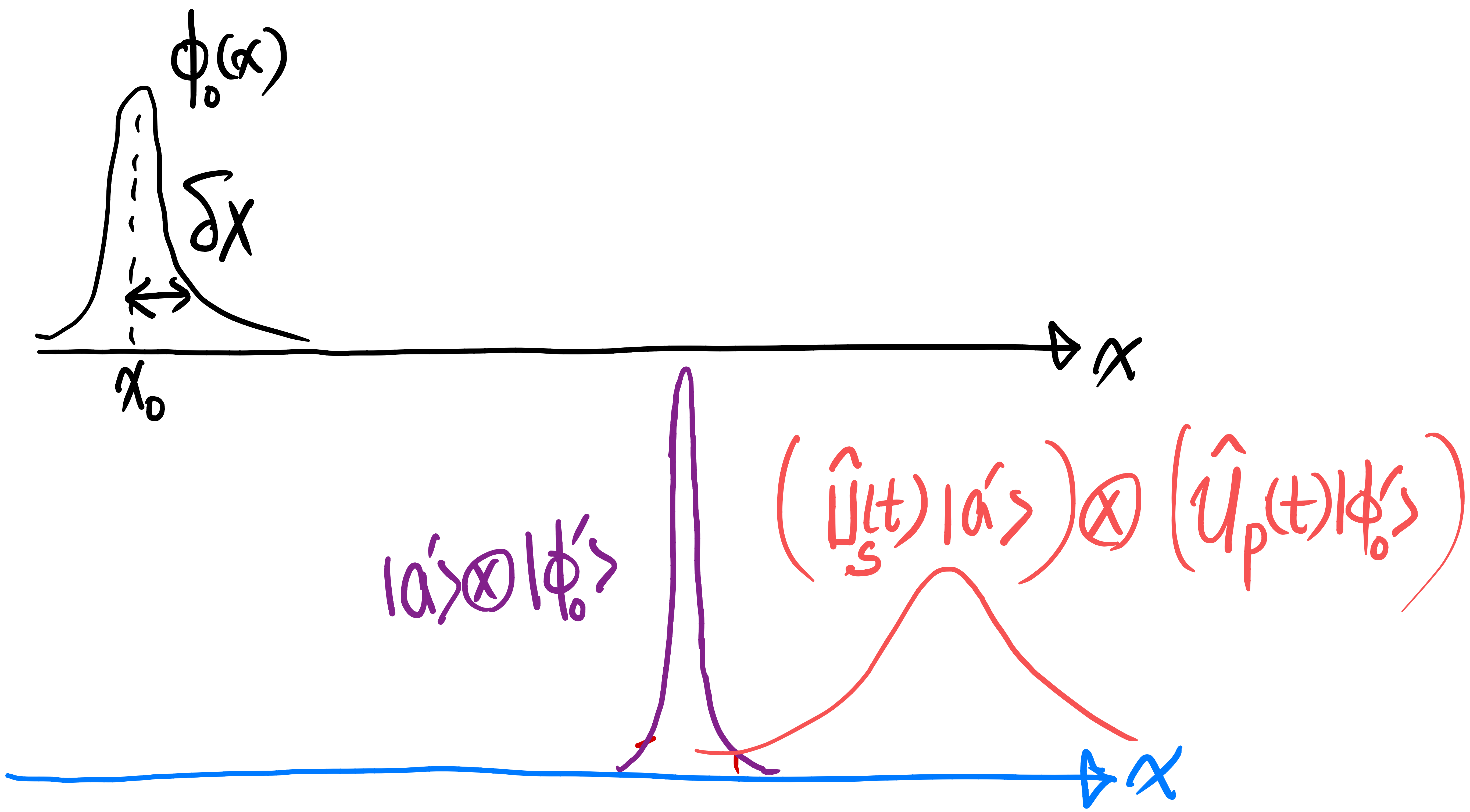
Right After One Measurement

Can one measure the system again?



Right Before Another Subsequent Measurement

Does it help?



Quantum Non-Demolition Measurement

Necessary Condition

$$\left[\hat{H}_S, \hat{A}\right] = 0$$

PROJECTIVE MEASUREMENT

VON NEUMANN MEASUREMENT

- Also known as "strong measurement" to be compared with the so-called "weak measurement"
- Sufficient information to distinguish *orthogonal* states
- Causes wave functions to "collapse"
- Lays base for the quantum mechanics

Weak Measurement

Weak Values

- Using the weak interaction of the system and the "probe"
- Insufficient to distinguish different (orthogonal) states
- Post selection is required
- Measured are "weak values," whose physical interpretation is not always transparent.

Weak Measurement

Information vs Disturbance

Weak values

Beyond Standard Quantum Limit

Entanglement Enhances Measurement Precision

Heisenberg Limit

The Ultimate Limit allowed in Quantum Mechanics

- Standard quantum limit (SQL, shot-noise limit) $\delta\phi \sim \frac{1}{\sqrt{N}}$
- Heisenberg limit $\delta\phi \sim \frac{1}{N}$