Internship Report

Fourth Year

Submitted by : Boris LAURENT

Strong measurement of a superconducting qubit in superconducting circuit QED systems

QCLab in Korea University

May 15th 2019 – August 15th 2019

Supervisors : Pr.Mahn Soo CHOI, Mr.Bernard GRUZZA

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Acknowledgements

Firstly, I would like to thank Yang Seongdeok, director of the college of science of Korea University, for allowing me in his institute.

Secondly, I would like to thank my supervisor, Prof. Mahn Soo Choi, for his teaching, for everything he taught me and for his availability throughout this internship.

Finally, I am very grateful for Prof. Nguyen, Thi Xuan Hoai, Mr. Seung Parg, Mr. Ha-Eum Kim and Mr. Hoang-Anh Le, whom I studied with in the laboratory QCLab of Korea University.

Résumé

Le but de ce travail est d'étudier la mesure forte d'un quantum bit supraconducteur. Pour ce faire, nous avons enquêté sur son fonctionnement et sur son environnement, c'est-à-dire les circuits électrodynamiques, auquel les qubits y sont couplés.

Suite aux propriétés de la superposition quantique sur lesquels repose le fonctionnement du quantum bit, nous ne pouvons pas lire les deux états du qubit en même temps. En réalisant une mesure forte, nous recherchons alors l'état immédiat de ce qubit.

Dans ce rapport, nous allons étudier les différents systèmes qui composent le qubit supraconducteur et le circuit. Puis, nous expliciterons l'énergie de ce circuit couplé au qubit. Enfin, nous utiliserons cette énergie pour déterminer la manière de mesurer la valeur d'un qubit.

Mots-clés

Transmon Qubit Circuit Electrodynamique Supraconducteur

Superposition quantique Mesure forte

Abstract

The aim of this work is to study the strong measurement of a superconducting quantum bit. To do so, we analysed both its operating system and its environment, which means we analysed the electrodynamic circuits the qubits are coupled into.

Due to the quantum superposition properties the qubit operates on, we are unable to read its two states at the same time. By realising a strong measurement, we are searching the instant state of this qubit.

In this report we will first study the different devices used to make the superconducting qubit and the QED system. Then we will determine the circuit energy coupled to the superconducting qubit. Lastly, we will use this energy to determine the way to measure the qubit state.

Keywords

Transmon Qubit QED Systems Superconduction

Quantum superposition Strong measurement

개요

본 연구는 초전도 회로 안에 있는 큐빗이 어떤 원리로 강한 측정이 이루어지는지에 대한 것이다.

양자 중첩 상태에 의해 큐빗은 중첩 상태를 가지게 되지만, 이 상태를 동시에 관측하기란 어렵다. 하지만 강한 축정을 통해 순간 큐빗 상태를 측정 할 수 있다.

이 보고서의 순서는 다음과 같다. 우선, 초전도회로 안에서 구현할 수 있는 두 종류의 큐빗들을 살펴본다. 그 후 다양한 실험 조건에서 큐빗과 초전도회로의 상호작용에서찾을 수 있는 의미있는 에너지 스케일을 정의하고 분석한다. 최종적으로 강한 축정을 이용할 수 있는 에너지 영역을 정의하고 이를 더 자세히 살펴본다.

키워드

트렌스몬 큐빗 QED 시스템 초전도 상태

양자 중첩 상태 강한 측정

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Glossary

TLS : Two-level system

A two-level system is a quantum system that can exist in any quantum superposition of two independent quantum states.

QED : Quantum Electrodynamics

Quantum electrodynamics is a quantum field theory of the electromagnetic force. It visualises the force between electrons or atoms as an exchange force arising from the exchange of virtual photons.

CPB : Cooper pair box

A Cooper pair box is an isolated superconducting volume that contains Cooper pair.

RF : Radio Frequency

Oscillation rate outside the cavity QED.

LO : Local Oscillator

Electronic oscillator used with a mixer to change the frequency signal.

IF : Intermediate Frequency

Frequency used as intermediate step for the transmission of the signal. It is also used with a mixer.

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Introduction

A quantum bit is the basic unit of quantum information. This bit works based on one of the quantum mechanics properties: quantum superposition. This gives the quantum bit a superposition of the state 1, and the state 0. Real applications in the coming years of the quantum bit are in super dense cooling, quantum teleportation and quantum computation. In this last topic, industries are interested into the quantum bit to have faster computation. Indeed, superposition of states means more combinations then shorter space to write information then faster computing.

Superconducting qubit is one of several types of qubit and this is the one we will study. According to its properties, we are interested in this work in measuring by strong measurement a superconducting qubit in a superconducting Quantum Electrodynamic (QED) system.

Doing research on this subject requires to know some scientific terms referring to the quantum electrodynamics. This is what we will introduce first. Then, we will determine the circuit QED energy coupled to the superconducting qubit and use this energy to determine the way to measure the qubit state by strong measurement.

I. Quantum information notions

In this part, we will introduce the different devices being studied and explain what a qubit is.

1. Devices

The superconductor circuit isn't only composed of a qubit. We then have to introduce the different components to make our superconducting qubit and also the components we need to realise our system.

1.1.1. Superconducting wire

A Cooper pair is a pair of electrons formed when the temperature of a metal is cooled down under critical temperature. Then the two electrons form a weak bond as we can see on the Figure 1, called a Cooper pair, which encounter less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way. Following the BCS[1] theory, it allows the superconductivity. The electrode carrying those Cooper pairs is a superconducting wire.

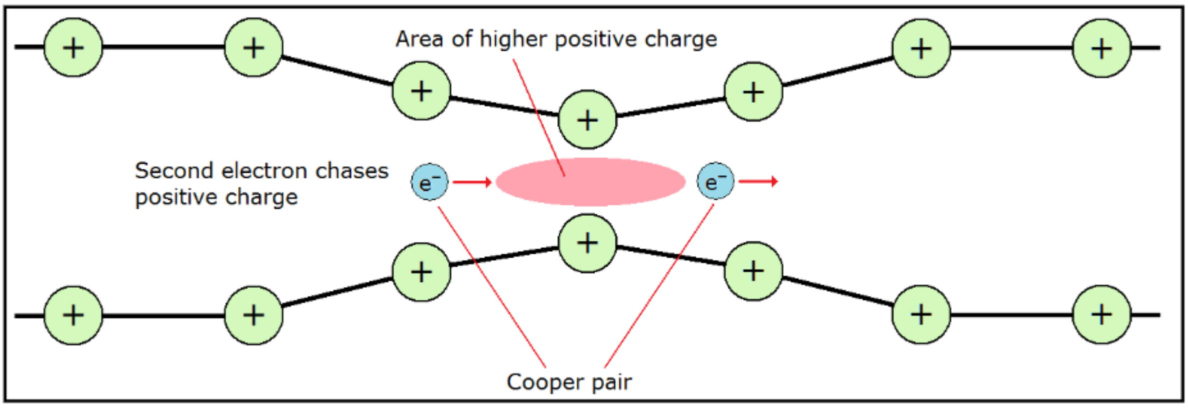

Figure 1 : Formation of a Cooper pair in a superconducting wire [1]

1 : Bardeen-Cooper-Schrieffer Theory

1.1.2. Josephson Junction

A Josephson junction is a device composed of two superconducting wires separated by a thin insulating layer as we can see on the second picture on the Figure 2.

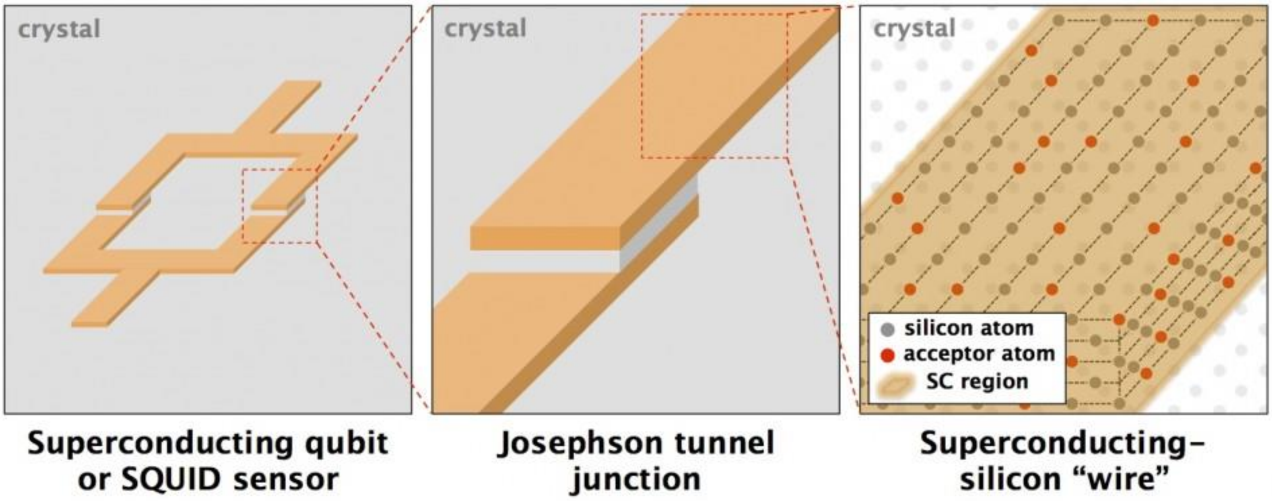

Figure 2 : Josephson junction [2]

Those two superconducting wires have a different wave function phase and the Cooper pair transfer from one superconductor to the other one by tunnelling effect. To control its effect, we use a Superconducting Quantum Interference Device, also wrote as SQUID that we can see on the first picture on the Figure 2, where we control the current of Cooper pair through the barrier with a magnetic.

2. Qubit

A qubit, or also called quantum bit, is a system that carries the digit in quantum information. Compared to the classical bit that takes a value of 0 or 1, the qubit uses the properties of quantum physics which means that the quantum bit can have two states: $|\,0>$ or $|\,1>$.

This property is called quantum superposition. It means a quantum system doesn't have one state but a combination of several basis states. For a qubit, the linear combination of the two basis states is:

$$|\,\psi> = \alpha\,|\,0> + \beta\,|\,1>$$

Equation 1: Linear combination of the qubit states [adapted from [2]]

When we measure a qubit, we get either the result 0, with probability $|\alpha|^2$, or the result 1, with probability $|\beta|^2$. Since the probabilities must sum to one, we have naturally $|\alpha|^2+|\beta|^2=1$. [3]

However, there are several types of qubits, for example, the quantum dots silicium whose the value of the qubit is made by the spin of an electron. In the following part, we will bring our attention to the charge qubit which will be essential in order to understand the type of quantum bit we use in our study.

1.2.1. Charge Qubit

To be able to have several states for our qubit, we need to get different energy levels. Those different energy levels correspond to an integer number of Cooper pairs N on a superconducting island, also known as cooper pair box.

To determine the energy of the system, we first need to know the energy in this Cooper pair box. Since there are many Cooper pairs and that the box is big, 100nm on each side, we can therefore model the Cooper pair box as a capacitor.

The expression of the energy in a capacitor is given by the following equation:

$$E = \frac{Q^2}{2C} = \frac{C.V^2}{2}$$

Equation 2 : Energy stored on a capacitor [4]

Where E stands for the energy, Q for the charge, C the capacity and V the voltage between the plates of the capacitor.

So for N Cooper pairs, the energy in the Cooper pair box is:

$$E = Ec.N^2 \qquad E = Ec.(N+1)^2 \qquad \text{with} \quad Ec = \frac{(2e)^2}{2C}$$

Equation 3 : Total energy in a Cooper pair box that contains N Cooper pair and N+1 Cooper pair..[from Equation 2]

However, since the Cooper pair box is an isolated system, we cannot add or remove a Cooper pair. Which means we cannot make a superposition of the state | 0 > and the state | 1 > to make the qubit. That's why we use a Josephson junction to make a tunnelling. The energy of the system doesn't depend only on the energy of the Cooper pair box but also on the energy of the Josephson junction E_J.

The Hamiltonian of the system becomes:

$$\hat{H} = Ec.\hat{N}^2 - E_J.\cos(\hat{\phi})$$

Equation 4 : Hamiltonian of the system comporting the Cooper pair box and the Josephson junction.[5]

To measure this Hamiltonian of the Equation 4, we can control the Josephson junction energy E_J with the magnetic field Φ. However, we cannot control the capacitive energy Ec. This energy could be controlled by changing the size of the Cooper pair box which is not possible. Then we have to introduce Ng, corresponding to the number of capacitor charge.

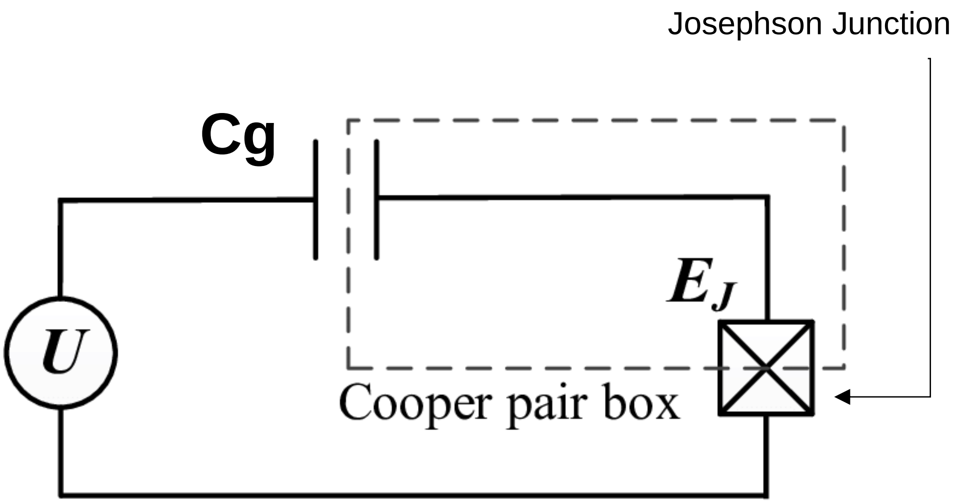

Figure 3 : Circuit of a charge qubit [adapted from [6]]

As illustrated on the Figure 3, the Cooper pair box is isolated by a capacitor of a capacity Cg and the Josephson junctions.

$$Qg = Ng.2e \iff Ng = \frac{Qg}{2e} = \frac{Cg.Vg}{2e}$$

Équation 5 : Number of capacitor charge Ng [from Equation 2]

2e represents twice the charge of an electron due to the Cooper pairs.

Subtracting Ng to N, the Hamiltonian from the Equation 4 becomes:

$$\hat{H} = Ec.(\hat{N} - Ng)^2 - Ej.\cos(\hat{\phi})$$

Equation 6 : Hamiltonian of a Charge Qubit [adapted from [7]]

We can now use the Equation 6 to display the energy levels of the charge qubit. This is what we are showing on the Figure 4.

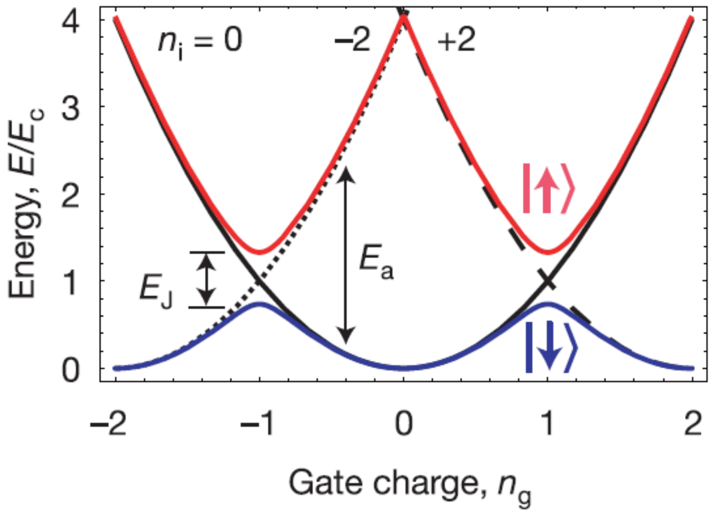

Figure 4 : Energy levels of a charge qubit [from [8]]

The black solid line represents the energy level of the Cooper pair box for N=1. The black dotted line for N=-2 and the dashed line for N=2. Coupling with the Josephson junction, the degeneracy opens up a gap proportional to the Josephson energy E_J.

To change the phase of the qubit, we then have to apply an activation Energy Ea over E_J but that depends on the gate charge Ng. This is the problem about the charge qubit. We will then study the qubit of our system : the superconducting qubit.

1.2.2. Superconducting Qubit

Also called transmon qubit, the superconducting qubit is similar to the charged qubit but here, the Cooper pair box is larger. The circuit being the same as the charge qubit, we have the same Hamiltonian as the Equation 6. A larger capacitance makes a larger E_J/E_C. The figure below shows the effect.

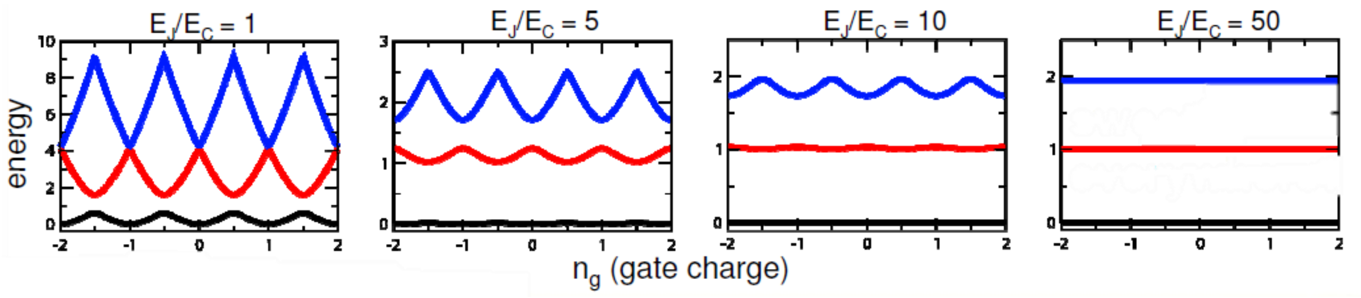

Figure 5 : Energy diagram of the Superconducting qubit depending on E_J/E_c [adapted from[6]]

We now can see with the Figure 5 how the superconducting qubit reduces the sensitivity to charge noises. Effectively, stronger Josephson coupling implies that more and more Cooper-pair number states will contribute to the eigenstates of the system, and if the eigenstates involve lots of different number states anyway, this will make them less sensitive to changes in charge number, and hence also fluctuations in the local charge environment. The activation Energy to access from the ground state to the excited state is about $\sqrt{E_J.E_C}$.[9]

II. Measurement of a superconducting qubit

To be able to measure the state of the superconducting qubit, we need to know the environment of this qubit, which is the superconducting circuit. We then have to know how the superconducting circuit is composed.

1. Superconducting circuit

2.1.1. Resonator

The resonator is an inductor-capacitor circuit, which can also be written as LC circuit. Like the one-dimensional harmonic oscillator problem, it can be quantised by using creation and annihilation operators. The Schrödinger equation using these operators for a one-dimensional time independent harmonic oscillator is:

$$\hbar\omega\left(a^{+}a+\frac{1}{2}\right)\psi(x)=E\psi(x)$$

Equation 7 Schrödinger equation of a harmonic oscillator [2]]

Then the Hamiltonian of the resonator can be approximated as:

$$H_{res}=\hbar\omega a^{+}a$$

Equation 8 : Hamiltonian of the resonator from Equation 7

The resonator makes a harmonic oscillator of evenly spaced energy levels of $\hbar\omega$ .We will use this Hamiltonian later in this report.

2.1.2. Two level system

As we said with the Equation 8, the resonator makes a harmonic oscillator with energy levels evenly spaced. To make our two-level system, we need to increase the energy between two levels. Increasing the energy prevents the electron from being driven to a third state but to keep the two lowest energy levels. Those two low energy levels are our qubit system.

We can assume we know the eigenstates of the upper level $|+\rangle\langle+|$ and down level $|-\rangle\langle-|$. We note there eigenenergies E_+ and E_-. Then we can write:

$$H_{TLS} = E_+|+\rangle\langle+| + E_-|-\rangle\langle-|$$

Equation 9 Two level system Hamiltonian written with the bra-ket [adapted from [2]]

Noting $\Omega=E_+ - E_-$, this equation leads to the two-level system Hamiltonian :

$$H_{TLS} = \frac{1}{2}\hbar\Omega\sigma^Z$$

Equation 10 Two level system Hamiltonian [from Equation 9]

With σ^Z , the Pauli matrix.

2.1.3. Qubit-Resonator coupling

The resonator produces a magnetic field E and interacts with the dipole moment of the two-level system. The energy of a magnetic moment is written as the Equation 11:

$$H_{coupling} = -\vec{d}.E$$

Equation 11 : Energy of a magnetic moment in a magnetic field [2]

This equation leads to:

$$H_{coupling} = \hbar \mathrm{g}(\sigma_+ + \sigma_-)(a + a^+)$$

Equation 12 : Qubit-Resonator coupling Hamiltonian [from Equation 11]

With σ_+ and σ_- , the Pauli matrices. They create, an excitation in the CPB. The term g represents the coupling strength between the resonator and the qubit.

2.1.4. Superconducting circuit

Thanks to the previous part the previous part of this report, we now understand the different components of our system. We then have our superconducting qubit composed by the large Cooper pair box and two Josephson junctions that we control with the magnetic flux Φ. This qubit is coupled to the resonator inside of the cavity QED delimited by C_{in}, the input capacitor and C_{out}, the output capacitor.

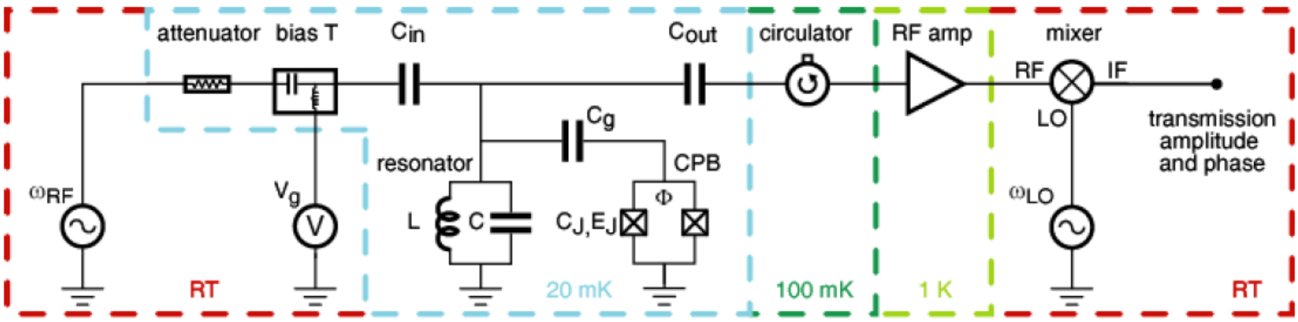

Figure 6 : Superconducting circuit. The dashed coloured zones represent the temperatures zone: RT : Room Temperature, other abbreviation in the glossary.[8]

The driving frequency ω_{RF} is the frequency that we control to read the qubit state. The cavity QED forms a Fabry-Perrot interferometer in which ω_{RF} controls the qubit state.

2. Hamiltonian

Finding the Hamiltonian of the system is necessary to be able to measure the qubit. However, the Hamiltonian of our system is computed from other Hamiltonians which we will study first.

2.2.1. Rabi Hamiltonian

The Rabi Hamiltonian is defined by the sum of the resonator Hamiltonian, the two-level system Hamiltonian and the coupling Hamiltonian but also, the Energy loss. Indeed, on the cavity QED, we have the rate of loss photon from the resonator κ and the rate of loss photon from the two-level system γ. By simplification, we write those two-energy loss as Hκ and Hγ.

We now have the full expression of the Rabi Hamiltonian for the QED system coupled with one qubit.

$$H_{Rabi} = \hbar\omega a^{+}a + \frac{1}{2}\hbar\Omega\sigma^{Z} + \hbar g(\sigma_{+} + \sigma_{-})(a + a^{+}) + H_{\kappa} + H_{\gamma}$$

Equation 13 : Rabi Hamiltonian [10]

2.2.2. Jaynes-Cummings model

The Jaynes-Cummings model is a simplification of the Rabi Hamiltonian. Indeed, we can neglect the two energy loss Hamiltonians because they have a very low behaviour compared to the other Hamiltonians.

Another term that we can simplify is the qubit-resonator coupling term which we can approximate with the rotating wave approximation.

In the rotating wave approximation, the terms that oscillate rapidly are neglected. This approximation is valid when the electromagnetic field is in resonance with the TLS transition. This is the case since we use a Fabry Perrot interferometer. This makes the detuning Δ=Ω-ω come very close to zero.

Setting the detuning to zero, the coupling Hamiltonian becomes:

$$H_{coupling} = \hbar \mathrm{g}(a.\sigma_+ + a^+.\sigma_-)$$

Equation 14 : Qubit-resonator coupling Hamiltonian at near-resonance [10]

Then the Hamiltonian of the Jaynes-Cummings model becomes:

$$H_{RWA} = \hbar\omega a^+ a + \frac{1}{2}\hbar\Omega\sigma^Z + \hbar \mathrm{g}(a^+.\sigma_- + a.\sigma_+)$$

Equation 15 : Jaynes-Cummings Hamiltonian [10]

We now have to represent the energy diagram of the superconducting circuit that we can realise from the Equation 15. Finding the energy of the system is equivalent of finding the eigenstates of the Hamiltonian.

When applying a basis of n photon level and a spin up and down to the Hamiltonian of the Equation 15, we get a system of 4 Hamiltonians depending on the Cooper pair number N in the CPB and the TLS spin. Written in a matrix form, we get the Equation 16 :

$$H^{RWA^{(n)}} = \begin{bmatrix} \frac{\Omega}{2} + \omega n & g\sqrt{n+1} \\ g\sqrt{n+1} & -\frac{\Omega}{2} + \omega(n+1) \end{bmatrix}$$

Equation 16 : Matrix form of the Jaynes-Cummings Hamiltonian depending on the photon level n [from Equation 15]

The Equation 16 leads us to the energy for n photon level, setting the detuning Δ to zero we get the following equation:

$$E_{\pm}^{(n)} = \left(n + \frac{1}{2}\right)\hbar\omega \pm g\hbar\sqrt{n+1}$$

Equation 17 : Energy of the superconducting circuit on resonance depending on the n photon level (from Equation 16)

First, we can study the system when the two-level system isn't coupled to the resonator which means g=0 on Equation 17. On the Figure 7, we represent the energy levels according to the states in photon level and the spin of the two-level system.

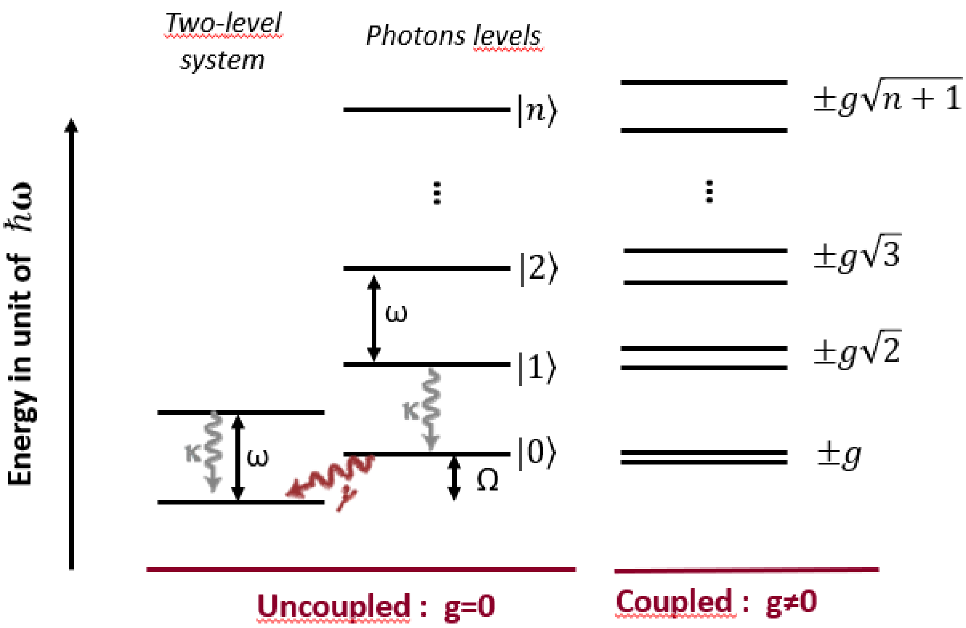

Figure 7 : Energy levels of the superconducting circuit when the qubit is not coupled to the resonator and when it is coupled [adapted from [10]]

Off-resonance, the state levels aren't dressed, furthermore, as we said before, the circuit is equivalent to a Fabry-Perot interferometer which means that the transmitted wave is on resonance. Being on resonance means setting the detuning Δ to zero which creates dressed states.

With the qubit coupled to the resonator, the energy levels of the superconducting circuit splits into two energy levels separated by $g\hbar\sqrt{n+1}$. This splitting makes an issue to control our two-level system.

2.2.3. Dispersive model

What we are interested in here is the dispersive limit, in which the qubit and the resonator are far detuned compared to the coupling strength: g<<|Δ|.

To get this dispersive Hamiltonian, we need to change the base of the Jaynes-Cummings Hamiltonian. We then have to use a unitary transformation.

When applying the Baker-Hausdorff formula to the Jaynes-Cumming Hamiltonian H_{RWA} with the unitary transformation matrix $D = e^{\lambda X_-}$, we get:

$$D^+ H_{RWA} D = H_{RWA} + \lambda\,[H_{RWA}, X_-] + \frac{\lambda^2}{2}\big[[H_{RWA}, X_-], X_-\big] + \dots + \frac{\lambda^n}{n!}\underbrace{\Big[\big[[H_{RWA}, X_-], X_-\big], \dots, X_-\Big]}_{\text{n times}}$$

Equation 18 : Application of the Baker-Hausdorff formula to H_{RWA} [11]

With $X_\pm = a^+\sigma_- \pm a\sigma_+$.

The dispersive Hamiltonian is defined to be equal to $D^+ H_{RWA} D$ to second order in $\lambda = \frac{g}{\Delta}$.

This leads to the Hamiltonian of the superconducting circuit, at the dispersive limit:

$$H_{Disp} = \frac{1}{2}\Omega\sigma^{Z} + \frac{g^{2}}{2\Delta}\sigma^{Z} + \left(\omega + \frac{g^{2}}{\Delta}\sigma^{Z}\right)a^{+}a$$

Equation 19 : Dispersive Hamiltonian [10]

We can now use this Hamiltonian to read-out the qubit state.

3. Qubit read-out

The cavity QED is a Fabry Perot interferometer. The signal being between two mirrors, the total of the signal transmitted through the cavity is a Lorentzian. Without qubit, the driving frequency ω_{RF} generates a Lorentzian on the resonator frequency ω.

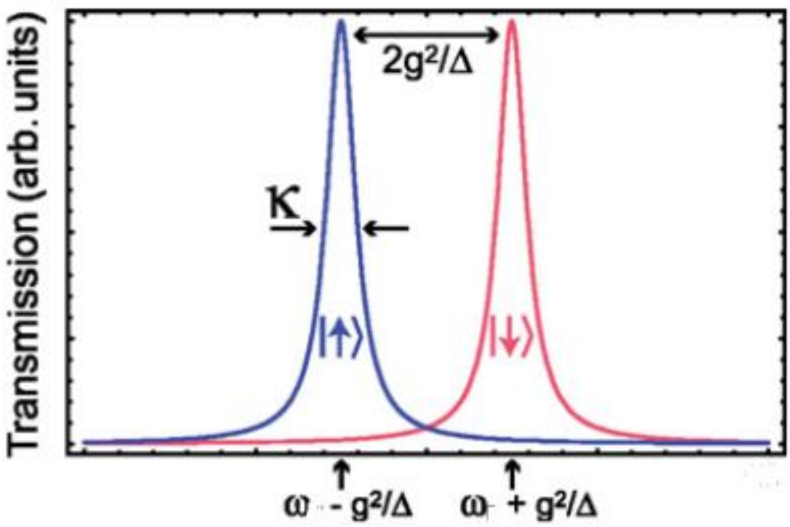

Figure 8 : Transmission spectrum of the cavity, depending on the state of the qubit : red for the excited state, blue for the ground state [9]

Performing a strong measurement of a qubit (g>>κ.) when pulsing a photon at a frequency $\omega_{RF}=\omega$, we get following to the Equation 19, a Lorentzian at $\omega - \frac{g^2}{\Delta}$ for the ground state or at $\omega + \frac{g^2}{\Delta}$ for the excited state. The half width at half maximum of the Lorentzian corresponds to the photon loss in the cavity κ.[10]

Conclusion

During this internship, we studied the strong measurement of a superconducting qubit which can be used to read the states of those artificial devices used in quantum computation.

We studied the superconducting qubit that uses a combination of two independent states. It needs then a Cooper-pair box and Josephson junctions to produce a two-level system where we control our quantum bit. The strong coupling allows us to read the instant qubit state. We get this strong measurement when the coupling strength between the qubit and the resonator is over the cavity photon rate loss. From this photon rate loss, we studied how we get the transmission spectrum where we can read the qubit state.

This internship was the studying of one single qubit in a superconducting circuit. Further research would mean investigating on the measurement of the circuit transmission with several qubits in the cavity. The difficulty is that the qubits interact with each other and we cannot neglect their coupling energy.

However, there is still an important difficulty to research further: When we have the superposition of both states, how to get the two complex number that describe the probability to get the states 0 and 1? This problem can be further explored and this measurement is called weak measurement.

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