



**Korea University, 27 March 2019**



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**MOST RECENT WORK**

**Simultaneous observation of  
particle and wave behaviors of  
entangled photons**

SCIENTIFIC REPORTS 

**Sci. Rep. 7, 42539 (2017)**



**Zhong-Xiao Man<sup>1</sup>, Yun-Jie Xia<sup>1</sup> & Nguyen Ba An<sup>2</sup>**

ARTICLE

DOI: 10.1038/s41467-017-01058-6

OPEN

# Entanglement of photons in their dual wave-particle nature

Adil S. Rab<sup>1</sup>, Emanuele Polino<sup>1</sup>, Zhong-Xiao Man<sup>2</sup>, Nguyen Ba An <sup>3</sup>,  
Rosario Lo Franco <sup>4,5</sup> & Fabio Sciarrino<sup>1</sup>  
Yun-Jie Xia<sup>2</sup>, Nicolò Spagnolo<sup>1</sup>,



**nature**  
**COMMUNICATIONS**

**Nature Communications 8, 915 (2017)**

**Deterministic joint remote preparation  
of an equatorial hybrid state  
via high-dimensional  
Einstein–Podolsky–Rosen pairs:  
active versus passive**

receiver

**Quantum Inf Process (2018) 17:75**

**C.T. Bich, L.T. Dat, N.V. Hop, N.B. An**

**Nonstandard protocols  
for joint remote preparation  
of a general quantum state and hybrid  
entanglement of any dimension**

**PHYSICAL REVIEW A 98, 042329 (2018)**

**N.B. An, L.T. Dat & J. Kim**



# Designs of interactions between discrete- and continuous-variable states for generation of hybrid entanglement

**Quantum Inf. Process. 18:685 (2019)**

**S. A. Podoshvedov, N.B. An**

**TODAY'S TALK**

**On**

**“QUANTUM DIALOGUE”**

**which belongs to**

**CRYPTOGRAPHY**



**Alice**



**Bob**



**Eve**

- **Alice want to send a meassge to Bob**
- **Eve's in the line to eavesdrop**
- **How to secure the communication?**

# PRIVATE-KEY SYSTEM

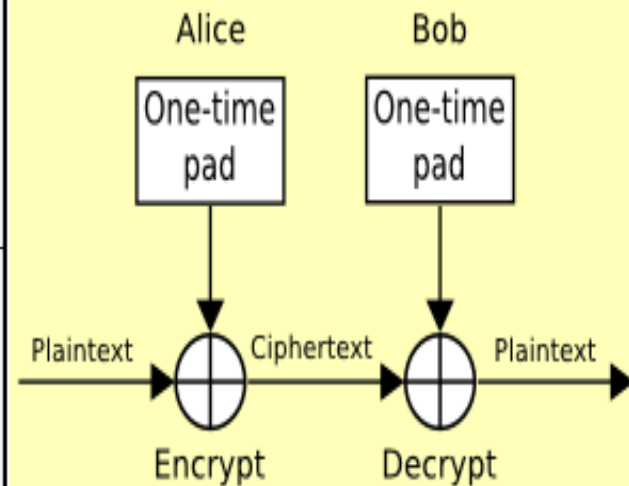
(Vernam 1917)

## ENCRYPT

$$\begin{array}{r} \oplus \\ \text{0 0 1 1 0 1 0 1} \text{ Plaintext} \\ \text{1 1 1 0 0 0 1 1} \text{ Secret Key} \\ \hline = \text{1 1 0 1 0 1 1 0} \text{ Ciphertext} \end{array}$$

## DECRYPT

$$\begin{array}{r} \oplus \\ \text{1 1 0 1 0 1 1 0} \text{ ciphertext} \\ \text{1 1 1 0 0 0 1 1} \text{ Secret Key} \\ \hline = \text{0 0 1 1 0 1 0 1} \text{ Plaintext} \end{array}$$



**Absolute Secure**

**BUT:**

**“One-time use” → INCONVENIENT**

# PUBLIC-KEY SYSTEM

(Rivest-Shamir-Adlman [RSA] 1997)

Alice: PA, SA

Bob: PB, SB

$$PX[SX[m]] = SX[PX[m]] = m$$

If Alice wants to send  $M = m + s$  to Bob

Alice encodes:  $PB[m] + SA[s]$  and sends to Bob

Bob decodes:  $SB[PB[m]] + PA[SA[s]] = m + s$

**Good: CONVENIENT (Now in use)**

**BAD: Unproven Security  
& Broken by Quantum Computer**

# QUANTUM CRYPTOGRAPHY

**Uses:** - QUANTUM RESOURCES

- QUANTUM LAWS

(Nocloning, Measure → Disturbance,...)

 **QKD (BB84, E91, B92, ...): UNCONDITIONAL SECURITY**

**(even in presence of qu-computers)**

**BUT:**

**Question:** Can ones communicate **without** a prior QKD (as in emergency situations)?

**Answer:** **YES, by QUANTUM DIALOGUE!**

## **QUANTUM DIALOGUE:**

**Way to securely exchange info**  
**(like to securely talk with each other)**  
**without a prior QKD**

**HOW TO PERFORM**  
**QUANTUM DIALOGUE?**

Alice prepares

$$|\Psi\rangle_{AB} = |0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B$$

Sends  $B$  to Bob. Bob applies  $I, X, Z$  or  $Y$  on  $B$  if wants to send 2 bits 00, 01, 10 or 11, then returns  $B$  to Alice

$$I \otimes I |\Psi\rangle_{AB} = |0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B$$

$$I \otimes X |\Psi\rangle_{AB} = |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B$$

$$I \otimes Z |\Psi\rangle_{AB} = |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B$$

$$I \otimes Y |\Psi\rangle_{AB} = |0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B$$

Alice makes Bell measurement to learn Bob's 2 bits.

Twice Holevo's bound (1973)  $\implies$  SUPERDENSE CODING (1992)



Now: Bob wants to send Alice  $i, j$  and Alice wants to send Bob  $m, n$

Alice prepares

$$|\Psi_{m,n}\rangle_{AB} : \begin{cases} |\Psi_{0,0}\rangle_{AB} = |0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B \\ |\Psi_{0,1}\rangle_{AB} = |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B \\ |\Psi_{1,0}\rangle_{AB} = |0\rangle_A |0\rangle_B - |1\rangle_A |1\rangle_B \\ |\Psi_{1,1}\rangle_{AB} = |0\rangle_A |0\rangle_B - |1\rangle_A |1\rangle_B \end{cases}$$

$$C_{00} = I, C_{01} = X, C_{10} = Z, C_{11} = Y$$

$$I \otimes C_{ij} |\Psi_{m,n}\rangle_{AB} = |\Psi_{m \oplus i, n \oplus j}\rangle_{AB}$$

- Alice sends B to Bob, Bob applies  $C_{ij}$  on B and returns to Alice
- Alice's Bell measurement  $\rightarrow |\Psi_{m \oplus i, n \oplus j}\rangle_{AB}$  to learn  $i, j$
- Alice announce  $m \oplus i, n \oplus j \implies$  Bob learns  $m, n$

QUANTUM DIALOGUE [NBA, PLA 328,6 (2004)]

# QUANTUM DIALOGUE



$(m,n)$



$(i,j)$

1) A  $|\Psi_{m,n}\rangle_{AB}$   $\rightarrow$  B

2) A  $\leftarrow$  B  $C_{ij}$

3) BM[ AB ] =  $|\Psi_{m\oplus i, n\oplus j}\rangle_{AB}$

$\rightarrow$  gets Bob's  $i,j$

Then sends  $\rightarrow$

$m \oplus i, n \oplus j$

4) Gets  $m,n$

Security to be discussed later!

# Another possible way to perform QUANTUM DIALOGUE

Exploiting NONSELECTIVE MEASUREMENT  
(= Measurement without reading the outcome)

- Classically,  
Measurement without reading outcome = Doing Nothing
- Quantumly,  
Measurement without reading outcome = Making Sense



## QUANTUM DIALOGUE

# MATHEMATICAL PRELIMINARIES

For  $N = \text{odd prime}$

$\exists$  complete set of  $N + 1$  MUBs (basis  $\beta = 0, 1, \dots, N - 1$  and basis " $N + 1$ ")

Each basis has  $N$  orthonormal states

A  $k$ th state ( $k = 0, 1, \dots, N - 1$ ) of a  $\beta$ th basis

$$|\beta; k\rangle = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \Omega^{\beta l^2 + kl} |l\rangle$$

$$|\langle \beta'; k' | \beta; k \rangle|^2 = \frac{1}{N}; \forall k, k'; \beta \neq \beta'$$

# QD by Nonselective measurement

- Alice:  $\beta \in \{0, 1, \dots, N - 1\}$
- Bob:  $\beta' \in \{0, 1, \dots, N - 1\}$
- They wish to exchange their numbers securely

# QD by Nonselective measurement

- Alice prepares (with  $\beta$  to be exchanged and  $k, q$  fixed)

$$|\beta; k, q\rangle_{AB} = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \Omega^{\beta l^2 + kl} |l\rangle_1 |q+l\rangle_2$$

Then Alice sends quNit  $B$  to Bob

- Bob measures  $B$  in basis  $\beta'$  (the to-be-exchanged number)

Then returns  $B$  to Alice without reading outcome (nonselective meas.)

$\Rightarrow$  QuNits  $A$  and  $B$  become in a mixed (NOT pure) state

$$\rho_{AB}(\beta, \beta', k, q) = \sum_{p=0}^{N-1} \Pi_A(\beta', p) |\beta; k, q\rangle_{AB} \langle \beta; k, q| \Pi_A(\beta', p),$$

$$\Pi_A(\beta', p) = |\beta'; p\rangle_A \langle \beta'; p|$$

# QD by Nonselective measurement

- Alice, having both  $A$  and  $B$ , measures them in  $\beta$ -basis (in which he prepared  $|\beta; k, q\rangle_{AB}$ ) to find  $|\beta; k', q'\rangle_{AB}$  with probability

$$P_{AB}(\beta, \beta', k, q, k', q') = \frac{1}{N} \delta_{k-k', 2(\beta-\beta')(q-q')}$$

$$\implies \beta' = \beta + \frac{k' - k}{2(q - q')}$$

That is, Alice gets Bob's number  $\beta'$

Then Alice announce  $\beta + \beta' \implies$  Bob gets Alice's  $\beta$ .

**THE INFO EXCHANGE IS DONE!**

**CAN BE REPEATED TO DO QUANTUM DIALOGUE**

# EVE's ATTACKS

## CAPTURE-AND-REPLACE ATTACK

- Eve creates her own  $|\beta''; k'', q''\rangle_{CD}$  ( $\beta'', k'', q''$  at her choice) and CAPTURES  $B$ , store it on the way Alice  $\rightarrow$  Bob  
Then REPLACES  $B$  by  $C$  to be sent to Bob.

- Bob nonselectively measure  $C$  in basis  $\beta'$ , then send it to Alice
- On the way Bob  $\rightarrow$  Alice, Eve CAPTURES  $C$ , measures  $CD$  in basis  $\beta''$  to get  $\beta'$   
Then, nonselectively measures  $B$  (from memory) in basis  $\beta'$  and sends it to Alice.
- Alice measures  $AB$  to get  $\beta'$  too, then announce  $\beta + \beta'$  to allows Bob know  $\beta$

THUS, Eve perfectly eavesdrops (If no SECURITY CHECK)



# SECURITY CHECK

To detect Eve in this and other kinds of attacks, two CONTROL MODES (CM) are introduced:

- CM1 (On the way Alice  $\rightarrow$  Bob)

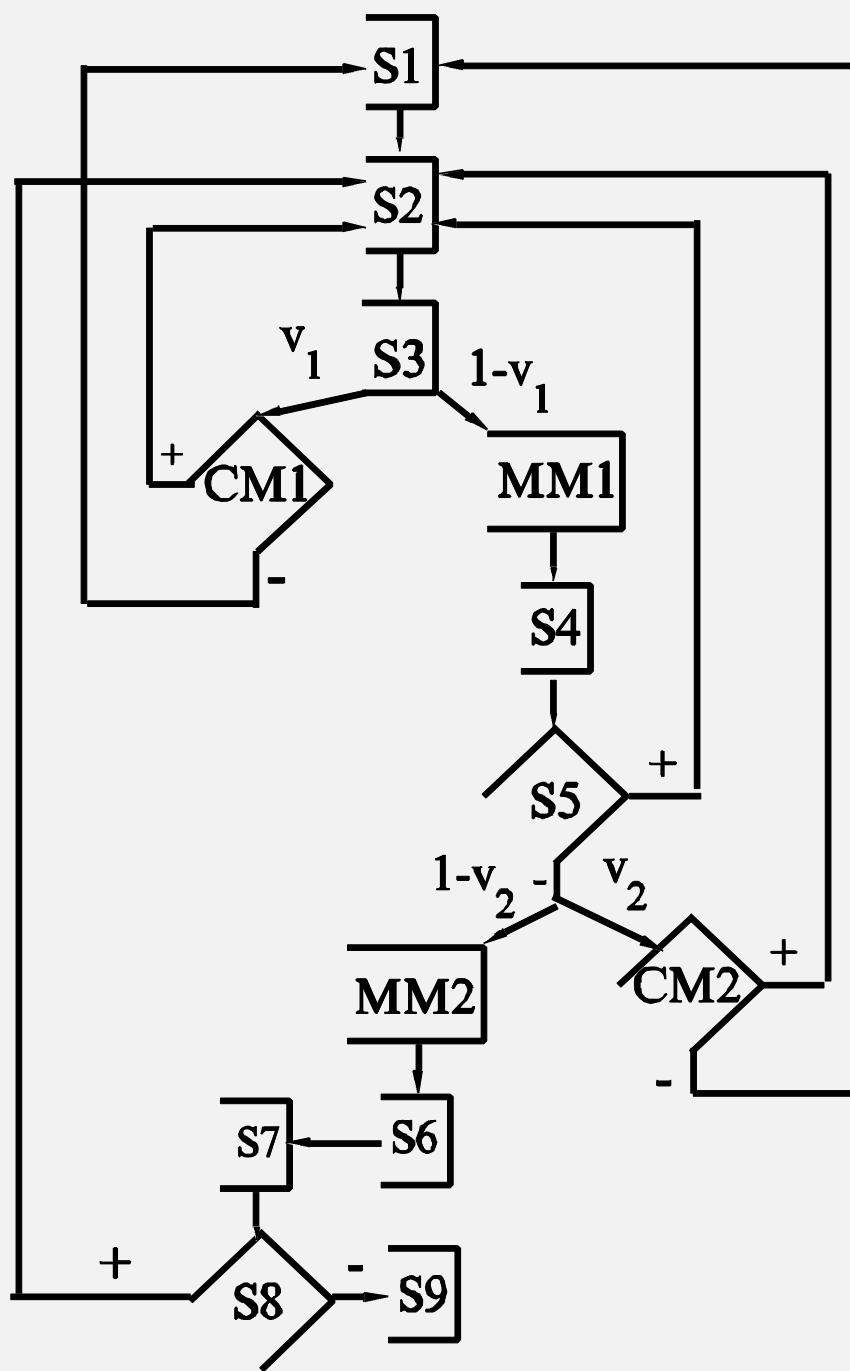
Bob measures  $B$  in computational basis and publishes outcome

Alice also measures  $A$  in computational basis.

Then they compare their outcomes to detect Eve

- CM2 (On the way Bob  $\rightarrow$  Alice).

Alice and Bob reveal their choices and check some equality to detect Eve



# CONCLUSION

- Exploiting Quantum Physics  
[Quantum resources,  
quantum unitary operations,  
quantum measurements  
(even nonselective measurements), ...]  
allows secure exchange of info without  
a prior QKD, like in a (quantum) dialogue
- The words “QUANTUM DIALOGUE”  
first appeared in [NBA, PLA 328, 6 (2004)]
- See also [NBA, J. Kor. Phys. Soc. 47, 562 (2005)  
and Adv. Nat. Sci.: Nanosci. Nanotech. 9, 025001  
(2018) ]
- There have been many publications that extend QD in  
various directions taking into accounts real factors (2005)

**Thank you**    감사 합니다 !

**and hope to see you again !**    또 보자

