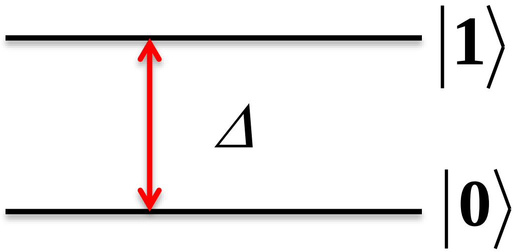
Research Training Workshop 2016
Quantum Control Lab

Introduction to decoherence effects

Presenter: NGUYEN LE DUC THINH

Alexandre Blais, Physical review A 69, 062320 (2004)
K.blum, Density Matrix theory and applications (2012)
Nathan K.Langford, Circuit QED-Lecture Notes (2013)

Superposition state



$$H = -\frac{1}{2}\hbar\Delta\sigma_z$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}\rangle + |\mathbf{1}\rangle\right)$$

Free evolution

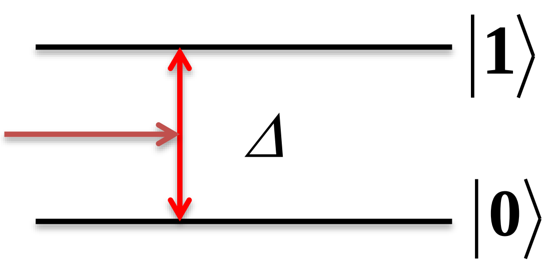
$$H = -\frac{1}{2}\hbar\Delta\sigma_z$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{\frac{1}{2}i\Delta t}|\mathbf{0}\rangle + e^{-\frac{1}{2}i\Delta t}|\mathbf{1}\rangle\right)$$

Stationary states

$$p_0(t) = p_1(t) = \frac{1}{2}\,\forall t$$

How can we know it was in the superposition state?



$$p_0(t), p_1(t)$$

Oscillating

Quantum information

Two level system

$|\mathbf{0}\rangle$ and $|\mathbf{1}\rangle$

$$\sigma_x \equiv X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y \equiv Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z \equiv Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_x|\mathbf{0}\rangle = |\mathbf{1}\rangle, \quad \sigma_x|\mathbf{1}\rangle = |\mathbf{0}\rangle, \quad \sigma_x|\pm\rangle = |\pm\rangle \quad \& \ \sigma_x|\pm i\rangle = |\mp i\rangle$$

$$|\pm\rangle \equiv (|\mathbf{0}\rangle \pm |\mathbf{1}\rangle)/\sqrt{2}$$

$$|\pm i\rangle \equiv (|\mathbf{0}\rangle \pm i|\mathbf{1}\rangle)/\sqrt{2}$$

Superposition states

$$|\mathbf{0}\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) = \frac{1}{\sqrt{2}i}(+|i\rangle + |-i\rangle)$$

$$|\mathbf{1}\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) == \frac{1}{\sqrt{2}i}(-|i\rangle - |-i\rangle)$$

Suppose Alice want to send Bob information by using two level system states→Bob can know by measure it in the right basis

$$p_\phi = |\langle\phi|\psi\rangle|^2$$

	Bob					
Alice	$\langle\mathbf{0}|$	$\langle\mathbf{1}|$	$\langle+|$	$\langle-|$	$\langle+i|$	$\langle-i|$
$|\mathbf{0}\rangle$	100%	0%	50%	50%	50%	50%
$|\mathbf{1}\rangle$	0%	100%	50%	50%	50%	50%

Other bases: Bob cannot know which state Alice sent

Correlated measurements

Mixed state

$|0\rangle$ and $|1\rangle$

Pure states

$$\sigma_x \equiv X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y \equiv Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z \equiv Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_x|\mathbf{0}\rangle = |\mathbf{1}\rangle, \quad \sigma_x|\mathbf{1}\rangle = |\mathbf{0}\rangle, \quad \sigma_x|\pm\rangle = |\pm\rangle \quad \& \ \sigma_x|\pm i\rangle = |\mp i\rangle$$

$$|\pm\rangle \equiv (|\mathbf{0}\rangle \pm |\mathbf{1}\rangle)/\sqrt{2}$$

$$|\pm i\rangle \equiv (|\mathbf{0}\rangle \pm i|\mathbf{1}\rangle)/\sqrt{2}$$

ensemble of $|\mathbf{0}$

Bob

Alice	$\langle \mathbf{0}\|$	$\langle \mathbf{1}\|$	$\langle +\|$	$\langle -\|$	$\langle +i\|$	$\langle -i\|$
$\|\mathbf{0}\rangle$	100%	0%	50%	50%	50%	50%
$\|\mathbf{1}\rangle$	0%	100%	50%	50%	50%	50%

$+i\|$	$\langle -i\|$
)%	50%

Mixed states: cannot write in a superposition form, or single wave vector

All uncorrelated measurements→Bob cannot know the information in any basis even if Alice tells him which basis she used

Entanglement

<u>Combined two-level systems quantum states</u>

$|0,0\rangle$ or $|1,1\rangle$

$$|0,0\rangle = |0\rangle \otimes |0\rangle$$
$$|1,1\rangle = |1\rangle \otimes |1\rangle$$

Suppose Xavier prepare the above quantum states. Alice and Bob want to know what it is by measure each component separatedly.

$$\psi = |\psi_1\rangle \otimes |\psi_2\rangle$$
$$p_{\phi_1\phi_2} = \langle \phi_1 | \psi_1 \rangle \otimes \langle \phi_2 | \psi_2 \rangle$$

Alice	$\langle 0\|$	$\langle 1\|$	$\langle +\|$	$\langle -\|$	$\langle +i\|$	$\langle -i\|$
$\langle 0\|$	50%	0%				
$\langle 1\|$	0%	50%				
$\langle +\|$			25%	25%		
$\langle -\|$			25%	25%		
$\langle +i\|$					25%	25%
$\langle -i\|$					25%	25%

There is one basis in which measurements of Bob and Alice are correlated

Entanglement

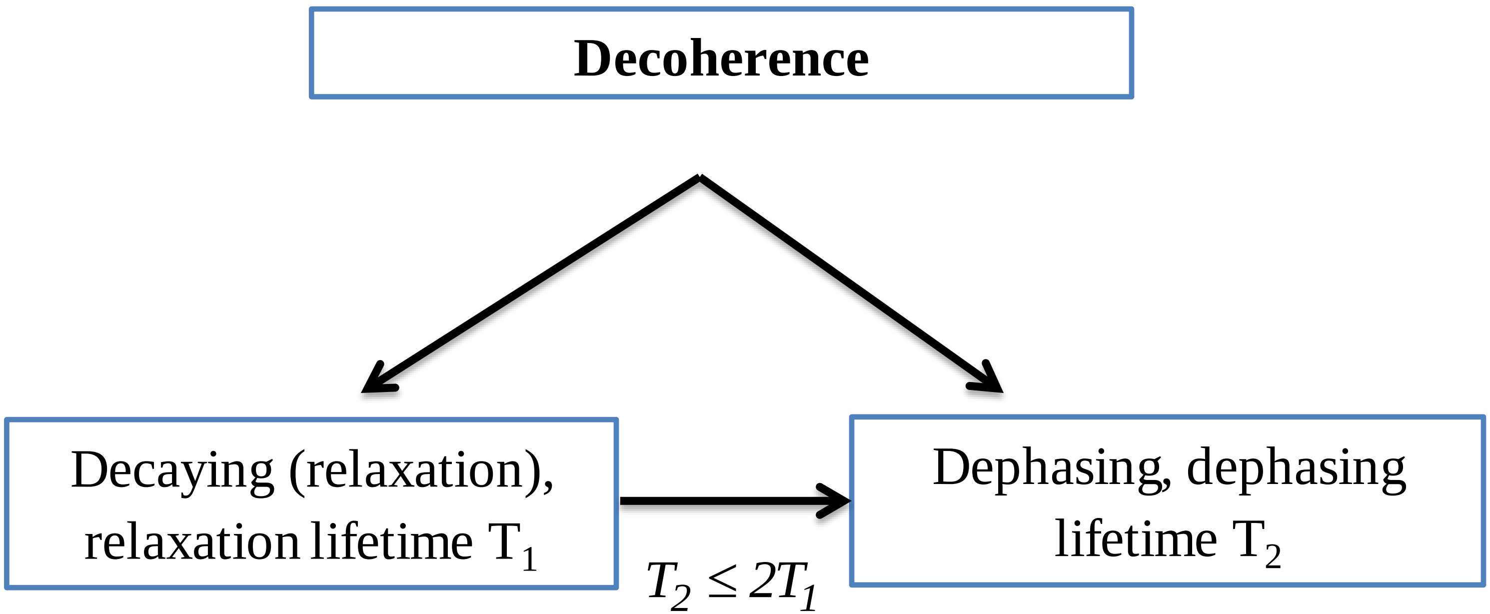
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\mathbf{0},\mathbf{0}\rangle + |\mathbf{1},\mathbf{1}\rangle) \equiv |\phi^+\rangle$$

	Bob					
Alice	$\langle\mathbf{0}\|$	$\langle\mathbf{1}\|$	$\langle+\|$	$\langle-\|$	$\langle+i\|$	$\langle-i\|$
$\langle\mathbf{0}\|$	50%	0%	25%	25%	25%	25%
$\langle\mathbf{1}\|$	0%	50%	25%	25%	25%	25%
$\langle+\|$	25%	25%	50%	0%	25%	25%
$\langle-\|$	25%	25%	0%	50%	25%	25%
$\langle+i\|$	25%	25%	25%	25%	0%	50%
$\langle-i\|$	25%	25%	25%	25%	50%	0%

There are **three bases** in which measurements of Bob and Alice are correlated

→ **Entanglement states**

Decoherence

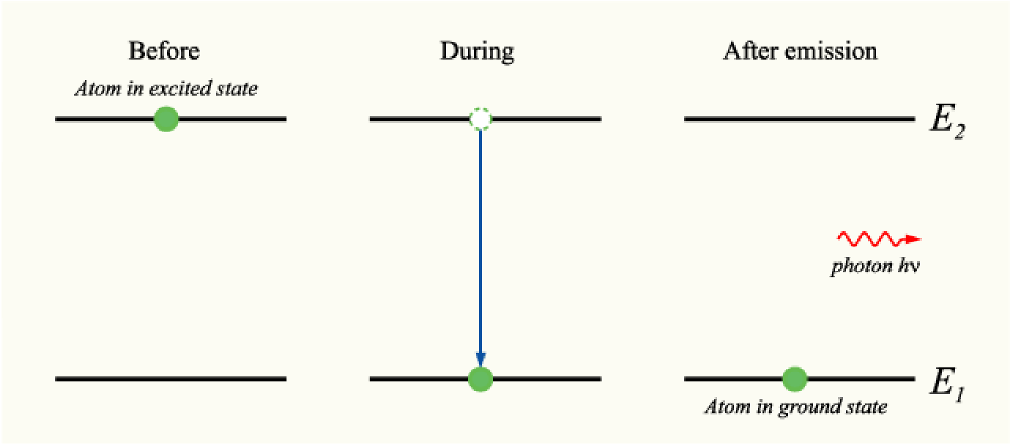


Decaying

Losing energy to the environment

Probability finding the system in excited state

$$p_1(t) = e^{-\gamma t} \rightarrow T_1 = \frac{1}{\gamma}$$



Dephasing

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}\rangle + |\mathbf{1}\rangle\right) \xrightarrow[H=-\frac{1}{2}\hbar\Delta\sigma_z]{} |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{\frac{1}{2}i\Delta t}|\mathbf{0}\rangle + e^{-\frac{1}{2}i\Delta t}|\mathbf{1}\rangle\right)$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}\rangle + |\mathbf{1}\rangle\right)$$

$$p_+(t) = \langle +||\psi(t)\rangle = \frac{1}{2} + \frac{1}{2}\cos(\Delta t)$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}\rangle + e^{-i\phi}|\mathbf{1}\rangle\right) \longrightarrow p_+(t) = \langle +||\psi(t)\rangle = \frac{1}{2} + \frac{1}{2}\cos(\Delta t - \phi)$$

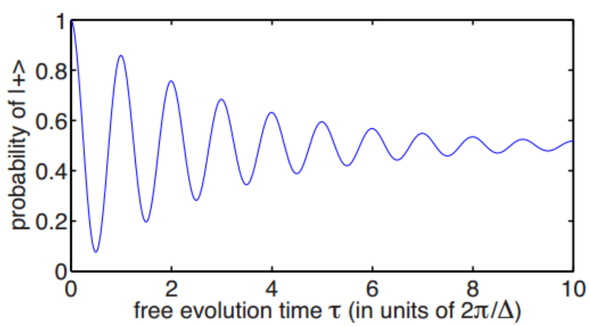
$+$ (sum of the two results above) $/2$

Random phase kick from environment

Average measurements

$$p_+(t) = \frac{1}{2} + \frac{1}{2}\cos(\Delta t - \phi/2)\cos(\phi/2)$$

Reduced amplitude



Treatment of decoherence

Density matrix

Pure state

$$\rho = |\chi\rangle\langle\chi|$$

Mix state

$$\rho = W_a|\chi_a\rangle\langle\chi_a| + W_b|\chi_b\rangle\langle\chi_b| + \dots$$

Pure state 2-level system

$$|\chi\rangle = a_1|\mathbf{0}\rangle + a_2|\mathbf{1}\rangle$$

$$\rho = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\begin{pmatrix} a_1^* & a_2^* \end{pmatrix} = \begin{pmatrix} |a_1|^2 & a_1 a_2^* \\ a_1^* a_2 & |a_2|^2 \end{pmatrix}$$

$$Tr(\rho) = 1$$

$$\langle A\rangle = Tr(\rho A)$$

Diagonal elements: probability to find the system in the pure state.

Relaxation: diagonal element decaying

Treatment of decoherence

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}\rangle + |\mathbf{1}\rangle\right) \xrightarrow[H=-\frac{1}{2}\hbar\Delta\sigma_z]{} |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{\frac{1}{2}i\Delta t}|\mathbf{0}\rangle + e^{-\frac{1}{2}i\Delta t}|\mathbf{1}\rangle\right)$$

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| = \frac{1}{2}\begin{pmatrix} 1 & e^{i\Delta t} \\ e^{-i\Delta t} & 1 \end{pmatrix}$$

Strongly depends of the basis representation!!!

$$p_+(t) = \langle +|\rho(t)|+\rangle = \frac{1}{4}(1 \quad 1)\begin{pmatrix} 1 & e^{i\Delta t} \\ e^{-i\Delta t} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} + \frac{1}{2}cos(\Delta t)$$

→Oscillation (inteference) term coming from the off-diagonal elements
→coherent superpostion

$$\rho(t) = \frac{1}{2}\left(|\mathbf{0}\rangle\langle\mathbf{0}| + |\mathbf{1}\rangle\langle\mathbf{1}|\right) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Mixed state→no coherent element in this basis

Master equation

Schrodinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t}=H(t)|\psi(t)\rangle$$

$$|\psi(t)\rangle=U(t)|\psi(0)\rangle$$

$$i\hbar\frac{\partial|U(t)}{\partial t}|\psi(0)\rangle=H(t)U(t)|\psi(0)\rangle$$

$$i\hbar\frac{\partial U(t)}{\partial t}=H(t)U(t)$$

$$\rho(t)=\sum_n W_n|\psi(t)_n\rangle\langle\psi(t)_n|$$

$$=\sum_n W_n U(t)|\psi(0)_n\rangle\langle\psi(0)_n|U(t)^\dagger$$

$$i\hbar\frac{\partial\rho(t)}{\partial t}=i\hbar\frac{\partial U(t)}{\partial t}\rho(0)U(t)^\dagger+i\hbar U(t)\rho(0)\frac{\partial U^\dagger}{\partial t}$$

$$=H(t)U(t)\rho(0)U(t)^\dagger-U(t)\rho(0)U(t)^\dagger H(t)$$

Liouville equation

$$i\hbar\frac{\partial\rho(t)}{\partial t}=[H(t),\ \rho(t)]$$

Master equation

Interaction picture

$$H(t) = H_0 + V(t)$$

$$|\psi(t)\rangle = e^{-(i/\hbar)H_0 t}|\psi(t)_I\rangle$$

$$|\psi(t)_I\rangle = e^{(i/\hbar)H_0 t}U(t)|\psi(0)\rangle$$

$$V(t)_I = e^{(i/\hbar)H_0 t}V(t)e^{-(i/\hbar)H_0 t}$$

$$\rho(t) = (e^{-(i/\hbar)H_0 t})\rho(t)_I e^{+(i/\hbar)H_0 t}$$

Liouville equation

$$i\hbar\frac{\partial\rho(t)_I}{\partial t} = [V(t)_I, \rho(t)_I]$$

↓

$$\rho(t)_I = \rho(0)_I - \frac{i}{\hbar}\int_0^t [V(\tau)_I, \rho(\tau)_I]d\tau$$

↓

$$\rho(t)_I = \rho(0)_I - \frac{i}{\hbar}\int_0^t [V(\tau)_I, \rho(0)_I]d\tau$$

↓

$$\dot{\rho}(t)_I = -(i/\hbar)[V(t)_I, \rho(0)] - (i/\hbar)^2\int_0^t dt'[V(t)_I, [V(t')_I, \rho(t')_I]]$$

Master equation

$$H_{tot} = H_S + H_R + H_{SR}(t)$$

$$H_{SR}(t) = V(t) \qquad \rho(t) = \underset{R}{Tr}\, \rho_{tot}(t)$$

$$\dot{\rho}_I(t) = \frac{-i}{\hbar} \underset{R}{Tr}\left[H_{SR,I}(t), \rho_{tot,I}(0)\right] - \left(\frac{i}{\hbar}\right)^2 \int_0^t dt' \underset{R}{Tr}\left[H_{SR,I}(t), \left[H_{SR,I}(t'), \rho_{tot,I}(t')\right]\right]$$

$$\rho_{tot}(0) = \rho(0) \otimes \rho_R$$

Suppose H_{SR} has no diagonal elements in the bath basis

$$\underset{R}{Tr}\left[H_{SR,I}(t), \rho_{tot,I}(0)\right] = \rho_I(0) \underset{R}{Tr}\left[H_{SR,I}(t), \rho_R\right] = 0$$

$$\dot{\rho}_I(t) = -\left(\frac{1}{\hbar}\right)^2 \int_0^t dt' \underset{R}{Tr}\left[H_{SR,I}(t), \left[H_{SR,I}(t'), \rho_{tot,I}(t')\right]\right]$$

Master equation

$$\rho_{tot,I}(t') = \rho_I(t') \otimes \rho_R$$

Weak coupling approximation

$$\rho_I(t') \rightarrow \rho_I(t)$$

Markoff approximation

$$\dot{\rho}_I(t) = -\left(\frac{1}{\hbar}\right)^2 \int_0^t dt' \, \underset{R}{Tr}\left[H_{SR,I}(t),\left[H_{SR,I}(t'),\rho_I(t)\otimes\rho_R\right]\right]$$

$$H_{SR} = \sum_\alpha A_\alpha \otimes B_\alpha \qquad H_{SR}(t) = \sum_\alpha A_\alpha(t) \otimes B_\alpha(t)$$

$$\langle Q \rangle = \underset{R}{Tr}(\rho Q) \qquad Tr(ABC) = Tr(CAB)$$

$$\langle B_\alpha(t) B_\beta(t') \rangle = \underset{R}{Tr}\left[e^{\frac{iH_R t}{\hbar}} B_\alpha e^{-\frac{iH_R t}{\hbar}} e^{+\frac{iH_R t'}{\hbar}} B_\beta e^{-\frac{iH_R t}{\hbar}} \rho_R \right] = \underset{R}{Tr}\left[e^{\frac{iH_R(t-t')}{\hbar}} B_\alpha e^{-\frac{iH_R(t-t')}{\hbar}} B_\beta \rho_R \right]$$

$$= \langle B_\alpha(t-t') B_\beta(0) \rangle$$

Master equation

$$\langle B_\alpha(t)\rangle = 0 \qquad t - t' \gg \tau \qquad \langle B_\alpha(t)B_\beta(t')\rangle \simeq \langle B_\alpha(t)\rangle\langle B_\beta(t')\rangle = 0$$

$$t'' = t - t', dt'' = -dt'$$

$$\dot{\rho}_I(t) = -\left(\frac{1}{\hbar}\right)^2 \sum_{\alpha,\beta}\int_0^\infty dt''\left\{\left[A_\alpha(t), A_\beta(t-t'')\rho_I(t)\right]\langle B_\alpha(t'')B_\beta(0)\rangle - \left[A_\alpha(t),\rho_I(t)A_\beta(t-t'')\right]\langle B_\beta(0)B_\alpha(t'')\rangle\right\}$$

Superoperator

$$H_{SR} = \sum_\alpha A_\alpha \otimes B_\alpha = \sum_\alpha \sum_\omega A_\alpha(\omega) B_\alpha$$

$$\left[H_S, A_\alpha(\omega)\right] = -\omega A_\alpha(\omega) \qquad A_\alpha(\omega,t) = A_\alpha(\omega)e^{-i\omega t}$$

$$\left[H_S, A_\alpha^\dagger(\omega)\right] = +\omega A_\alpha^\dagger(\omega) \qquad H_{SR} = \sum_\alpha \sum_\omega A_\alpha(\omega) B_\alpha^\dagger = \sum_\alpha \sum_\omega A_\alpha^\dagger(\omega) B_\alpha$$

Master equation

$$\dot{\rho}_I(t) = -\left(\frac{1}{\hbar}\right)^2 \sum_{\alpha,\beta}\int_0^\infty dt''\left\{\left[A_\alpha(t), A_\beta(t-t'')\rho_I(t)\right]\left\langle B_\alpha(t'')B_\beta(0)\right\rangle - \left[A_\alpha(t), \rho_I(t)A_\beta(t-t'')\right]\left\langle B_\beta(0)B_\alpha(t'')\right\rangle\right\}$$

$$H_{SR} = \sum_\alpha\sum_\omega A^\dagger_\alpha(\omega)B_\alpha \quad H_{SR} = \sum_\beta\sum_{\omega'} A_\beta(\omega')B^\dagger_\beta \quad H_{SR} = \sum_\alpha\sum_\omega A_\alpha(\omega)B^\dagger_\alpha \quad H_{SR} = \sum_\beta\sum_{\omega'} A^\dagger_\beta(\omega')B_\beta$$

$$\dot{\rho}_I(t) = \left(\frac{1}{\hbar}\right)^2 \sum_{\omega,\omega'} e^{i(\omega-\omega')t}\sum_{\alpha,\beta}\Gamma_{\alpha\beta}(\omega')\left[A_\beta(\omega')\rho_I(t)A^\dagger_\alpha(\omega) - A^\dagger_\alpha(\omega)A_\beta(\omega')\rho_I(t)\right] + h.c$$

$$\Gamma_{\alpha\beta}(\omega') = \int_0^\infty dt'' e^{i\omega' t''}\left\langle B_\alpha(t'')B^\dagger_\beta(0)\right\rangle = \int_0^\infty dt e^{i\omega' t}\left\langle B_\alpha(t)B^\dagger_\beta(0)\right\rangle$$

$$\dot{\rho}_I(t) = \left(\frac{1}{\hbar}\right)^2 \sum_{\omega,\omega'}\sum_{\alpha,\beta}\Gamma_{\alpha\beta}(\omega')\left[A_\beta(\omega',t)\rho_I(t)A^\dagger_\alpha(\omega,t) - A^\dagger_\alpha(\omega,t)A_\beta(\omega',t)\rho_I(t)\right]$$

$$+\left(\frac{1}{\hbar}\right)^2 \sum_{\omega,\omega'}\sum_{\alpha,\beta}\Gamma^*_{\beta\alpha}(\omega')\left[A_\alpha(\omega,t)\rho_I(t)A^\dagger_\beta(\omega',t) - \rho_I(t)A^\dagger_\beta(\omega',t)A_\alpha(\omega',t)\right]$$

Master equation

$$\dot{\rho}_I(t)=\left(\frac{1}{\hbar}\right)^2\sum_{\omega,\omega'}\sum_{\alpha,\beta}\Gamma_{\alpha\beta}(\omega')\left[A_\beta(\omega',t)\rho_I(t)A_\alpha^\dagger(\omega,t)-A_\alpha^\dagger(\omega,t)A_\beta(\omega',t)\rho_I(t)\right]$$

$$+\left(\frac{1}{\hbar}\right)^2\sum_{\omega,\omega'}\sum_{\alpha,\beta}\Gamma^*_{\beta\alpha}(\omega')\left[A_\alpha(\omega,t)\rho_I(t)A_\beta^\dagger(\omega',t)-\rho_I(t)A_\beta^\dagger(\omega',t)A_\alpha(\omega',t)\right]$$

$$\Gamma_{\alpha\beta}(\omega')=\frac{1}{2}\gamma_{\alpha\beta}(\omega')+iS_{\alpha\beta}(\omega')$$

$$\Gamma^*_{\beta\alpha}(\omega')=\frac{1}{2}\gamma_{\alpha\beta}(\omega')-iS_{\alpha\beta}(\omega')$$

$$\dot{\rho}_I(t)=-i\left(\frac{1}{\hbar}\right)^2\sum_{\omega,\omega'}\sum_{\alpha,\beta}S_{\alpha\beta}(\omega')\left[A_\alpha^\dagger(\omega,t)A_\beta(\omega',t),\rho_I(t)\right]$$

$$+\left(\frac{1}{\hbar}\right)^2\sum_{\omega,\omega'}\sum_{\alpha,\beta}\gamma_{\alpha\beta}(\omega')\left[A_\beta(\omega',t)\rho_I(t)A_\alpha^\dagger(\omega,t)-\frac{1}{2}\left\{A_\alpha^\dagger(\omega,t)A_\beta(\omega',t),\rho_I(t)\right\}\right]$$

Master equation

$$\dot{\rho}_I(t) = -i\left(\frac{1}{\hbar}\right)^2 \sum_{\omega,\omega'}\sum_{\alpha,\beta} S_{\alpha\beta}(\omega')\left[A^{\dagger}_{\alpha}(\omega,t)A_{\beta}(\omega',t),\rho_I(t)\right]$$

$$+\left(\frac{1}{\hbar}\right)^2 \sum_{\omega,\omega'}\sum_{\alpha,\beta} \gamma_{\alpha\beta}(\omega')\left[A_{\beta}(\omega',t)\rho_I(t)A^{\dagger}_{\alpha}(\omega,t) - \frac{1}{2}\left\{A^{\dagger}_{\alpha}(\omega,t)A_{\beta}(\omega',t),\rho_I(t)\right\}\right]$$

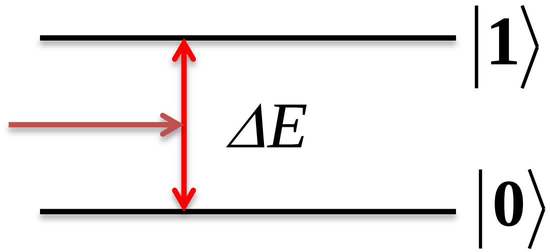
$$\omega \neq \omega' \quad \tau_S = \frac{1}{|\omega-\omega'|} \gg \tau \rightarrow \omega \approx \omega'$$

Master equation in Schrodinger Picture

$$\dot{\rho}(t) = -i\left[H_S,\rho\right] - i\left(\frac{1}{\hbar}\right)^2 \sum_{\omega}\sum_{\alpha,\beta} S_{\alpha\beta}(\omega)\left[A^{\dagger}_{\alpha}(\omega)A_{\beta}(\omega),\rho(t)\right]$$

$$+\left(\frac{1}{\hbar}\right)^2 \sum_{\omega}\sum_{\alpha,\beta} \gamma_{\alpha\beta}(\omega)\left[A_{\beta}(\omega)\rho(t)A^{\dagger}_{\alpha}(\omega) - \frac{1}{2}\left\{A^{\dagger}_{\alpha}(\omega)A_{\beta}(\omega),\rho(t)\right\}\right]$$

Master equation



$$\sigma^{+} = |\mathbf{0}\rangle\langle\mathbf{1}| \quad \sigma^{-} = |\mathbf{1}\rangle\langle\mathbf{0}|$$

The system interacts with the environment

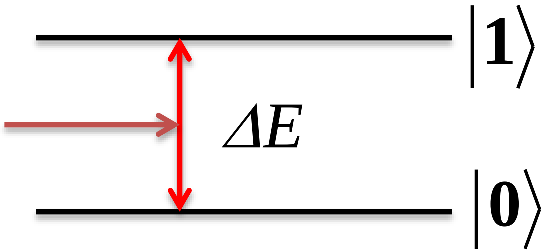
$\hbar = 1$

$$\dot{\rho}_I(t) = -iS\left[\sigma^{+}\sigma^{-}, \rho_I(t)\right] + \gamma\left[\sigma^{-}\rho_I(t)\sigma^{+} - \frac{1}{2}\left(\sigma^{+}\sigma^{-}\rho_I(t) + \rho_I(t)\sigma^{+}\sigma^{-}\right)\right]$$

$$\sigma^{-} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \sigma^{+} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{\rho}_{11} & \dot{\rho}_{12} \\ \dot{\rho}_{21} & \dot{\rho}_{22} \end{pmatrix} = \begin{pmatrix} -\gamma\rho_{11} & -iS\rho_{12} - \frac{1}{2}\gamma\rho_{12} \\ iS\rho_{12} - \frac{1}{2}\gamma\rho_{21} & \gamma\rho_{11} \end{pmatrix}$$

Master equation



$$\sigma^{+},\sigma^{-} \qquad \hbar = 1$$

$$\begin{cases} \rho_{11} = 1 - \rho_{22}(0)e^{-\gamma t} \\ \rho_{12} = \rho_{12}(0).e^{\left(iS - \frac{1}{2}\gamma\right)t} \\ \rho_{12} = \rho_{12}(0).e^{\left(-iS - \frac{1}{2}\gamma\right)t} \\ \rho_{22} = \rho_{22}(0)e^{-\gamma t} \end{cases}$$

$$T_2 \le 2T_1 = \frac{2}{\gamma}$$

THANK YOU FOR YOUR LISTENING!