# Fluctuation Effects on the Interaction of Two-Dimensional Vortices with Strong-Pinning Sites in High-Temperature Cuprates

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#### Abstract

We investigate the angular dependence of the flux-flow resistance in highly anisotropic layered high-temperature cuprates with columnar, ion-track pinning centers. The two-dimensional pancake vortices on a layer interacts with others on weakly coupled adjacent layers as well as with the strong pinning sites of columnar defects. The effects of the thermal fluctuations are shown to be significant.

### 1 Introduction

The highly anisotropic nature of the high- $T_c$  superconductors (HTSC) has given rise the issue of the dimensionality of the vortices [?]. Vortices in moderately anisotropic superconductors such as YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> is widely believed to be three-dimensional (3D) and form continuous vortex lines. Many experiments in more anisotropic superconductors like  $Bi_2Sr_2CaCu_2O_x$  and  $Tl_2Ba_2CaCu_2O_x$ , on the other hand, exhibit drastic change from 3D-like vortex behavior at low fields to two-dimensional pancake vortices [?, ?]. Recently, the dimensionality of the vortices has been explored by considering the interaction of vortices with the strong pinning centers in ion-irradiated layered HTSCs [?, ?, ?, ?]. In such a sample, the transport properties are likely to depend on the angle between the external magnetic field and the columnar defects. It has been demonstrated that the angular dependence may be a useful probe of the dimensionality of the vortices.

In this article, we investigate the angular dependence of the flux-flow resistance in highly anisotropic layered high-temperature cuprates with columnar, ion-track pinning centers. We introduce a simple model, where the 2D pancake vortices on a layer interacts with others on weakly coupled adjacent layers as well as with the strong pinning sites. We also discuss the effects of the thermal fluctuations within the approximations of the Gaussian fluctuations.

#### 2 Model

We consider 2D pancake vortices on layers  $n = 0, 1, 2, \dots, N$  with centers at  $x_n$ , whose partition function can be written in the form:

$$Z = \prod_{n=0}^{N} \int dx_n \ e^{-\mathcal{F}[x]/T} \tag{1}$$

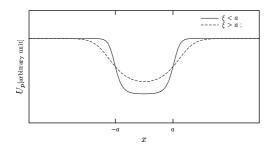


Figure 1: A model of the pinning potential near a strong pinning center.

with the free energy functional given by [?, ?]

$$\frac{\mathcal{F}}{T} = \frac{1}{2} \frac{\epsilon_J}{T} \sum_{n=0}^{N-1} (x_n - x_{n+1})^2 
+ \frac{1}{2} \frac{E_p}{T} \sum_{n=0}^{N} U_p(x_n) 
+ \frac{1}{2} \frac{E_p}{T} \sum_{n=0}^{N} U_p(x_n - a_{\phi}),$$
(2)

where  $\epsilon_J$  is the interlayer Josephson-coupling energy per unit area of pancake vortices,  $E_p$  is the pinning energy of the columnar defects, and the temperature T is in units such that  $k_B=1$ . Here we ignored the degrees of freedom in y-direction on each layer only for simplicity.  $a_\phi$  is the average distance between two columnar defects:  $a_\phi^2 = \Phi_0/B_\phi$ , where  $\Phi_0 \equiv hc/2|e|$  is the flux quantum and  $B_\phi$  is the matching field. As far as we are concerned with the angular dependence this should not affect the analysis qualitatively. As a simple but physically appealing model for the dimensionless pinning potential  $U_p$  in Eq. (2), we adopt the following functional form (see Fig. 1):

$$U_p(x) \equiv \tanh\left[\frac{x^2 - a^2}{\xi^2}\right],$$
 (3)

where a is the radius of the columnar defects and  $\xi$  is the coherence length in the Cu-O plane. It is interesting to notice that as  $\xi$  exceeds (e.g., as temperature approach the transition point  $T_{c0}$ ) the column radius, the pinning potential is significantly smoothened out (Fig. 1).

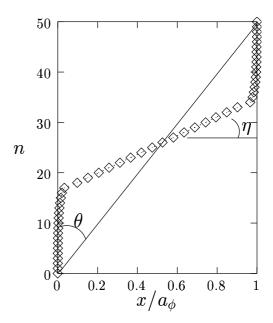


Figure 2: The profile of the position of the pancake vortex on each layer n.

## 3 Angular Dependence

In this section, we calculate the fraction of the unpinned vortices in a string of pancake vortices, which is proportional to the flux flow resistance, as a function of the angle  $\theta$  between the external magnetic field and the columnar defects. Apart from the fluctuation effects, the segments of the unpinned vortices is determined by the configuration minimizing the free energy functional  $(\delta \mathcal{F} = 0)$ : for  $n = 1, 2, \dots, N-1$ 

$$E_{J}(x_{n+1} + x_{n-1} - 2x_{n})$$

$$- E_{p}x_{n}\operatorname{sech}^{2}\left[x_{n}^{2} - a^{2}\right]$$

$$- E_{p}(x_{n} - a_{\phi})\operatorname{sech}^{2}\left[(x_{n} - a_{\phi})^{2} - a^{2}\right]$$

$$= 0.$$
(4)

We impose the boundary conditions at the two layers n=0 and n=N

$$x_0 = 0, \quad x_N = a_{\phi}. \tag{5}$$

In other words, the field angle  $\theta$  is given by

$$\theta = \arccos\left[\frac{N}{N^2 + a_{\phi}^2/d^2}\right],\tag{6}$$

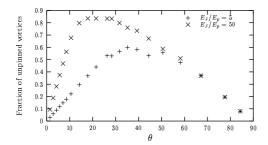


Figure 3: The fraction of the unpinned vortices as a function of the angle between the field and the columnar defect at the low field limit ( $B \ll B_{\phi}$ ). The fraction is proportional to the flux flow resistance in the thermal activated flux flow regime.

where d is the interlayer distance. In Eq. (4), we rescaled all the lengths by the coherence length  $\xi$ , and defined  $E_J \equiv \epsilon_J \xi^2$ . The finite difference equation Eq. (4) can be solved numerically.

Figure 2 is a typical profile of the vortex positions on different layers. The angle  $\eta$  between the Cu-O plane and the unpinned vortex segment, which is independent of the field direction for small  $\theta$  [?], determines the  $\theta_{max}$  at which the unpinned vortex fraction has the maximum value (see Fig. 3): for  $\theta \ll (\pi/2 - \eta)$ ,  $\eta$  remains constant and the fraction increase linearly with  $\theta$ ; for larger angles, however, the stacking more than one pancake vortices in the columnar defects does not gain any energy, and the whole vortex string align itself almost parallel to the external field, decreasing the fraction with  $\theta$  increasing.

 $\eta$  is a monotonically increasing function of the ratio  $E_J/E_p$  (Fig. 3). Thus the  $\theta_{max}$  decreases as  $E_J/E_p$  increase. Note that as we already seen in Sec. 2, the pinning potential is substantially smoothened as  $\xi$  exceeds a. This effectively renormalize  $E_p$  to reduce its value. Thus it is expected that the maximum-fraction angle  $\theta_{max}$  shifts towards zero as temperature increases. (Here  $\varepsilon_j$  is assumed not to depend strongly on temperature.)

## 4 Fluctuation Effects

The effects of the fluctuations from the freeenergy minimizing solution  $\{\bar{x}_n\}$  in Eq. (4)  $(x_n \to \bar{x}_n + x_n; |x_n| \ll |\bar{x}_n|)$  can be taken into account within the Gaussian approximations:

$$\frac{\delta \mathcal{F}}{T} \simeq \frac{1}{2} \frac{E_J}{T} \sum_n (x_n - x_{n+1})^2 
+ \frac{1}{2} \frac{E_p}{T} \sum_n U_p''(\bar{x}_n) x_n^2 
+ \frac{1}{2} \frac{E_p}{T} \sum_n U_p''(\bar{x}_n - a_\phi) x_n^2 
+ \mathcal{O}(x_n^3)$$
(7)

with the second-order derivative of the pinning potential

$$U_p''(x) = {\rm sech}^2[{\bf x}^2 - {\bf a}^2] \left\{ 1 - 4{\bf x}^2 \tanh[{\bf x}^2 - {\bf a}^2] \right\}.$$

For  $|\bar{x}_n| \lesssim a$  or  $|\bar{x}_n - a_\phi| \lesssim a$ ,  $U_p''(\bar{x}_n) \sim \mathcal{O}(1)$ . On such layers,  $\sigma_n \equiv \sqrt{\langle x_n^2 \rangle} \sim \sqrt{T/E_p}$ . It thus seems that around the temperature  $T^*$  given by  $\sqrt{T/E_p} \simeq a$ , the fluctuations play a significant role: Below  $T^*$ , the pinning is strong enough and the unpinned vortex segments are still determined by the mean-field equation Eq. (4). On the other hand, above  $T^*$ , the fluctuations further disturb the vortices pinned to columnar defects, and thereby increase the unpinned vortex fraction. This crossover behavior with respect to temperature has recently been observed in Hg-1212 thin films [?].

### 5 Conclusion

We have investigated the angular dependence of the flux-flow resistance in highly anisotropic layered high-temperature cuprates with strong columnar, ion-track pinning centers. The angular dependence displayed the interplay between the interlayer vortex interaction and the interaction of the vortices with the strong pinning centers. The effects of the thermal fluctuations lead to the crossover behavior with respect to the temperature.

For the present, the discussion is just qualitative and more thorough quantitative analysis

is needed. Furthermore, the model in Eq. (2) does not include the interaction of vortices in the same layer, which becomes important at higher fields.